EXCLUSIVE DECAYS OF HEAVY FLAVOURS*

T. Mannel

Institut für Theoretische Teilchenphysik, Universität Karlsruhe
D–76128 Karlsruhe, Germany

(Received March 15, 1995)

Recent progress in the theoretical description of exclusive heavy flavour decays is reviewed. After a general discussion of heavy quark symmetries some applications are studied.

PACS numbers: 12.39.Hg

1. Introduction

In the past five years substantial progress has been made in the theoretical description of systems containing heavy quarks. This progress on the theoretical side has been accompanied by an enormous improvement of experimental data, which made the field of heavy quark physics one of the most interesting and prosperous fields in high energy physics.

The theoretical breakthrough was triggered by the observation that for a heavy quark one may take advantage of the fact that one may treat such a quark to first approximation as infinitely heavy [1, 2]. This infinite mass limit has two important properties. It may be formulated as an effective field theory, which is called Heavy Quark Effective Theory (HQET) [3]. This implies that the corrections to this limit may be treated systematically in the framework of a combined $\alpha_s(m_Q)$ and $\bar{\Lambda}/m_Q$ expansion, where m_Q is the mass of the heavy quark and $\bar{\Lambda}$ is some scale related to the light degrees of freedom, typically of the order of a few hundred MeV.

The second and even more important point is that the leading term (corresponding to the infinite mass limit) exhibits two additional symmetries which are not present in full QCD [2]. It was this observation which started the whole development, since these symmetries of the effective theory allow

* Presented at the Cracow Epiphany Conference on Heavy Quarks, held in honour of the 60th birthday of Kacper Zalewski, Kraków, Poland, January 5–6, 1995.

to restrict the non-perturbative input needed to describe *e.g.* weak decay matrix elements. In other words, the hadronic uncertainties may be reduced substantially and even a few completely model independent statements are possible. The progress in this field is documented in more or less extensive reviews [4].

Heavy quark symmetries work most efficiently for transitions among heavy quarks; treating both b and c as heavy the weak transition $b \to c$ are precisely of this type and thus these weak transitions are strongly constrained by heavy quark symmetries. In particular, one may even obtain the absolute normalization of the transition matrix elements at a certain kinematic point, allowing for a model independent determination of the CKM matrix element V_{cb}. If the final state quark is light as *e.g.* in the weak transitions of the type $b \to u$ or $c \to s$ heavy quark symmetries may still be applied but are less restrictive [5].

In the present review I try to summarize some of the basics of heavy quark symmetries and to indicate how to deal with HQET. Numerous papers have appeared since the pioneering work of Voloshin, Shifman and Eichten, Hill [1], Isgur, Wise [2] and it is impossible to summarize everything in this short review. Heavy quark symmetries lie at the heart of HQET, and in Section 2 we consider these additional symmetries. In Section 3 the method of how to calculate corrections in the framework of HQET is outlined. Section 4 is dedicated to heavy to heavy transitions, where we shall study the decay $B \to D^{(*)}\ell\bar{\nu}_\ell$ in some detail. Heavy to light decays will be considered in Section 5, where purely leptonic and semileptonic decays are studied.

2. Heavy quark symmetry

The main impact of the heavy quark limit is due to two additional symmetries which are not present in full QCD; the first is a heavy flavour symmetry and the second one is the so called spin symmetry.

We shall first study the heavy flavour symmetry. The interaction of the quarks with the gluons is flavour independent; all flavour dependence in QCD is only due to the different quark masses. In the $1/m_Q$ expansion the leading order Lagrangian is mass independent and hence a flavour symmetry appears relating heavy quarks moving with the same velocity.

For the case of two heavy flavours b and c one has to leading order the Lagrangian [3]

$$\mathcal{L}_{\text{heavy}} = \bar{b}_v(v \cdot D)b_v + \bar{c}_v(v \cdot D)c_v \,, \tag{1}$$

where b_v (c_v) is the field operator h_v for the b (c) quark moving with velocity v and $D = \partial + igA$ is the QCD covariant derivative. This Lagrangian is obviously invariant under the $SU(2)_{HF}$ rotations

$$\begin{pmatrix} b_v \\ c_v \end{pmatrix} \to U_v \begin{pmatrix} b_v \\ c_v \end{pmatrix} \quad U_v \in SU(2)_{HF} . \tag{2}$$

We have put a subscript v for the transformation matrix U, since this symmetry only relates heavy quarks moving with the same velocity.

The second symmetry is the heavy-quark spin symmetry. As is clear form the Lagrangian in the heavy-mass limit, both spin degrees of freedom of the heavy quark couple in the same way to the gluons; we may rewrite the leading-order Lagrangian as

$$\mathcal{L} = \bar{h}_v^{+s}(ivD)h_v^{+s} + \bar{h}_v^{-s}(ivD)h_v^{-s} , \tag{3}$$

where $h_v^{\pm s}$ are the projections of the heavy quark field on a definite spin direction s

$$h_v^{\pm s} = \tfrac{1}{2}(1 \pm \gamma_5 \not{s})h_v , \quad s \cdot v = 0 . \tag{4}$$

This Lagrangian has a symmetry under the rotations of the heavy quark spin and hence all the heavy hadron states moving with the velocity v fall into spin-symmetry doublets as $m_Q \to \infty$. In Hilbert space this symmetry is generated by operators $S_v(\varepsilon)$ as

$$[h_v, S_v(\varepsilon)] = i\not{\varepsilon}\not{v}\gamma_5 h_v , \tag{5}$$

where ε with $\varepsilon^2 = -1$ is the rotation axis. The simplest spin-symmetry doublet in the mesonic case consists of the pseudoscalar meson $H(v)$ and the corresponding vector meson $H^*(v, \varepsilon)$, since a spin rotation yields

$$\exp\left(iS_v(\varepsilon)\tfrac{\pi}{2}\right) |H(v)\rangle = (-i)|H^*(v, \varepsilon)\rangle , \tag{6}$$

where we have chosen an arbitrary phase to be $(-i)$.

In the heavy-mass limit the spin symmetry partners have to be degenerate and their splitting has to scale as $1/m_Q$. In other words, the quantity

$$\lambda_2 = \frac{1}{4}(M_{H^*}^2 - M_H^2) \tag{7}$$

has to be the same for all spin symmetry doublets of heavy ground state mesons. This is well supported by data: For both the (B, B^*) and the (D, D^*) doublets one finds a value of $\lambda_2 \sim 0.12$ GeV^2. This shows that the spin-symmetry partners become degenerate in the infinite mass limit and the splitting between them scales as $1/m_Q$.

In the infinite mass limit the symmetries imply relations between matrix elements involving heavy quarks. For a transition between heavy ground-state mesons H (either pseudoscalar or vector) with heavy flavour $f(f')$ moving with velocities v (v'), one obtains in the heavy-quark limit

$$\langle H^{(f')}(v')|\bar{h}_{v'}^{(f')}\Gamma h_v^{(f)}|H^{(f)}(v)\rangle = \xi(vv')\mathrm{Tr}\left\{\overline{\mathcal{H}(v)}\Gamma\mathcal{H}(v)\right\} , \qquad (8)$$

where Γ is some arbitrary Dirac matrix and $H(v)$ are the representation matrices for the two possibilities of coupling the heavy quark spin to the spin of the light degrees of freedom, which are in a spin-$^1/_2$ state for ground state mesons

$$\mathcal{H}(v) = \frac{\sqrt{M_H}}{2}\begin{cases} (1+\not{v})\gamma_5 & 0^-, (\bar{q}Q) \text{ meson} \\ (1+\not{v})\not{\varepsilon} & 1^-, (\bar{q}Q) \text{ meson} \\ & \text{with polarization } \varepsilon . \end{cases} \qquad (9)$$

Due to the spin and flavour independence of the heavy mass limit the Isgur–Wise function ξ is the only nonperturbative information needed to describe all heavy to heavy transitions within a spin-flavour symmetry multiplet.

Excited mesons have been studied in [6]. They may be classified by the angular momentum of the light degrees of freedom j_l, which is coupled with the heavy quark spin S to the total angular momentum J of the meson. Furthermore, the orbital angular momentum ℓ determines the parity $P = (-1)^{\ell+1}$ of the meson. For a given $\ell > 0$ we can have $j_l = \ell \pm 1/2$ and the coupling of the heavy quark spin yields two spin symmetry doublets $(J = \ell - 1, J = \ell)$ and $(J = \ell, J = \ell + 1)$. For example, the lowest positive parity $\ell = 1$ mesons are two spin symmetry doublets $(0^+, 1^+)$ and $(1^+, 2^+)$. In the D meson system these states have been observed [7] and behave as predicted by heavy quark symmetry [8].

Similarly as for the mesons heavy-quark symmetries imply that only one form factor is needed to describe heavy to heavy transitions within a spin flavour symmetry multiplet; in other words, there is an Isgur–Wise function for each multiplet.

The ground state baryons have been studied in [9–11]. According to the particle data group they are classified as follows

$$\Lambda_h = [(qq')_0 h]_{1/2} \qquad \Xi'_h = [(qs)_0 h]_{1/2} \qquad (10)$$

$$\Sigma_h = [(qq')_1 h]_{1/2} \qquad \Xi_h = [(qs)_1 h]_{1/2} \qquad \Omega_h = [(ss)_1 h]_{1/2} \qquad (11)$$

$$\Sigma_h^* = [(qq')_1 h]_{3/2} \qquad \Xi_h^* = [(qs)_1 h]_{3/2} \qquad \Omega_h^* = [(ss)_1 h]_{3/2} . \qquad (12)$$

Here, q, q' refer to u and d quarks, $q \neq q'$ for the Λ_h, but q may be the same as q' for the Σ_h and Σ_h^*. The first subscript (0, 1) is the total spin of the light degrees of freedom, while the second subscript (1/2, 3/2) is the total spin of the baryon.

Spin symmetry forces these baryons into spin symmetry doublets. For the Λ-type baryons (10) the spin rotations are simply a subset of the Lorentz transformations, since the light degrees of freedom are in a spin-0 state. The corresponding spin symmetry doublet is in this case given by the two polarization directions of the heavy baryon. From the point of view of heavy quark symmetries the Λ-type baryons are the simplest hadrons, although from the quark model point of view they are composed of three quarks.

The baryons with the light degrees of freedom in a spin one state may be represented by a pseudovector-spinor object R^μ with $v_\mu R^\mu = 0$ [1]. In general $\gamma_\mu R^\mu \neq 0$ because R^μ contains spin-$^1/_2$ contributions as well as spin-$^3/_2$ parts. In other words, R^μ contains a Rarita–Schwinger field as well as a Dirac field. Under Lorentz transformations R^μ behaves as

$$R^\mu(v) \to \Lambda^\mu{}_\nu D(\Lambda) R^\nu(\Lambda v)\,, \tag{13}$$

where $\Lambda_{\mu\nu}$ and $D(\Lambda)$ are the Lorentz transformations in the vector and spinor representation respectively, while under spin rotations we have

$$R^\mu(v) \to -\gamma_5 \not{v} \not{\epsilon} R^\mu(v)\,. \tag{14}$$

The spin-$^3/_2$ component of the pseudovector-spinor object corresponding to the Σ_h^* is projected out by contracting with γ_μ

$$\gamma_\mu R^\mu_{\Sigma_h^*} = 0\,. \tag{15}$$

The rest of the independent components of R correspond to Σ_h baryon:

$$R^\mu_{\Sigma_h} = \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 u_{\Sigma_h}\,. \tag{16}$$

where u_{Σ_h} is the Dirac spinor of the Σ_h state. Similar expressions hold for the nonstrange baryons $\Xi_h^{(*)}$ and $\Omega_h^{(*)}$.

The spin rotation (14) transform the Σ-like baryons into the Σ^* states and vice versa. Thus the spin symmetry doublets for the ground state baryons are given by the two polarization directions of the baryons in (10), and by the two states with corresponding light quark flavour numbers in (11) and (12).

---

[1] One could as well represent the light degrees of freedom by an antisymmetric tensor instead by a pseudovector; this is a completely equivalent formulation [11].

Similar to the case of mesons one may derive a Wigner–Eckart theorem for the spin symmetry doublets of the baryons

$$\langle \Lambda_h(v)|\bar{h}\Gamma h'|\Lambda_{h'}(v')\rangle = A(v\cdot v')\bar{u}_{\xi_h}(v)\Gamma u_{\xi_{h'}}(v')\,, \tag{17}$$

where we have allowed for the possibility of two heavy quark flavours h and h'. In the same way, one obtains two form factors for the $\Sigma_h^{(*)} \to \Sigma_{h'}^{(*)}$.

$$\begin{aligned}&\langle \Sigma_h^{(*)}(v)|\bar{h}_v\Gamma h_{v'}|\Sigma_h^{(*)}\rangle \\ &= \bar{R}^{\mu}_{\Sigma_h^{(*)}}(v)\Gamma R^{\nu}_{\Sigma_h^{(*)}}(v')\ \left[B(v\cdot v')g_{\mu\nu} + C(v\cdot v')v'_\mu v_\nu\right]\,.\end{aligned} \tag{18}$$

Finally, parity does not allow for transitions between Λ and $\Sigma^{(*)}$ type baryons

$$\langle \Sigma_h^{(*)}(v)|\bar{h}_v\Gamma h_{v'}|\Lambda_h(v)\rangle = 0\,, \tag{19}$$

and hence these transitions are not only suppressed by the flavour symmetry of the light degrees of freedom, but additionally by heavy quark symmetry.

Excited baryons may be studied along the same lines as for the mesons. The spin symmetry doublets as well as the restrictions on transition matrix elements have been studied in [6].

Heavy quark symmetries thus lead to a strong reduction of the number of independent from factors that describe current induced transitions among heavy hadrons. In addition to that the symmetries even allow us to obtain the normalization of some of these form factors. Since the currents

$$J^{hh'} = \bar{h}_v\gamma_\mu h'_v = v_\mu \bar{h}_v h'_v \tag{20}$$

are the generators of heavy flavour symmetry in the velocity sector v, the normalization of the Wigner–Eckart theorems (8), (17), (18) is known at the nonrecoil point $v = v'$. By standard arguments one obtains for the mesons

$$\xi(vv' = 1) = 1\,, \tag{21}$$

while the corresponding relation for the baryons is

$$A(vv' = 1) = \sqrt{m_{\Lambda_h} m_{\Lambda_{h'}}} \tag{22}$$

$$B(vv' = 1) = \sqrt{m_{\Sigma_h^{(*)}} m_{\Sigma_{h'}^{(*)}}}\,, \tag{23}$$

where the factor involving the square root of the masses means that the hadron states in (17) are normalized relativistically.

3. Corrections to the heavy mass limit

The heavy quark symmetries allow us to obtain relations which hold in the heavy mass limit, such as the Wigner–Eckart theorems (8), (17), (18) and the normalization statements (21), (22), (23). Corrections due to finite masses may be calculated systematically using the machinery of HQET. In this section we consider the strategy of how the corrections are studied.

In general there are two types of corrections. The short-distance corrections may be calculated in perturbation theory, based on the leading order of the $1/m_Q$ expansion of the Lagrangian. The logarithmic ultraviolet divergences in the effective theory correspond to logarithmic dependences on the heavy-quark mass m_Q in the full theory, and renormalization group methods may be employed to perform resummations of these logarithms.

The starting point of a QCD corrections calculation are the Feynman rules of full QCD and the ones of HQET. In HQET there are only two of Feynman rules modified compared to full QCD:

	Full QCD		HQET	
Propagator of the heavy quark	$\frac{i}{\not p - m_Q + i\varepsilon}$	$\longrightarrow$	$\frac{i}{vk + i\varepsilon}$,	$p = mv + k$
Heavy quark gluon vertex	$ig\gamma_\mu T^a$	$\longrightarrow$	$igv_\mu T^a$	

For the sake of simplicity and clarity we shall consider the matrix element of some operator $\mathcal{O}$ corresponding to some observable quantity, *e.g.* a current mediating a weak decay. Since HQET is an effective theory, the machinery of effective theory guarantees the factorization of long distance effects from the short distance ones, which are related to the large mass m_Q. Neglecting $1/m_Q$ corrections, this factorization takes the form

$$\langle \mathcal{O} \rangle_{full} = Z\left(\frac{m_Q}{\mu}\right) \langle \mathcal{O} \rangle_{\text{static}}(\mu) + \mathcal{O}(1/m_Q)\,, \tag{24}$$

where the dependence on m_Q of the coefficient Z is given as a combined expansion in the coupling strength $\alpha_s = g^2/(4\pi)$ and logarithms of m_Q

$$\begin{aligned} Z\left(\frac{m_Q}{\mu}\right) &= a_{00} \\ &+ a_{11}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{10}\alpha_s \\ &+ a_{22}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{21}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{20}\alpha_s^2 \\ &+ a_{33}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^3 + a_{32}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{31}\alpha_s^2\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)\,. \end{aligned} \tag{25}$$

This factorization theorem corresponds to the statement that the ultraviolet divergencies in the effective theory have to match the logarithmic mass dependences of full QCD. The factorization scale μ is an arbitrary parameter, and the physical quantity $\langle\mathcal{O}\rangle_{\text{full}}$ does not depend on this parameter. However, calculating the matrix element of this operator in the effective theory and studying its ultraviolet behaviour allows us to access the mass dependence of the observable $\langle\mathcal{O}\rangle$.

The ultraviolet behaviour of the effective theory is investigated by the renormalization group equations. Differentiating (24) with respect to the factorization scale μ yields the renormalization group equation

$$\frac{d}{d\ln\mu}\left\{Z\left(\frac{m_Q}{\mu}\right)\langle\mathcal{O}\rangle_{\text{static}}(\mu)\right\} = 0\,, \tag{26}$$

from which we may obtain an equation which determines the change of the coefficient Z when the scale is changed

$$\left(\frac{d}{d\ln\mu} + \gamma_{\mathcal{O}}(\mu)\right) Z\left(\frac{m_Q}{\mu}\right) = 0\,,$$
$$\gamma_{\mathcal{O}}(\mu) = \frac{d}{d\ln\mu}\ln(\langle\mathcal{O}\rangle_{static}(\mu))\,. \tag{27}$$

The quantity $\gamma_{\mathcal{O}}$ is called the anomalous dimension of the operator $\mathcal{O}$ which is universal for all matrix elements of $\mathcal{O}$, since it is connected with the short distance behaviour of the insertion of the operator $\mathcal{O}$.

Eq. (27) describes the renormalization group scaling in the effective theory. It allows to shift logarithms of the large mass scale from the matrix element of $\mathcal{O}$ into the coefficient Z: If the matrix element is renormalized at the large scale m_Q the logarithms of the type $\ln m_Q$ will appear in the matrix element of $\mathcal{O}$ while the coefficient Z at this scale will simply be

$$Z(1) = a_{00} + a_{10}\alpha_s(m_Q) + a_{20}\alpha_s^2(m_Q) + a_{30}\alpha_s^3(m_Q) + \cdots\,, \tag{28}$$

The renormalization group equation (27) allows to lower the renormalization point from m_Q to μ; the matrix element renormalized at μ will not contain any logarithms of M any more, they will appear in the coefficient Z in the way shown in (25).

In all cases relevant in the present context the matrix elements will be matrix elements involving hadronic states, which are in most cases impossible to calculate from first principles. However, Eq. (27) allows to extract the short distance piece, *i.e.* the logarithms of the large mass M and to separate it into the Wilson coefficients.

The anomalous dimension may be calculated in perturbation theory in powers of the coupling constant g of the theory. In general, in a renormalizable theory the coupling constant depends on the scale μ at which the theory is renormalized. The scale dependence of the coupling constant is determined by the β function

$$\frac{d}{d\ln\mu} g(\mu) = \beta(\mu) . \tag{29}$$

In a mass independent scheme the renormalization group functions γ_O and β will depend on the scale μ only through their dependence on the coupling constant

$$\beta = \beta(g(\mu)) \qquad \gamma_O = \gamma_O(g(\mu)) . \tag{30}$$

Hence we may rewrite the renormalization group equation (27) as

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_O(g)\right) Z\left(\frac{m_Q}{\mu}, g\right) = 0 . \tag{31}$$

The renormalization group functions β and γ_O are calculated in perturbation theory; the first term of the β function on QCD is obtained from a one-loop calculation and is given by

$$\beta(g) = -\frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}n_f\right) g^3 + \cdots , \tag{32}$$

where n_f is the number of active flavors, i.e. the number of flavors with a mass less than m_Q.

With this input the renormalization group equation may be solved to yield

$$Z\left(\frac{m_Q}{\mu}\right) = a_{00}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{48\pi^2}{33-2n_f}\gamma_1} , \tag{33}$$

where γ_1 is the first coefficient in the perturbative expansion of the anomalous dimension $\gamma_O = \gamma_1 g^2 + \cdots$ and $\alpha_s(\mu)$ is the one loop expression for the running coupling constant of QCD

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2n_f)\ln(\mu^2/\Lambda^2_{QCD})} , \tag{34}$$

which is obtained from solving (29) using (32). This expression corresponds to a summation of the leading logarithms $(\alpha_s \ln m_Q)^n$ which is achieved by a one-loop calculation of the renormalization group functions *beta* and γ_Q; in other words, in this way a resummation of the first column of the expansion (25) is obtained.

In a similar way one may also resum the second column of (25), if the renormalization group functions β and γ are calculated to two loops and the finite terms of the one loop expression are included. Below we present some results up to two loops.

The second type of corrections are the power corrections of order $1/m_Q^n$, which in general involve long-distance physics and hence may in general not be calculated, but have to be parametrized. As an example, consider a matrix element of a current $\bar{q}\Gamma Q$ mediating a transition between a heavy meson and some arbitrary state $|A\rangle$. The full QCD Lagrangian $\mathcal{L}$ and the fields Q are expanded in terms of a power series in $1/m_Q$ and one obtains [2]

$$\mathcal{L} = \bar{h}_v(ivD)h_v+\tfrac{1}{2m}\bar{h}_v i\not{D}P_- i\not{D}h_v+\left(\tfrac{1}{2m}\right)^2\bar{h}_v i\not{D}P_-(-ivD)i\not{D}h_v+\cdots \quad (35)$$

$$Q(x) = e^{-im_Q vx}\left[1+\tfrac{1}{2m_Q}i\not{D}_\perp+\left(\tfrac{1}{2m_Q}\right)^2(-ivD)P_- i\not{D}+\cdots\right]h_v\,, \quad (36)$$

where $P_\pm = (1\pm\not{v})/2$ is the projector on the upper/lower component of the spinors. For the matrix element under consideration one obtains up to order $1/m_Q$:

$$\langle A|\bar{q}\Gamma Q|M(v)\rangle = \langle A|\bar{q}\Gamma h_v|H(v)\rangle \qquad + \frac{1}{2m_Q}\langle A|\bar{q}\Gamma P_- i\not{D}h_v|H(v)\rangle$$
$$- i\int d^4x\langle A|T\{L_1(x)\bar{q}\Gamma h_v\}|H(v)\rangle + \mathcal{O}(1/m^2)\,, \quad (37)$$

where L_1 are the first-order corrections to the Lagrangian as given in (35). Furthermore, $|M(v)\rangle$ is the state of the heavy meson in full QCD, including all its mass dependence, while $|H(v)\rangle$ is the corresponding state in the infinite mass limit.

Expression (37) displays the generic structure of the higher-order corrections as they appear in any HQET calculation. There will be local contributions coming from the expansion of the full QCD field; these may be interpreted as the corrections to the currents. The nonlocal contributions, *i.e.* the time-ordered products, are the corresponding corrections to the states and thus in the r.h.s. of (37) only the states of the infinite-mass limit appear.

Finally we shall review briefly an important result concerning $1/m_Q$ corrections, which is called Lukes theorem. It is a generalization of the Ademollo–Gatto theorem, which states that in the presence of explicit symmetry breaking the matrix elements of the currents that generate the symmetry are still normalized up to terms which are second order in the symmetry breaking interaction.

---

[2] It has been pointed out repeatedly [12] that these expansions are not unique; one may always shift terms from the fields into the Lagrangian, which then appear as terms that would vanish by a naive application of the equations of motion.

For the case at hand the relevant symmetry is the heavy flavor symmetry. This symmetry is an SU(2) symmetry and is generated by three operators $Q_\pm$ and Q_3 with

$$Q_+ = \int d^3x\, \bar{b}_v(x) c_v(x) \quad Q_- = \int d^3x\, \bar{c}_v(x) b_v(x)\,,$$
$$Q_3 = \int d^3x\, (\bar{b}_v(x) b_v(x) - \bar{c}_v(x) c_v(x))\,,$$
$$[Q_+, Q_-] = Q_3 \qquad [Q_+, Q_3] = -2Q_+ \qquad (Q_+)^\dagger = Q_-\,. \tag{38}$$

Let us denote the ground state flavour symmetry multiplet as $|B\rangle$ and $|D\rangle$. Then the operators act in the following way

$$\begin{aligned} Q_3|B\rangle &= |B\rangle \qquad Q_3|D\rangle = -|D\rangle\,,\\ Q_+|D\rangle &= |B\rangle \qquad Q_-|B\rangle = |D\rangle\,,\\ Q_+|B\rangle &= Q_-|D\rangle = 0\,. \end{aligned} \tag{39}$$

The Hamiltonian of this system has a $1/m_Q$ expansion of the form

$$\begin{aligned} H &= H_0^{(b)} + H_0^{(c)} + \frac{1}{2m_b} H_1^{(b)} + \frac{1}{2m_c} H_1^{(c)} + \cdots \\ &= H_0^{(b)} + H_0^{(c)} + \frac{1}{2}\left(\frac{1}{2m_b} + \frac{1}{2m_c}\right)(H_1^{(b)} + H_1^{(c)}) \\ &\qquad + \frac{1}{2}\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)(H_1^{(b)} - H_1^{(c)}) + \cdots \\ &= H_{\text{symm}} + H_{\text{break}}\,. \end{aligned} \tag{40}$$

In the second equation, the first line is still symmetric under heavy flavour SU(2) while the term in the second line does not commute any more with $Q_\pm$, but it still commutes with Q_3. In other words, to order $1/m_Q$ we still have common eigenstates of H and Q_3, which we shall denote as $|\tilde{B}\rangle$ and $|\tilde{D}\rangle$. Sandwiching the commutation relation we get

$$\begin{aligned} 1 &= \langle\tilde{B}|Q_3|\tilde{B}\rangle = \langle\tilde{B}|[Q_+, Q_-]|\tilde{B}\rangle \\ &= \sum_n \left[\langle\tilde{B}|Q_+|\tilde{n}\rangle\langle\tilde{n}|Q_-|\tilde{B}\rangle - \langle\tilde{B}|Q_-|\tilde{n}\rangle\langle\tilde{n}|Q_+|\tilde{B}\rangle\right] \\ &= \sum_n \left[|\langle\tilde{B}|Q_+|\tilde{n}\rangle|^2 - |\langle\tilde{B}|Q_-|\tilde{n}\rangle|^2\right]\,, \end{aligned} \tag{41}$$

where $|\tilde{n}\rangle$ form a complete set of states of the Hamiltonian $H_{\text{symm}} + H_{\text{break}}$. The matrix elements may be written as

$$\langle\tilde{B}|Q_\pm|\tilde{n}\rangle = \frac{1}{E_B - E_n}\langle\tilde{B}|[H_{\text{break}}, Q_\pm]|\tilde{n}\rangle\,, \tag{42}$$

where E_B and E_n are the energies of the states $|\tilde{B}\rangle$ and $|\tilde{n}\rangle$, repectively. In the case $|\tilde{n}\rangle \doteq |\tilde{D}\rangle$ the matrix element will be of order unity, since both the numerator as well as the energy difference in the denominator are of the order of the symmetry breaking. For all other states the energy difference in the denominator is nonvanishing in the symmetry limit, and hence this difference is of order unity; thus the matrix element for these states will be of the order of the symmetry breaking. From this we conclude

$$\langle\tilde{B}|Q_+|\tilde{D}\rangle = 1 + \mathcal{O}\left[\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)^2\right] . \tag{43}$$

In particular, the weak transition currents at the nonrecoil point $v = v'$ are proportional to these symmetry generators and hence we may conclude that for some of these matrix elements we only have corrections of the order $1/m_Q^2$.

4. Heavy to heavy transitions

For the case of a heavy to heavy transition the Wigner–Eckart theorem (8) implies that there is only a single form factor which describe the weak decays of heavy hadrons; furthermore, the heavy mass limit yields the normalization of this form factor at the kinematic point $v = v'$.

Treating both the b and the c quark as heavy, the semileptonic decays $B \to D^{(*)}\ell\nu$ are the phenomenologically relevant examples. The matrix elements for these transitions are in general parametrized in terms of six form factors

$$\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_B m_D}\,\left[\xi_+(y)(v_\mu + v'_\mu) + \xi_-(y)(v_\mu - v'_\mu)\right] \tag{44}$$

$$\langle D^*(v',\varepsilon)|\bar{c}\gamma_\mu b|B(v)\rangle = i\sqrt{m_B m_{D^*}}\xi_V(y)\varepsilon_{\mu\alpha\beta\rho}\varepsilon^{*\alpha}v'^\beta v^\rho \tag{45}$$

$$\langle D^*(v',\varepsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\left[\xi_{A1}(y)(vv'+1)\varepsilon^*_\mu - \xi_{A2}(y)(\varepsilon^* v)v_\mu - \xi_{A2}(y)(\varepsilon^* v)v'_\mu\right] , \tag{46}$$

where we have defined $y = vv'$. Due to the Wigner–Eckart theorem (8) these six from factors are related to the Isgur–Wise function by

$$\xi_i(y) = \xi(y) \text{ for } i = +, V, A1, A3, \qquad \xi_i(y) = 0 \text{ for } i = -, A2 . \tag{47}$$

Since heavy quark symmetries also yield the normalization of the Isgur–Wise function, we know the absolute value of the differential rate at the point $v = v'$ in terms of the meson masses and V_{cb}. Hence we may use this to extract V_{cb} from these decays in a model independent way by extrapolating the lepton spectrum to the kinematic endpoint $v = v'$. Using the mode $B \to D^{(*)}\ell\nu$ one obtains the relation

$$\lim_{v\to v'} \frac{1}{\sqrt{(vv')^2-1}}\frac{d\Gamma}{d(vv')} = \frac{G_F^2}{4\pi^3}|V_{cb}|^2(m_B - m_{D^*})^2 m_{D^*}^3 |\xi_{A1}(1)|^2 , \quad (48)$$

where ξ_{A1} is equal to the Isgur–Wise function in the heavy mass limit, and hence $\xi_{A1}(1) = 1$.

Corrections to this relation have been calculated along the lines outlined above in leading and subleading order. A complete discussion may be found in more extensive review articles (see *e.g.* Neubert's review [4]), including reference to the original papers. Here we only state the final result

$$\xi_{A1}(1) = x^{6/25}\left[1 + 1.561\frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi} - \frac{8\alpha_s(m_c)}{3\pi} + z\left\{\frac{25}{54} - \frac{14}{27}x^{-9/25} + \frac{1}{18}x^{-12/25} + \frac{8}{25}\ln x\right\} - \frac{\alpha_s(\bar{m})}{\pi}\frac{z^2}{1-z}\ln z\right] + \delta_{1/m^2} , \quad (49)$$

where we use the abbreviations

$$x = \frac{\alpha_s(m_c)}{\alpha_s(m_b)}, \quad z = \frac{m_c}{m_b}$$

and $\bar{m}$ is a scale somewhere between m_b and m_c.

Up to the term δ_{1/m^2} all these contributions may be calculated perturbatively, including the dependence on z. The quantity δ_{1/m^2} parametrizes the nonperturbative contributions, which enter here at order $1/m^2$. These corrections may be expressed in terms of the kinetic energy λ_1, the chromomagnetic moment λ_2, which are given in terms of matrix elements of higher-order terms of the Lagrangian

$$\lambda_1 = \frac{\langle H(v)|\bar{h}_v(iD)^2 h_v|H(v)\rangle}{2M_H} , \quad (50)$$

$$\lambda_2 = \frac{\langle H(v)|\bar{h}_v\sigma_{\mu\nu}iD^\mu iD^\nu h_v|H(v)\rangle}{2M_H} , \quad (51)$$

where the normalization of the states is chosen to be $\langle H(v)|\bar{h}_v h_v|H(v)\rangle = 2M_H$, where M_H is the mass of the heavy meson in the static limit. In terms of these parameters δ_{1/m^2} may be written as

$$\delta_{1/m^2} = -\left(\frac{1}{2m_c}\right)^2 \frac{1}{2}\Big(-\lambda_1 + \lambda_2 + (-i)^2\frac{1}{2\sqrt{M_B M_D}}\int d^4x\, d^4y\, \langle B^*(v,\varepsilon)|T\left[\mathcal{L}_b^{(1)}(x)\bar{b}_v c_v \mathcal{L}_c^{(1)}(y)\right]|D^*(v,\varepsilon)\rangle\Big) + \mathcal{O}(1/m_c^3, 1/m_b^2, 1/(m_c m_b)) , \quad (52)$$

where $\mathcal{L}_Q^{(1)}$ is the first order Lagrangian for the quark Q as given in (35) and M_B (M_D) are the masses of the B (D) meson in the heavy quark limit. Here we display only the largest contribution of order $1/m_c^2$; the complete expression, including the $1/m_b^2$ and $1/(m_c m_b)$ terms, may be found in [13, 14].

The matrix elements λ_1, λ_2 and the one of the time-ordered product have to be estimated in a model or need to be taken from data. The parameter λ_2 has been discussed above and is given by the mass splitting between the ground state spin symmetry partners. The kinetic energy λ_1 is currently subject of intensive discussions; it may not be read off from the hadron spectrum and thus it is not easy to access. It is not yet determined from data and only theoretical estimates exist; from its definition one is led to assume $\lambda_1 < 0$; a more restrictive inequality

$$-\lambda_1 > 3\lambda_2 \tag{53}$$

has been derived in a quantum mechanical framework in [15] and using heavy-flavour sum rules [16]. Furthermore, there exists also a sum rule estimate [17] for this parameter

$$\lambda_1 = -0.52 \pm 0.12 \text{ GeV}^2 \,. \tag{54}$$

Similarly it is not easy to obtain information on the matrix element involving the time-ordered product, and thus the corrections of order $1/m^2$ will finally limit our ability to determine the CKM matrix element V_{cb} in a model independent way, at least along the lines as described above.

Various estimates for δ_{1/m^2} have been given in the literature. The first estimate of this correction has been given in [13] using the GISW model [18], which is based on a wave function for the light quark. In this work $\delta_{m^2} = -2\% \ldots - 3\%$ has been obtained. Another estimate with weaker assumptions yields $\delta_{m^2} = 0 \ldots - 5\%$ [14], but both estimates have been criticised recently as being too small. Based on heavy flavour sum rules it has been argued in [19] that the $1/m^2$ corrections can be quite large $\delta_{m^2} = 0\% \ldots - 8\%$ [19]. These various estimates indicate the size of the theoretical error involved in the determination of V_{cb} from the exclusive channel $B \to D^* \ell \bar{\nu}_\ell$.

This result has been used to extract V_{cb} from data. In Fig. 1 the latest data [20] are shown. From this fit one obtains [20]

$$|V_{cb}| = 0.0362 \pm 0.0019 \pm 0.0020 \pm 0.0014 \,, \tag{55}$$

where $\xi(1) = 0.97 \pm 0.04$ has been used. The third error in $|V_{cb}|$ is due to the theoretical uncertainties, which by now almost match the experimental ones.

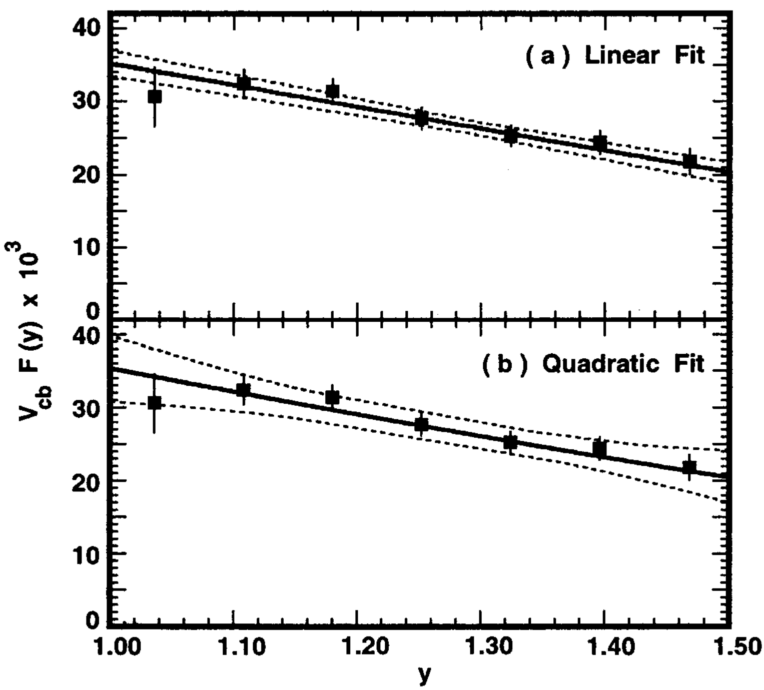

Fig. 1. Latest data [20] for the product $|V_{cb}|\xi(vv')$ as a function of $y = vv'$.

Similar statements can be made for the semileptonic decays of heavy baryons. The decay $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ is again parametrized by a single form factor, and this even remains true for the $1/m_Q$ correction terms [21, 22]. However, at present data on these decays are still sparse, although first measurements have been performed [23].

The Σ-type baryons will in general decay either strongly or electromagnetically and hence a weak decay will be completely unobservable. The only candidate where a weak decay may be observable is the Ω_Q baryon, since a strong decay would require the emission of a kaon and this may be suppressed due to the too small phase space.

5. Heavy to light transitions

Heavy quark symmetries may also be used to restrict the independent form factors appearing in heavy to light decays. For the decays of heavy mesons into light 0^- and 1^- particles heavy quark symmetries restrict the number of independent form factors to six, which is just the number needed to parametrize the semileptonic decays of this type. Furthermore, no absolute normalization of form factors may be obtained from heavy quark symmetries in the heavy to light case; only the relative normalization of B meson decays heavy to light transitions may be obtained from the corresponding D decays.

In general we shall discuss matrix elements of a heavy to light current which have the following structure

$$J = \langle A|\bar{\ell}\Gamma h_v|H(v)\rangle , \tag{56}$$

where Γ is an arbitrary Dirac matrix, ℓ is a light quark (u, d or s) and A is a state involving only light degrees of freedom.

Spin symmetry implies that the heavy quark index hooks directly the to the heavy quark index of the Dirac matrix of the current. Thus one may write for the transition matrix element (62)

$$\langle A|\bar{\ell}\Gamma h_v|H(v)\rangle = \text{Tr } (\mathcal{M}_A \Gamma H(v)) , \tag{57}$$

where the matrix $H(v)$ representing the heavy meson has been given in (9). The matrix $\mathcal{M}_A$ describes the light degrees of freedom and is the most general matrix which may be formed from the kinematical variables involved. Furthermore, if the energies of the particles in the state A are small, *i.e.* of the order of Λ_{QCD}, the matrix $\mathcal{M}_A$ does not depend on the heavy quark; in particular it does not depend on the heavy mass m_H. In the following we shall discuss some examples.

The first example is the heavy meson decay constant, where the state A is simply the vacuum state. The heavy meson decay constant is defined by

$$\langle 0|\bar{\ell}\gamma_\mu\gamma_5 h_v|H(v)\rangle = f_H m_H v_\mu , \tag{58}$$

and since $|A\rangle = |0\rangle$ the matrix $\mathcal{M}_0$ is simply the unit matrix times a dimensionful constant[3] and one has, using (57)

$$\langle 0|\bar{\ell}\gamma_\mu\gamma_5 h_v|H(v)\rangle = \kappa \text{ Tr } (\gamma\gamma_5 H(v)) = 2\kappa\sqrt{m_H} v_\mu . \tag{59}$$

As discussed above the constant κ does not depend on the heavy mass and thus on infers the well-known scaling law for the heavy meson decay constant from the last two equation

$$f_H \propto \frac{1}{\sqrt{m_H}} . \tag{60}$$

[3] Note that contributions proportional to $\not{v}$ may be eliminated using

$$H(v)\not{v} = -H(v).$$

Including the leading and subleading QCD radiative corrections one obtains a relation between f_B and f_D

$$f_B = \sqrt{\frac{m_c}{m_b}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} \left[1 + 0.894 \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi} \right] f_D \sim 0.69 f_D \,. \tag{61}$$

The second example are transitions of a heavy meson into a light pseudoscalar meson, which we shall denote as π. The matrix element corresponding to (56) is

$$J_P = \langle \pi(p) | \bar{\ell} \Gamma h_v | H(v) \rangle \,, \tag{62}$$

where p is the momentum of the light quark.

The Dirac matrix $\mathcal{M}_P$ for the light degrees of freedom appearing now in (57) depends on p and v. It may be expanded in terms of the sixteen independent Dirac matrices 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$ taking into account that it has to behave like a pseudoscalar. The form factors appearing in the decomposition of $\mathcal{M}_P$ depend on the variable $v \cdot p$, the energy of the light meson in the rest frame of the heavy one. In order to compare different heavy to light transition by employing heavy flavor symmetry this energy must be sufficiently small, since the typical scale for the light degrees of freedom has to be of the order of Λ_{QCD} to apply heavy quark symmetry[4]. For the case of a light pseudoscalar meson the most general decomposition of $\mathcal{M}_P$ is

$$\mathcal{M}_P = \sqrt{v \cdot p} A(\eta) \gamma_5 + \frac{1}{\sqrt{v \cdot p}} B(\eta) \gamma_5 \not{p} \,, \tag{63}$$

where we have defined the dimensionless variable

$$\eta = \frac{v \cdot p}{\Lambda_{\rm QCD}} \,. \tag{64}$$

The form factors A and B are universal in the kinematic range of small energy of the light meson, *i.e.* where the momentum transfer to the light degrees of freedom is of the order $\Lambda_{\rm QCD}$; in this region η is of order unity. This universality of the form factors may be used to relate various kinds of heavy to light transitions, *e.g.* the semileptonic decays like $D \to \pi e \nu$, $D \to K e \nu$ or $B \to \pi e \nu$ and also the rare decays like $B \to K \ell^+ \ell^-$ or $B \to \pi \ell^+ \ell^-$, where ℓ denotes an electron or a muon.

---

[4] Note that in this case the variable $v \cdot p$ ranges between 0 and $m_H/2$ where we have neglected the pion mass. Thus at the upper end of phase space the variable $v \cdot p$ scales with the heavy mass and heavy quark symmetries are not applicable any more.

As an example we give the relations between exclusive semileptonic heavy to light decays. The relevant hadronic current for this case may be expressed in terms of two form factors

$$\begin{aligned}\langle\pi(p)|\bar\ell\gamma(1-\gamma_5)h_v|H(v)\rangle &= F_1(v\cdot p)m_H v_\mu + F_2(v\cdot p)p_\mu \\ &= F_+(v\cdot p)(m_H v_\mu + p_\mu) + F_-(v\cdot p)q_\mu\,,\end{aligned} \tag{65}$$

where

$$F_\pm(v\cdot p) = \tfrac{1}{2}\left(F_1(v\cdot p) \pm F_2(v\cdot p)\right). \tag{66}$$

Inserting this into (62) one may express $F_\pm$ in terms of the universal form factors A and B

$$F_1(v\cdot p) = F_+(v\cdot p) + F_-(v\cdot p) = -2\sqrt{\frac{v\cdot p}{m_H}}A(\eta)\,, \tag{67}$$

$$F_2(v\cdot p) = F_+(v\cdot p) - F_-(v\cdot p) = -2\sqrt{\frac{m_H}{v\cdot p}}B(\eta)\,. \tag{68}$$

From these relations one may read off the scaling of the form factors with the heavy mass which was already derived in [5].

This may be used to normalize the semileptonic B decays into light mesons relative to the semileptonic D decays. One obtains

$$F_\pm^B(v\cdot p) = \tfrac{1}{2}\left(\sqrt{\frac{m_D}{m_B}} \pm \sqrt{\frac{m_B}{m_D}}\right)F_+^D(v\cdot p) + \tfrac{1}{2}\left(\sqrt{\frac{m_D}{m_B}} \mp \sqrt{\frac{m_B}{m_D}}\right)F_-^D(v\cdot p)\,. \tag{69}$$

Note that F_+ for the B decay is expressed in terms of F_+ *and* F_- for the D decays. In the limit of vanishing fermion masses only F_+ contributes, which means that the F_- contribution to the rate is of the order of $m_{\rm lepton}/m_H$. Thus it will be extremely difficult to determine experimentally.

The case of a heavy meson decaying into a light vector meson may be treated similarly. The matrix element for the transition of a heavy meson into a light vector meson (denoted generically as ρ in the following) is given again by (56) and is in this case

$$J_V = \langle\rho(p,\varepsilon)|\bar\ell\Gamma h_v|H(v)\rangle.. \tag{70}$$

Using (57) one has

$$\langle\rho(p,\varepsilon)|\bar\ell\Gamma h_v|H(v)\rangle = \mathrm{Tr}\ \left(\mathcal{M}_V\Gamma H(v)\right)\,, \tag{71}$$

where now the Dirac matrix $\mathcal{M}_V$ has to be a linear function of the polarization of the light vector meson.

The most general decomposition is given in terms of four dimensionless form factors

$$\mathcal{M}_V = \sqrt{v\cdot p}\,C(\eta)(v\cdot\varepsilon) + \frac{1}{\sqrt{v\cdot p}}D(\eta)(v\cdot\varepsilon)\not{p} + \sqrt{v\cdot p}\,E(\eta)\not{\varepsilon} + \frac{1}{\sqrt{v\cdot p}}F(\eta)\not{p}\not{\varepsilon}\,, \tag{72}$$

where the variable η has been defined in (64).

Similar to the case of the decays into a light pseudoscalar meson (71) may be used to relate various exclusive heavy to light processes in the kinematic range where the energy of the outgoing vector meson is small. For example, the semileptonic decays $D \to \rho e\nu$, $D \to K^* e\nu$ and $B \to \rho e\nu$ are related among themselves and all of them may be related to the rare heavy to light decays $B \to K^*\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ with $\ell = e, \mu$.

Finally we comment on the heavy to light transitions of baryons. For the Λ-type heavy baryons (10) spin symmetry relates different polarizations of the same particle and thus imposes interesting constraints. Consider for example the matrix element of an operator $\bar{\ell}\Gamma h_v$ between a heavy Λ_Q and a light spin-$^1/_2$ baryon B_ℓ. It is described by only two form factors,

$$\langle B_\ell(p)|\bar{\ell}\Gamma h_v|\Lambda_Q(v)\rangle = \bar{u}_\ell(p)\{F_1(v\cdot p) + \not{p}F_2(v\cdot p)\}\Gamma u_{\Lambda_Q}(v)\,. \tag{73}$$

Thus in this particular case spin symmetry greatly reduces the number of independent Lorentz-invariant amplitudes which describe the heavy to light transitions.

This has some interesting implications for exclusive semileptonic Λ_c decays. For the case of a left handed current $\Gamma = \gamma_\mu(1-\gamma_5)$, the semileptonic decay $\Lambda_c \to \Lambda\ell\bar{\nu}_\ell$ is in general parametrized in terms of six form factors

$$\begin{aligned}\langle\Lambda(p)|\bar{q}\gamma_\mu(1-\gamma_5)c|\Lambda_c(v)\rangle =& \bar{u}(p)\left[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3q^\mu\right]u(p')\\ &+ \bar{u}(p)\left[g_1\gamma_\mu + ig_2\sigma_{\mu\nu}q^\nu + g_3q^\mu\right]\gamma_5u(p')\,,\end{aligned} \tag{74}$$

where $p' = m_{\Lambda_c}v$ is the momentum of the Λ_c whereas $q = m_{\Lambda_c}v - p$ is the momentum transfer. From this one defines the ratio G_A/G_V by

$$\frac{G_A}{G_V} = \frac{g_1(q^2=0)}{f_1(q^2=0)}\,. \tag{75}$$

In the heavy c quark limit one may relate the six form factors f_i and g_i $(i = 1,2,3)$ to the two form factors F_j $(j = 1,2)$

$$f_1 = -g_1 = F_1 + \frac{m_\Lambda}{m_{\Lambda_c}}F_2\,, \tag{76}$$

$$f_2 = f_3 = -g_2 = -g_3 = \frac{1}{m_{\Lambda_c}}F_2\,, \tag{77}$$

from which one reads off $G_A/G_V = -1$. This ratio is accessible by measuring in semileptonic decays $\Lambda_c \to \lambda \ell \bar{\nu}_\ell$ the polarization variable α

$$\alpha = \frac{2G_A G_V}{G_A^2 + G_V^2}, \tag{78}$$

which is predicted to be $\alpha = -1$ in the heavy c quark limit. The subleading corrections to the heavy c quark limit have been estimated and found to be small [24]

$$\alpha < -0.95, \tag{79}$$

and recent measurements yield

$$\alpha = -0.91 \pm 0.49 \quad \text{ARGUS [25]} \tag{80}$$

$$\alpha = -0.89^{+0.17+0.09}_{-0.11-0.05} \quad \text{CLEO [26]} \tag{81}$$

and are in satisfactory agreement with the theoretical predictions.

Recently the CLEO collaboration also measured the ratio of the form factors F_1 and F_2, averaged over phase space. Heavy quark symmetries do not fix this form factor ratio, at least not for a heavy to light decay, while for a heavy to heavy decay the form factor F_2 vanishes in the heavy mass limit for the final state quark. CLEO measures [28]

$$\left\langle \frac{F_2}{F_2} \right\rangle_{\text{phase space}} = -0.25 \pm 0.14 \pm 0.08, \tag{82}$$

which is in good agreement with model estimates [27].

6. Conclusions

The field of heavy quark physics has gone through a remarkable development over the last few years due to new theoretical ideas as well as to a major improvement of data. In particular the progress in the technology of detectors (*e.g.* silicon vertex detectors) opened the possibility to study b physics even at machines which originally were not designed for this kind of research. In this way also the high energy colliders (in particular LEP and TEVATRON) could contribute substantially in this area, since they allow to measure states (such as the B_s and the b flavoured baryons) which lie above the threshold of the $\Upsilon(4s)$-B-factories.

From the theoretical side the heavy quark limit and HQET brought an important success, since it provides a model independent and QCD based framework for the description of processes involving heavy quarks. As far as exclusive heavy to heavy decays are concerned, the additional symmetries of the heavy mass limit restrict the number of nonperturbative functions in a model independent way; furthermore, heavy quark symmetries fix the absolute normalization of some of the transition amplitudes at the point of maximum momentum transfer. In heavy to light decays heavy quark symmetries do not work as efficiently; in this case only the relative normalization of B decays versus the corresponding D decays may be obtained.

Corrections may be studied systematically HQET. As in any effective field theory this framework allows a clean separation between short distance effects, connected with the large mass m_Q, and the long distance pieces, which are related to small hadronic scales of order Λ_{QCD}. The short distance part may be calculated in renormalization group improved perturbation theory, while the long distance piece needs to be parametrized, but is restricted by heavy quark symmetries.

The phenomenological impact of the heavy quark limit is tremendous. Its main field of application are the semileptonic decays of b and c hadrons, where the hadronic matrix elements are studied in the heavy mass expansion. In particular for $b \to c$ decays, when both quarks are treated as heavy, one has many model independent statements concerning the decay rates; in addition, also the absolute normalization of the matrix elements is known, allowing us the extraction of V_{cb} without strong model dependences.

Heavy quark symmetries also relate exclusive semileptonic transitions with the exclusive rare decays, which are based on $b \to s\gamma$ or $b \to s\ell^+\ell^-$ decays. These are, however, of the heavy to light type and thus are not as strongly restricted as the heavy to heavy ones.

HQET does not yet have much to say about exclusive nonleptonic decays; even for the decays $B \to D^{(*)}D_s^{(*)}$, which involves three heavy quarks, heavy quark symmetries are not sufficient to yield useful relations between the decay rates [29]. Of course, with additional assumptions such as factorization one can go ahead and relate the nonleptonic decays to the semileptonic ones; however, this is a very strong assumption and it is not clear in what sense factorization is an approximation. On the other side, the data of the nonleptonic B decays supports factorization, and first attempts to understand this from QCD and HQET have been undertaken [30]; however, the problem of the exclusive nonleptonic decays still needs clarification and hopefully the heavy mass limit will also be useful here.

It was great pleasure and honour for me to be invited and to talk at the symposium celebrating Kacper Zalewski's 60^{th} birthday. I want to thank the organizers of the conference for the invitation and their hospitality during my stay in Cracow.

REFERENCES

[1] M. Voloshin, M. Shifman, *Sov. J. Nucl. Phys.* **45**, 292 (1987); **47**, 511 (1988); E. Eichten, B. Hill, *Phys. Lett.* **B234**, 511 (1990).

[2] N. Isgur, M. Wise, *Phys. Lett.* **B232**, 113 (1989); **B237**, 527 (1990).

[3] B. Grinstein, *Nucl. Phys.* **B339**, 253 (1990) 253; H. Georgi, *Phys. Lett.* **B240**, 447 (1990); A. Falk, H. Georgi, B. Grinstein, M. Wise, *Nucl. Phys.* **B343**, 1 (1990); T. Mannel, W. Roberts, Z. Ryzak, *Nucl. Phys.* **B 368**, 204 (1992).

[4] H. Georgi: contribution to the Proceedings of TASI–91, ed. R.K. Ellis *et al.*, World Scientific, Singapore 1991; B. Grinstein, contribution to *High Energy Phenomenology*, ed. R. Huerta and M.A. Peres, World Scientific, Singapore 1991; N. Isgur, M. Wise, contribution to *Heavy Flavors*, ed. A. Buras and M. Lindner, World Scientific, Singapore 1992); M. Neubert, *Phys. Rep.* **245**, 259 (1994); T. Mannel, contribution to *QCD–20 Years Later*, ed. P. Zerwas and H. Kastrup, World Scientific, Singapore 1993.

[5] N. Isgur, M. Wise, *Phys. Rev.* **D42**, 2388 (1990).

[6] A. Falk, *Nucl. Phys.* **B 378**, 79 (1992).

[7] T. Bergfeld *et al.* CLEO Collaboration, *Phys. Lett.* **B340**, 194 (1994).

[8] E. Eichten, C. Hill, C. Quigg, *Phys. Rev. Lett.* **71**, 4116 (1993).

[9] N. Isgur, M. Wise, *Nucl. Phys.* **B348**, 276 (1991).

[10] H. Georgi, *Nucl. Phys.* **B348**, 293 (1991).

[11] T. Mannel, W. Roberts, Z. Ryzak, *Nucl. Phys.* **B355**, 38 (1991).

[12] S. Balk, J. Körner, D. Pirjol, *Nucl. Phys.* **B428**, 499 (1994); W. Kilian, T. Ohl, *Phys. Rev.* **D50**, 4649 (1994).

[13] A. Falk, M. Neubert, *Phys. Rev.* **D47**, 2965 (1993).

[14] T. Mannel, *Phys. Rev.* **D50**, 428 (1994).

[15] I. Bigi *et al.*, *Int. J. Mod. Phys.* **A9**, 2467 (1994).

[16] I. Bigi *et al.*, preprint CERN-TH-7250/94 (1994), hep-ph/9405410.

[17] P. Ball, V. Braun, *Phys. Rev.* **D49**, 2472 (1994).

[18] N. Isgur, D. Scora, B. Grinstein, M. Wise, *Phys. Rev.* **D39**, 799 (1989).

[19] M. Shifman, N.G. Uraltsev, A. Vainshtein, Minnesota Preprint TPI-MINN-94-13-T (1994).

[20] B. Barish *et al.*, CLEO Collaboration, Cornell preprint CLNS-94-1285, (1994).

[21] H. Georgi, B. Grinstein, M. Wise, *Phys. Lett.* **B252**, 456 (1990).

[22] P. Cho, B. Grinstein, *Phys. Lett.* **B285**, 153 (1992).

[23] C. Albajar *et al.*, UA1 Collaboration, *Phys. Lett.* **B273**, 540 (1991); D. Buskulic *et al.*, ALEPH Collaboration, *Phys. Lett.* **B294**, 145 (1992); P. Abreu *et al.*, DELPHI Collaboration, *Phys. Lett.* **B311**, 379 (1993); R. Akers *et al.*, OPAL Collaboration, *Phys. Lett.* **B316**, 435 (1993).

[24] G. Lin, T. Mannel, *Phys. Lett.* **B321**, 417 (1994).

[25] H. Albrecht *et al.*, ARGUS Collaboration, *Phys. Lett.* **B326**, 320 (1994).

[26] T. Bergfeld, *et al.*, CLEO Collaboration, *Phys. Lett.* **B323**, 219 (1994).
[27] J. Körner, M. Krämer, *Phys. Lett.* **B275**, 495 (1992).
[28] G. Crawford *et al.*, CLEO COllaboration, Cornell preprint CLNS 94/1306, CLEO 94/24 (1994).
[29] T. Mannel, W. Roberts, Z. Ryzak, *Phys. Lett.* **B248**, 392 (1990).
[30] M. Dugan, B. Grinstein, *Phys. Lett.* **B255**, 583 (1991).