RARE DECAYS, CP VIOLATION AND QCD*

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We discuss several aspects of rare decays and CP violation in the standard model including the impact of the recent top quark discovery. In particular we review the present status of next-to-leading QCD calculations in this field stressing their importance in the determination of the parameters in the Cabibbo–Kobayashi–Maskawa matrix. We emphasize that the definitive tests of the standard model picture of rare decays and CP violation will come through a *simultaneous* study of CP asymmetries in $B^0_{d,s}$ decays, the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, and $(B^0_d - \bar{B}^0_d)/(B^0_s - \bar{B}^0_s)$.

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1. Preface

It is a great privilege and a great pleasure to give this talk at the symposium celebrating the 60th birthday of Kacper Zalewski. I have known Kacper during the last 20 years admiring him, his research and his constructive criticism. I do hope very much to give another talk on this subject in 2015 at a symposium celebrating Kacper's 80th birthday. I am convinced that the next 20 years in the field of rare decays and CP violation will be very exciting and hopefully full of surprises. A 1995 view of this field is given below.

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2. Setting the scene

An important target of particle physics is the determination of the unitary 3 × 3 Cabibbo–Kobayashi–Maskawa matrix [1, 2] which parametrizes the charged current interactions of quarks:

$$J^{cc}_{\mu} = (\bar{u}, \bar{c}, \bar{t})_L \gamma_{\mu} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L . \tag{1}$$

The CP violation in the standard model is supposed to arise from a single phase in this matrix. It is customary these days to express the CKM-matrix in terms of four Wolfenstein parameters [3] $(\lambda, A, \varrho, \eta)$ with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and η representing the CP violating phase:

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \tag{2}$$

Because of the smallness of λ and the fact that for each element the expansion parameter is actually λ^2, it is sufficient to keep only the first few terms in this expansion.

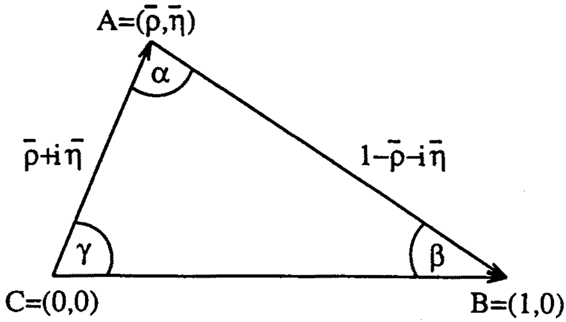

Fig. 1.

Following [4] one can define the parameters $(\lambda, A, \varrho, \eta)$ through

$$s_{12} \equiv \lambda, \qquad s_{23} \equiv A\lambda^2, \qquad s_{13}e^{-i\delta} \equiv A\lambda^3(\varrho - i\eta), \tag{3}$$

where s_{ij} and δ enter the standard exact parametrization [5] of the CKM matrix. This specifies the higher orders terms in (2).

The definition of $(\lambda, A, \varrho, \eta)$ given in (3) is useful because it allows to improve the accuracy of the original Wolfenstein parametrization in an elegant manner. In particular

$$V_{us} = \lambda, \qquad V_{cb} = A\lambda^2, \tag{4}$$

$$V_{ub} = A\lambda^3(\varrho - i\eta), \qquad V_{td} = A\lambda^3(1-\bar{\varrho}-i\bar{\eta}), \tag{5}$$

where

$$\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}), \qquad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}), \tag{6}$$

turn out [4] to be excellent approximations to the exact expressions.

A useful geometrical representation of the CKM matrix is the unitarity triangle obtained by using the unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\,, \tag{7}$$

rescaling it by $\mid V_{cd}V_{cb}^* \mid = A\lambda^3$ and depicting the result in the complex $(\bar{\rho}, \bar{\eta})$ plane as shown in Fig. 1. The lengths CB, CA and BA are equal respectively to 1,

$$R_b \equiv \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2})\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \quad \text{and} \quad R_t \equiv \sqrt{(1-\bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{8}$$

The triangle in Fig. 1 is one of the important targets of the contemporary particle physics. Together with $\mid V_{us} \mid$ and $\mid V_{cb} \mid$ it summarizes the structure of the CKM matrix. In particular the area of the unrescaled triangle gives a measure of CP violation in the standard model [6]:

$$\mid J_{\rm CP} \mid = 2(\text{Area of } \Delta) = \mid V_{ud} \mid\mid V_{us} \mid\mid V_{ub} \mid\mid V_{cb} \mid \sin\delta = A^2\lambda^6\bar{\eta} = \mathcal{O}(10^{-5})\,. \tag{9}$$

This formula shows an important feature of the KM picture of CP violation: the smallness of CP violation in the standard model is not necessarily related to the smallness of η but to the fact that in this model the size of CP violating effects is given by products of small mixing parameters.

Looking at the expressions for R_b and R_t we also observe that within the standard model the measurements of four CP *conserving* decays sensitive to $\mid V_{us} \mid$, $\mid V_{ub} \mid$, $\mid V_{cb} \mid$ and $\mid V_{td} \mid$ can tell us whether CP violation is predicted in the standard model. This is a very remarkable property of the Kobayashi–Maskawa picture of CP violation: quark mixing and CP violation are closely related to each other.

There is of course the very important question whether the KM picture of CP violation is correct and more generally whether the standard model offers a correct description of weak decays of hadrons. In order to answer these important questions it is essential to calculate as many branching ratios as possible, measure them experimentally and check if they all can be described by the same set of the parameters $(\lambda, A, \varrho, \eta)$. In the language of the unitarity triangle this means that the various curves in the $(\bar{\varrho}, \bar{\eta})$ plane extracted from different decays should cross each other at a single point as shown in Fig. 2. Moreover the angles (α, β, γ) in the resulting

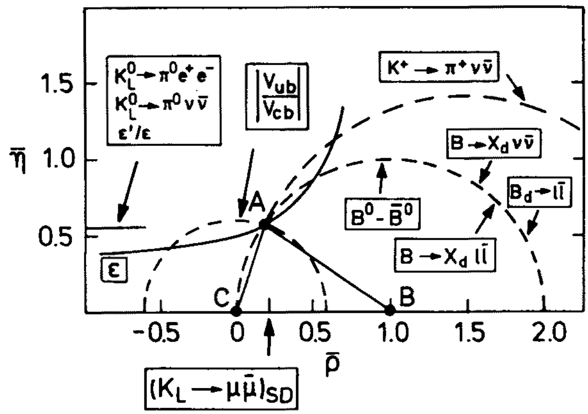

Fig. 2.

triangle should agree with those extracted one day from CP-asymmetries in B-decays. More about this below.

There is a common belief that during the coming fifteen years we will certainly witness a dramatic improvement in the determination of the CKM-parameters analogous to, although not as precise as, the determination of the parameters in the gauge boson sector which took place during the recent years. To this end, however, it is essential not only to perform difficult experiments but also to have accurate formulae which would allow a confident and precise extraction of the CKM-parameters from the existing and future data. We will review what progress has been done in this direction in Section 4.

Finally it is important to stress that the discovery of the top quark [7, 8] and its mass measurement had an important impact on the field of rare decays and CP violation reducing considerably one potential uncertainty. In loop induced K and B decays the relevant mass parameter is the running current quark mass. With the pole mass measurement of CDF, $m_t^{\rm pole} = 176 \pm 13$ GeV, one has $m_t^* = \bar{m}_t(m_t) \approx 168 \pm 13$ GeV. Similarly the D0 value $m_t^{\rm pole} = 199 \pm 30$ GeV corresponds to $m_t^* = \bar{m}_t(m_t) \approx 190 \pm 30$ GeV. In this review we will simply denote m_t^* by m_t.

3. Basic framework

3.1. OPE and renormalization group

The basic framework for weak decays of hadrons containing u, d, s, c and b quarks is the effective field theory relevant for scales $\mu \ll M_W, M_Z, m_t$. This framework brings in local operators which govern "effectively" the transitions in question. From the point of view of the decaying hadrons containing the lightest five quarks this is the only correct picture we know and also the most efficient one in studying the presence of QCD. Furthermore it represents the generalization of the Fermi theory. In this connection it should be mentioned that the usual Feynman diagram drawings containing full W-propagators or Z^0−propagators and top-quark propagators represent really the happening at scales $\mathcal{O}(M_W)$ whereas the true picture of a decaying hadron is more correctly described by effective vertices which are represented by local operators in question.

Thus whereas at scales $\mathcal{O}(M_W)$ we deal with the full six-quark theory containing photon, weak gauge bosons and gluons, at scales $\mathcal{O}(1\ \text{GeV})$ the relevant effective theory contains only three light quarks u, d and s, gluons and the photon. At intermediate energy scales, $\mu = \mathcal{O}(m_b)$ and $\mu = \mathcal{O}(m_c)$, relevant for b and charm decays effective five-quark and effective four-quark theories have to be considered respectively.

The usual procedure then is to start at a high energy scale $\mathcal{O}(M_W)$ and consecutively integrate out the heavy degrees of freedom (heavy with respect to the relevant scale μ) from explicitly appearing in the theory. The word "explicitly" is very essential here. The heavy fields did not disappear. Their effects are merely hidden in the effective gauge coupling constants, running masses and most importantly in the coefficients describing the "effective" strength of the operators at a given scale μ, the Wilson coefficient functions.

Operator Product Expansion (OPE) combined with the renormalization group approach can be regarded as a mathematical formulation of the picture outlined above. In this framework the amplitude for a decay $M \to F$ is written as

$$A(M \to F) = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_i C_i(\mu) \langle F \mid Q_i(\mu) \mid M \rangle \,, \tag{10}$$

where M stands for the decaying meson, F for a given final state and V_{CKM} denotes the relevant CKM factor. $Q_i(\mu)$ denote the local operators generated by QCD and electroweak interactions. $C_i(\mu)$ stand for the Wilson coefficient functions (c-numbers). The scale μ separates the physics contributions in the "short distance" contributions (corresponding to scales higher than μ) contained in $C_i(\mu)$ and the "long distance" contributions (scales lower than μ) contained in $\langle F \mid Q_i(\mu) \mid M \rangle$. By evolving the scale from $\mu = \mathcal{O}(M_W)$ down to lower values of μ one transforms the physics information at scales higher than μ from the hadronic matrix elements into $C_i(\mu)$. Since no information is lost this way the full amplitude cannot depend on μ. This is the essence of renormalization group equations which govern the evolution (μ-dependence) of $C_i(\mu)$. This μ-dependence must be cancelled by the one present in $\langle Q_i(\mu) \rangle$. It should be stressed, however, that this cancellation generally involves many operators due to the operator mixing under renormalization.

The general expression for $C_i(\mu)$ is given by:

$$\vec{C}(\mu) = \hat{U}(\mu, M_W)\vec{C}(M_W), \tag{11}$$

where $\vec{C}$ is a column vector built out of C_i's. $\vec{C}(M_W)$ are the initial conditions which depend on the short distance physics at high energy scales. In particular they depend on m_t. $\hat{U}(\mu, M_W)$, the evolution matrix, is given as follows

$$\hat{U}(\mu, M_W) = T_g \exp\Big(\int_{g(M_W)}^{g(\mu)} dg' \frac{\hat{\gamma}^T(g')}{\beta(g')}\Big), \tag{12}$$

with g denoting QCD effective coupling constant. $\beta(g)$ governs the evolution of g and $\hat{\gamma}$ is the anomalous dimension matrix of the operators involved. The structure of this equation makes it clear that the renormalization group approach goes beyond the usual perturbation theory. Indeed $\hat{U}(\mu, M_W)$ sums automatically large logarithms $\log M_W/\mu$ which appear for $\mu \ll M_W$. In the so called leading logarithmic approximation (LO) terms $(g^2 \log M_W/\mu)^n$ are summed. The next-to-leading logarithmic correction (NLO) to this result involves summation of terms $(g^2)^n(\log M_W/\mu)^{n-1}$ and so on. This hierarchic structure gives the renormalization group improved perturbation theory.

As an example let us consider only QCD effects and the case of a single operator. Keeping the first two terms in the expansions of $\gamma(g)$ and $\beta(g)$ in powers of g:

$$\gamma(g) = \gamma^{(0)}\frac{\alpha_{QCD}}{4\pi} + \gamma^{(1)}\frac{\alpha^2_{QCD}}{16\pi^2}, \quad \beta(g) = -\beta_0\frac{g^3}{16\pi^2} - \beta_1\frac{g^5}{(16\pi^2)^2} \tag{13}$$

and inserting these expansions into (12) gives:

$$U(\mu, M_W) = \left[1 + \frac{\alpha_{QCD}(\mu)}{4\pi}J\right]\left[\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(\mu)}\right]^P\left[1 - \frac{\alpha_{QCD}(M_W)}{4\pi}J\right], \tag{14}$$

where

$$P = \frac{\gamma^{(0)}}{2\beta_0}, \qquad J = \frac{P}{\beta_0}\beta_1 - \frac{\gamma^{(1)}}{2\beta_0}. \tag{15}$$

General formulae for $\hat{U}(\mu, M_W)$ in the case of operator mixing and valid also for electroweak effects can be found in Ref. [16]. The leading logarithmic approximation corresponds to setting $J = 0$ in (14).

3.2. Classification of operators

Below we give six classes of operators which play the dominant role in the phenomenology of weak decays. Typical diagrams in the full theory from which these operators originate are indicated and shown in Fig. 3 . The cross in Fig. 3(d) indicates that magnetic penguins originate from the mass-term on the external line in the usual QCD or QED penguin diagrams. The six classes are given as follows:

Current–Current (Fig. 3(a)):

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}, \qquad Q_2 = (\bar{s}u)_{V-A} (\bar{u}d)_{V-A}. \tag{16}$$

QCD–Penguins (Fig. 3(b)):

$$Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A},$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}, \tag{17}$$

$$Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V+A},$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}. \tag{18}$$

Electroweak–Penguins (Fig. 3(c)):

$$Q_7 = \tfrac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A},$$

$$Q_8 = \tfrac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}, \tag{19}$$

$$Q_9 = \tfrac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A},$$

$$Q_{10} = \tfrac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A}. \tag{20}$$

Magnetic–Penguins (Fig. 3(d)):

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) b_\alpha F_{\mu\nu},$$

$$Q_{8G} = \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu}. \tag{21}$$

$\Delta S = 2$ and $\Delta B = 2$ Operators (Fig. 3(e)):

$$Q(\Delta S = 2) = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}\,,$$
$$Q(\Delta B = 2) = (\bar{b}d)_{V-A}(\bar{b}d)_{V-A}\,. \tag{22}$$

Semi–Leptonic Operators (Fig. 3(f)):

$$Q_{9V} = (\bar{b}s)_{V-A}(\bar{e}e)_{V}\,, \quad Q_{10A} = (\bar{b}s)_{V-A}(\bar{e}e)_{A}\,, \tag{23}$$
$$Q(\nu\bar{\nu}) = (\bar{s}d)_{V-A}(\nu\bar{\nu})_{V-A}\,, \quad Q(\mu\bar{\mu}) = (\bar{s}d)_{V-A}(\mu\bar{\mu})_{V-A}\,. \tag{24}$$

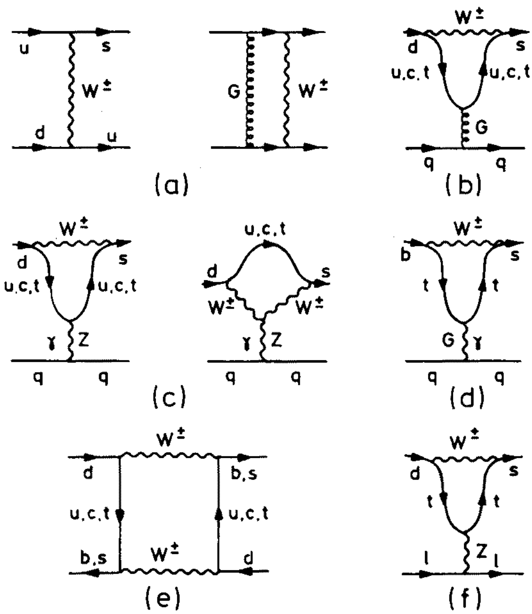

Fig. 3.

3.3. Towards phenomenology

The rather formal expression for the decay amplitudes given in (10) can always be cast in the form [9]:

$$A(M \to F) = \sum_i B_i V^i_{\rm CKM} \eta^i_{\rm QCD} F_i(m_t, m_c) \tag{25}$$

which is more useful for phenomenology. In writing (25) we have generalized (10) to include several CKM factors. $F_i(m_t, m_c)$, the Inami–Lim functions, result from the evaluation of loop diagrams with internal top and charm exchanges (see Fig. 3) and may also depend solely on m_t or m_c. In the case of current-current operators F_i are mass independent. The factors $\eta^i_{\rm QCD}$ summarize the QCD corrections which can be calculated by formal methods discussed above. Finally B_i stand for nonperturbative factors related to the hadronic matrix elements of the contributing operators: the main theoretical uncertainty in the whole enterprise. In semi-leptonic decays such as $K \to \pi\nu\bar{\nu}$, the non-perturbative B-factors can fortunately be determined from leading tree level decays such as $K^+ \to \pi^0 e^+ \nu$ reducing or removing the non-perurbative uncertainty. In non-leptonic decays this is generally not possible and we have to rely on existing non-perturbative methods. A well known example of a B_i-factor is the renormalization group invariant parameter B_K [10] defined by

$$B_K = B_K(\mu)\left[\alpha_s(\mu)\right]^{-2/9}\,, \quad \langle \bar{K}^o \mid Q(\Delta S = 2) \mid K^o \rangle = \tfrac{8}{3} B_K(\mu) F_K^2 m_K^2\,, \tag{26}$$

B_K plays an important role in the phenomenology of CP violation in $K \to \pi\pi$. We will encounter several examples of (25) below.

4. Weak decays beyond leading logarithms

4.1. General remarks

Until 1989 most of the calculations in the field of weak decays were done in the leading logarithmic approximation. An exception was the important work of Altarelli *et al.* [11] who in 1981 calculated NLO QCD corrections to the Wilson coefficients of the current-current operators.

Today the effective Hamiltonians for weak decays are available at the next-to-leading level for the most important and interesting cases due to a series of publications devoted to this enterprise beginning with the work of Peter Weisz and myself in 1989 [12]. The list of the existing calculations is given in Table I. We will discuss this list briefly below. A detailed review of the existing NLO calculations will appear soon [13].

Let us recall why NLO calculations are important for the phenomenology of weak decays.

- The NLO is first of all necessary to test the validity of the renormalization group improved perturbation theory.
- Without going to NLO the QCD scale $\Lambda_{\overline{MS}}$ extracted from various high energy processes cannot be used meaningfully in weak decays.
- Due to renormalization group invariance the physical amplitudes do not depend on the scales μ present in α_s or in the running quark masses, in particular $m_t(\mu)$, $m_b(\mu)$ and $m_c(\mu)$. However in perturbation theory this property is broken through the truncation of the perturbative series.

Consequently one finds sizable scale ambiguities in the leading order, which can be reduced considerably by going to NLO.

- In several cases the central issue of the top quark mass dependence is strictly a NLO effect.

TABLE I

References to NLO Calculations

Decay	Reference
$\Delta F = 1$ Decays	
current-current operators	[11, 12]
QCD penguin operators	[14, 16–18]
electroweak penguin operators	[15–18]
magnetic penguin operators	[19]
$Br(B)_{SL}$	[11, 20, 21]
Particle-Antiparticle Mixing	
η_1	[22]
η_2, η_B	[23]
η_3	[24]
Rare K- and B-Meson Decays	
$K_L^0 \to \pi^0 \nu\bar{\nu}$, $B \to l^+ l^-$, $B \to X_s \nu\bar{\nu}$	[25, 26]
$K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \mu^+ \mu^-$	[27]
$K^+ \to \pi^+ \mu\bar{\mu}$	[28]
$K_L \to \pi^0 e^+ e^-$	[29]
$B \to X_s e^+ e^-$	[30, 31]

4.2. Current-current operators

The NLO corrections to the coefficients of Q_1 and Q_2 have been first calculated by Altarelli *et al.* [11] using the Dimension Reduction Scheme (DRED) for γ_5. In 1989 these coefficients have been calculated in DRED, NDR and HV schemes for γ_5 by Peter Weisz and myself [12]. The result for DRED obtained by the Italian group has been confirmed. The coefficients C_1 and C_2 show a rather strong renormalization scheme dependence which in physical quantities should be cancelled by the one present in the matrix elements of Q_1 and Q_2. This cancellation has been shown explicitly in [12] demonstrating thereby the compatibility of the results for C_1 and C_2 in DRED, NDR and HV schemes. A recent discussion of $C_1(\mu)$ and $C_2(\mu)$ in these schemes can be found in [32].

4.3. NLO corrections to B_{SL}

A direct physical application of the NLO corrections to C_1 and C_2 discussed above is the calculation of the non-leptonic width for B-Mesons which is relevant for the theoretical prediction of the inclusive semileptonic branching ratio:

$$B_{SL} = \frac{\Gamma(B \to Xe\nu)}{\Gamma_{SL}(B) + \Gamma_{NL}(B)}\,. \qquad (27)$$

This calculation can be done within the spectator model corrected for small non-perturbative corrections [33] and more important gluon bremsstrahlung and virtual gluon corrections. The latter cancel the scheme and μ dependances of $C_i(\mu)$. The calculation of B_{SL} for massless final quarks has been done by Altarelli *et al.* [11] in the DRED scheme and by Buchalla [20] in the HV scheme. The results of these papers agree with each other.

It is well known that the inclusion of QCD corrections in the spectator model, lowers B_{SL} which otherwise would be roughly 16%. Unfortunately the theoretical branching ratio based on the QCD calculation of Refs [11, 20] give typically $B_{SL} = 12.5 - 13.5\%$ [34] whereas the experimental world average [5] is

$$B_{SL}^{\rm exp} = (10.43 \pm 0.24)\%\,. \qquad (28)$$

The inclusion of the leading non-perturbative correction $\mathcal{O}(1/m_b^2)$ lowers slightly the theoretical prediction but gives only $\Delta_{NP}B_{SL} = -0.2\%$ [33]. On the other hand mass effects in the QCD corrections to B_{SL} seem to play an important role. Bagan *et al.* [21] using partially the results of Hokim and Pham [35] have demonstrated that the inclusion of mass effects in the QCD calculations of Refs [11, 20] (in particular in the decay $b \to c\bar{c}s$ (see also [36])) and taking into account various renormalization scale uncertainties improves the situation considerably. Bagan *et al.* find [21]:

$$B_{SL} = (12.0 \pm 1.4)\% \quad \text{and} \quad \bar{B}_{SL} = (11.2 \pm 1.7)\%\,, \qquad (29)$$

for the pole quark masses and $\overline{MS}$ masses respectively. Within existing uncertainties, this result does not disagree significantly with the experimental value, although it is still somewhat on the high side.

4.4. $\Delta S = 2$ and $\Delta B = 2$ transitions

The M_{12} amplitude describing the $K^0 - \bar{K}^0$ mixing is given as follows

$$M_{12}(\Delta S = 2) = \frac{G_F^2}{12\pi^2} F_K^2 B_K m_K M_W^2 \times \left[\lambda_c^{*2}\eta_1 S(x_c) + \lambda_t^{*2}\eta_2 S(x_t) + 2\lambda_c^*\lambda_t^*\eta_3 S(x_c, x_t)\right] \qquad (30)$$

with $x_i = m_i^2/M_W^2$, $\lambda_i = V_{id}V_{is}^*$, $S(x_i)$ denoting the Inami–Lim functions resulting from box diagrams and η_i representing QCD corrections. The parameter B_K is defined in (26). The corresponding amplitude for the $B_d^o - \bar{B}_d^o$ mixing is dominated by the box diagrams with top quark exchanges and given by

$$| M_{12}(\Delta B = 2) | = \frac{G_F^2}{12\pi^2} F_B^2 B_B m_B M_W^2 \mid V_{td} \mid^2 \eta_B S(x_t) , \qquad (31)$$

where we have set $V_{tb} = 1$. A similar formula exists for $B_s^o - \bar{B}_s^o$. For $m_t \ll M_W$ and in the leading order η_i have been calculated by Gilman and Wise [37]. Generalization to $m_t = \mathcal{O}(M_W)$ gives roughly [38–41]

$$\eta_1 = 0.85 , \qquad \eta_2 = 0.62 , \qquad \eta_3 = 0.36 , \qquad \eta_B = 0.60 . \qquad (32)$$

As of 1995 the coefficients η_i and η_B are known including NLO corrections. The coefficients η_2 and η_B have been calculated in [23] and η_1 and η_3 in [22] and [24] respectively. It has been stressed in these papers that the LO results for η_i in (32) suffer from sizable scale uncertainties, as large as $\pm 20\%$ for η_1 and $\pm 10\%$ for the remaining η_i. As demonstrated in [22–24] these uncertainties are considerably reduced in the products like $\eta_1 S(x_c)$, $\eta_2 S(x_t)$, $\eta_3 S(x_c, x_t)$ and $\eta_B S(x_t)$ provided NLO corrections are taken into account. For $m_c = \bar{m}_c(m_c) = 1.3 \pm 0.1$ GeV and $m_t = \bar{m}_t(m_t) = 170 \pm 15$ GeV one finds:

$$\begin{aligned} \eta_1 &= 1.3 \pm 0.2 , & \eta_2 &= 0.57 \pm 0.01 , \\ \eta_3 &= 0.** \pm 0.04 , & \eta_B &= 0.55 \pm 0.01 , \end{aligned} \qquad (33)$$

where the "**" in η_3 will be public soon [24]. It should be stressed that η_i given here are so defined that the relevant B_K and B_B non-perturbative factors (see (26)) are renormalization group invariant.

Let us list the main implications of these results:

- The enhancement of η_1 implies the enhancement of the short distance contribution to the $K_L - K_S$ mass difference so that for $B_K = 3/4$ as much as 80% of the experimental value can be attributed to this contribution [22].
- The improved calculations of η_2 and η_3 combined with the analysis of the CP violating parameter ε_K allow an improved determination of the parameters η and ϱ in the CKM matrix [4, 24].
- Similarly the improved calculation of η_B combined with the analysis of $B_d^0 - \bar{B}_d^0$ mixing allows an improved determination of the element

$| V_{td} |$ [4]:

$$| V_{td} | = 8.7 \cdot 10^{-3} \left[\frac{200\ \text{MeV}}{\sqrt{B_B} F_B}\right] \left[\frac{170\ \text{GeV}}{\bar{m}_t(m_t)}\right]^{0.76} \left[\frac{x_d}{0.72}\right]^{0.5} \left[\frac{1.50\ ps}{\tau_B}\right]^{0.5} . \tag{34}$$

This using all uncertainties (see below) gives:

$$| V_{td} | = (9.6 \pm 3.0) \cdot 10^{-3} \quad \Rightarrow \quad (9.3 \pm 2.5) \cdot 10^{-3} \tag{35}$$

with the last number obtained after the inclusion of the ε-analysis [4].

Concerning the parameter B_K, the most recent analyses using the lattice methods [42, 43] ($B_K = 0.83 \pm 0.03$) and the $1/N$ approach of [44] modified somewhat in [45] give results in the ball park of the $1/N$ result $B_K = 0.70 \pm 0.10$ obtained long time ago [44]. In particular the analysis of Bijnens and Prades [46] seems to have explained the difference between these values for B_K and the lower values obtained using the QCD Hadronic Duality approach [46] ($B_K = 0.39 \pm 0.10$) or using SU(3) symmetry and PCAC ($B_K = 1/3$) [47]. This is gratifying because such low values for B_K would require $m_t > 250$ GeV in order to explain the experimental value of ε [48, 4, 24].

There is a vast literature on the lattice calculations of F_B. The most recent results are somewhat lower than quoted a few years ago. Based on a review by Chris Sachrajda [49], the recent extensive study by Duncan *et al.* [50] and the analyses in [51] we conclude: $F_{B_d} = (180 \pm 40)$ MeV. This together with the earlier result of the European Collaboration for B_B, gives $F_{B_d}\sqrt{B_{B_d}} = 195 \pm 45$ MeV. The reduction of the error in this important quantity is desirable. These results for F_B are compatible with the results obtained using QCD sum rules (*e.g.* [52]). An interesting upper bound $F_{B_d} < 195\ MeV$ using QCD dispersion relations has also recently been obtained [53].

4.5. $\Delta S = 1$ Hamiltonian and ε'/ε

The effective Hamiltonian for $\Delta S = 1$ transitions is given as follows:

$$\mathcal{H}_{\text{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu)\right] Q_i\,, \tag{36}$$

where $\tau = -(V_{td}V_{ts}^*)/(V_{ud}V_{us}^*)$. The coefficients of all ten operators are known including NLO QCD and QED effects in NDR and HV schemes due to the independent work of Munich and Rome groups [14–18]. The results of both groups agree with each other. A direct application of these results is the calculation of $\mathrm{Re}(\varepsilon'/\varepsilon)$ which measures the ratio of direct to indirect CP violation in $K \to \pi\pi$ decays. In the standard model ε'/ε is governed by QCD penguins and electroweak (EW) penguins [54]. In spite of being suppressed by α/α_s relative to QCD penguin contributions, the electroweak penguin contributions have to be included because of the additional enhancement factor $\mathrm{Re}\,A_0/\,\mathrm{Re}\,A_2 = 22$ relative to QCD penguins. Moreover with increasing m_t the EW-penguins become increasingly important [55, 38] and entering ε'/ε with the opposite sign to QCD-penguins suppress this ratio for large m_t. For $m_t \approx 200\ GeV$ the ratio can even be zero [38]. This strong cancellations between these two contributions was one of the prime motivations for the NLO calculations performed in Munich and Rome. Although these calculations can be regarded as an important step towards a reliable theoretical prediction for ε'/ε the situation is clearly not satisfactory at present. Indeed ε'/ε is plagued with uncertainties related to non-perturbative B-factors which multiply m_t dependent functions in a formula like (25). Several of these B-factors can be extracted from leading CP-conserving $K \to \pi\pi$ decays [16]. Two important B-factors (B_6 = the dominant QCD penguin (Q_6) and B_8 = the dominant electroweak penguin (Q_8)) cannot be determined this way and one has to use lattice or $1/N$ methods to predict $\mathrm{Re}(\varepsilon'/\varepsilon)$.

An analytic formula for $\mathrm{Re}(\varepsilon'/\varepsilon)$ as a function of m_t, $\Lambda_{\overline{MS}}$, B_6, B_8, m_s and V_{CKM} can be found in [56]. A very simplified version of this formula is given as follows

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = 12\cdot 10^{-4}\left[\frac{\eta\lambda^5 A^2}{1.7\cdot 10^{-4}}\right]\left[\frac{150\ MeV}{\bar{m}_s(m_c)}\right]^2 \times\left[\frac{\Lambda^{(4)}_{\overline{\mathrm{MS}}}}{300\ MeV}\right]^{0.8}\left[B_6 - Z(x_t)B_8\right], \tag{37}$$

where $Z(x_t)$ is given in (41). Note the strong dependence on $\Lambda_{\overline{\mathrm{MS}}}$ pointed out in [16]. For $m_t = 170 \pm 13$ GeV and $\bar{m}_s(m_c) \approx 150 \pm 20$ MeV [57] and using ε_K-analysis to determine η one finds using the formulae in [16, 56] roughly

$$-1\cdot 10^{-4} \le \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) \le 15\cdot 10^{-4}\,, \tag{38}$$

if $B_6 = 1.0 \pm 0.2$ and $B_8 = 1.0 \pm 0.2$ are used. Such values are found in the $1/N$ approach [59] and using lattice methods: [60] and [60, 61] for B_6 and B_8 respectively. A very recent analysis of the Rome group [58] gives a smaller range, $\mathrm{Re}(\varepsilon'/\varepsilon) = (3.1 \pm 2.5)\cdot 10^{-4}$, which is however compatible with (38).

Similar results are found with hadronic matrix elements calculated in the chiral quark model [62]. However ε'/ε obtained in [63] is substantially larger and about $2 \cdot 10^{-3}$.

The experimental situation on $\mathrm{Re}(\varepsilon'/\varepsilon)$ is unclear at present. While the result of NA31 collaboration at CERN with $\mathrm{Re}(\varepsilon'/\varepsilon) = (23 \pm 7) \cdot 10^{-4}$ [65] clearly indicates direct CP violation, the value of E731 at Fermilab, $\mathrm{Re}(\varepsilon'/\varepsilon) = (7.4 \pm 5.9) \cdot 10^{-4}$ [65] is compatible with superweak theories [66] in which $\varepsilon'/\varepsilon = 0$. The E731 result is in the ball park of the theoretical estimates. The NA31 value appears a bit high compared to the range given in (38).

Hopefully, in about five years the experimental situation concerning ε'/ε will be clarified through the improved measurements by the two collaborations at the 10^{-4} level and by experiments at the Φ factory in Frascati. One should also hope that the theoretical situation of ε'/ε will improve by then to confront the new data.

4.6. $\Delta B = 1$ effective Hamiltonian

The effective Hamiltonian for $\Delta B = 1$ transitions involving operators $Q_1, \ldots Q_{10}$ (with corresponding changes of flavours) is also known including NLO corrections [16]. It has been used in the study of CP asymmetries in B-decays [67].

4.7. $K \to \pi^o e^+ e^-$

The effective Hamiltonian for $K \to \pi^0 e^+ e^-$ is given as follows:

$$\mathcal{H}_{\mathrm{eff}}(K \to \pi^0 e^+ e^-) =$$
$$\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left[\sum_{i=1}^{6,9V} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i + \tau y_{10A}(M_W) Q_{10A} \right], \quad (39)$$

where Q_{9V} and Q_{10A} are given by (23) with $\bar{b}s$ replaced by $\bar{s}d$.

Whereas in $K \to \pi\pi$ decays the CP violating contribution is a tiny part of the full amplitude and the direct CP violation is expected to be at least by three orders of magnitude smaller than the indirect CP violation, the corresponding hierarchies are very different for the rare decay $K_L \to \pi^0 e^+ e^-$. At lowest order in electroweak interactions (single photon, single Z-boson or double W-boson exchange), this decay takes place only if CP symmetry is violated [68]. Moreover, the direct CP violating contribution is predicted to be larger than the indirect one. The CP conserving contribution to the amplitude comes from a two photon exchange, which although higher order in α could in principle be sizable. The studies in [69, 70] indicate however that the CP conserving part is smaller than the direct CP violating contribution.

The size of the indirect CP violating contribution will be known once the CP conserving decay $K_S \to \pi^0 e^+ e^-$ has been measured [71]. On the other hand the direct CP violating contribution can be fully calculated as a function of m_t, CKM parameters and the QCD coupling constant α_s. There are practically no theoretical uncertainties related to hadronic matrix elements in this part, because the relevant matrix elements of the operators Q_{9V} and Q_{10A} can be extracted from the well-measured decay $K^+ \to \pi^0 e^+ \nu$.

Restricting the attention to the CP violating parts of the coefficients C_{9V} and C_{10V} and factoring out the relevant CKM factor as well as $\alpha/2\pi$ one finds [29]

$$\tilde{y}_{9V} = P_0 + \frac{Y(x_t)}{\sin^2\theta_W} - 4Z(x_t)\,, \qquad \tilde{y}_{10A} = -\frac{Y(x_t)}{\sin^2\theta_W}\,. \tag{40}$$

where, to a very good approximation for 140 GeV $\leq m_t \leq$ 230 GeV,

$$Y(x_t) = 0.315 \cdot x_t^{0.78}\,, \qquad Z(x_t) = 0.175 \cdot x_t^{0.93}\,. \tag{41}$$

The next-to-leading QCD corrections to P_0 have been calculated in [29] reducing certain ambiguities present in leading order analyses [72] and enhancing the leading order value typically from $P_0(LO) = 1.9$ to $P_0(NLO) = 3.0$ The final result for the branching ratio is given by

$$Br(K_L \to \pi^0 e^+ e^-)_{\rm dir} = 6.3 \cdot 10^{-6} (\mathrm{Im}\,\lambda_t)^2 \left(\tilde{y}_{7A}^2 + \tilde{y}_{7V}^2\right), \tag{42}$$

where $\mathrm{Im}\,\lambda_t = \mathrm{Im}(V_{td}V_{ts}^*)$. For $m_t = 170 \pm 10$ GeV one finds [29]

$$Br(K_L \to \pi^0 e^+ e^-)_{\rm dir} = (5. \pm 2.) \cdot 10^{-12}\,, \tag{43}$$

where the error comes dominantly from the uncertainties in the CKM parameters. This should be compared with the present estimates of the other two contributions: $Br(K_L \to \pi^o e^+ e^-)_{\rm indir} \leq 1.6 \cdot 10^{-12}$ and $Br(K_L \to \pi^o e^+ e^-)_{\rm cons} \approx (0.3 - 1.8) \cdot 10^{-12}$ for the indirect CP violating and the CP conserving contributions respectively [70]. Thus direct CP violation is expected to dominate this decay.

The present experimental bounds

$$Br(K_L \to \pi^0 e^+ e^-) \leq \begin{cases} 4.3 \cdot 10^{-9} & [73] \\ 5.5 \cdot 10^{-9} & [74] \end{cases} \tag{44}$$

are still by three orders of magnitude away from the theoretical expectations in the Standard Model. Yet the prospects of getting the required sensitivity of order 10^{-11}–10^{-12} in five years are encouraging [75].

4.8. $B \to X_s\gamma$

The effective Hamiltonian for $B \to X_s\gamma$ at scales $\mu = O(m_b)$ is given by

$$\mathcal{H}_{\rm eff}(b \to s\gamma) = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\sum_{i=1}^{6} C_i(\mu) Q_i + C_{7\gamma}(\mu) Q_{7\gamma} + C_{8G}(\mu) Q_{8G}\right], \tag{45}$$

where in view of $|V_{us}^* V_{ub}/V_{ts}^* V_{tb}| < 0.02$ we have neglected the term proportional to $V_{us}^* V_{ub}$.

The perturbative QCD effects are very important in this decay. They are known [76, 77] to enhance $B \to X_s\gamma$ in the SM by 2–3 times, depending on the top quark mass. Since the first analyses in [76, 77] a lot of progress has been made in calculating the QCD effects beginning with the work in [78, 79]. We will briefly summarize this progress.

A peculiar feature of the renormalization group analysis in $B \to X_s\gamma$ is that the mixing under infinite renormalization between the set $(Q_1 \ldots Q_6)$ and the operators $(Q_{7\gamma}, Q_{8G})$ vanishes at the one-loop level. Consequently in order to calculate the coefficients $C_{7\gamma}(\mu)$ and $C_{8G}(\mu)$ in the leading logarithmic approximation, two-loop calculations of $\mathcal{O}(eg_s^2)$ and $\mathcal{O}(g_s^3)$ are necessary. The corresponding NLO analysis requires the evaluation of the mixing in question at the three-loop level.

At present, the coefficients $C_{7\gamma}$ and C_{8G} are only known in the leading logarithmic approximation. However the peculiar feature of this decay mentioned above caused that the first fully correct calculation of the leading anomalous dimension matrix has been obtained only in 1993 [80, 81]. It has been confirmed subsequently in [82, 83, 30]. In order to extend these calculations beyond the leading order one would have to calculate $\hat{\gamma}_s^{(1)}$ and $O(\alpha_s)$ corrections to the initial conditions $\vec{C}(M_W)$. The 6×6 two-loop submatrix of $\hat{\gamma}_s^{(1)}$ involving the operators $Q_1 \ldots Q_6$ is the same as in Section 4.5. The two-loop mixing in the sector $(Q_{7\gamma}, Q_{8G})$ has been calculated only last year [19]. The three loop mixing between the set $(Q_1 \ldots Q_6)$ and the operators $(Q_{7\gamma}, Q_{8G})$ has not be done. The $O(\alpha_s)$ corrections to $C_{7\gamma}(M_W)$ and $C_{8G}(M_W)$ have been considered in [84]. Gluon corrections to the matrix elements of magnetic penguin operators have been calculated in [85, 86].

The leading logarithmic calculations [78, 81, 82, 30, 85, 87] can be summarized in a compact form, as follows:

$$\frac{Br(B \to X_s\gamma)}{Br(B \to X_c e\bar{\nu}_e)} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{QED}}{\pi f(z)} |C_{7\gamma}^{(0)\rm eff}(\mu)|^2, \tag{46}$$

where $C_{7\gamma}^{(0)\mathrm{eff}}(\mu)$ is the effective coefficient for which an analytic expression can be found in [87], $z = m_c/m_b$, and $f(z)$ is the phase space factor in the semileptonic b-decay. The expression given above is based on the spectator model corrected for short-distance QCD effects. Support for this approximation comes from the $1/m_b$ expansions. Indeed the spectator model has been shown to correspond to the leading order approximation in the $1/m_b$ expansion. The next corrections appear at the $\mathcal{O}(1/m_b^2)$ level. The latter terms have been studied by several authors [88, 89, 93] with the result that they affect $Br(B \to X_s\gamma\ )$ and $Br(B \to X_c e\bar{\nu}_e)$ by only a few percent.

A critical analysis of theoretical and experimental uncertainties present in the prediction for $Br(B \to X_s\gamma\ )$ based on the formula (46) has been made in [87] giving

$$Br(B \to X_s\gamma)_{TH} = (2.8 \pm 0.8) \times 10^{-4}. \tag{47}$$

where the error is dominated by the uncertainty in choice of the renormalization scale $m_b/2 < \mu < 2m_b$ as first stressed by Ali and Greub [85] and confirmed in [87]. Since $B \to X_s\gamma$ is dominated by QCD effects, it is not surprising that this scale-uncertainty in the leading order is particularly large.

The $B \to X_s\gamma$ decay has already been measured and as such appears to be the only unquestionable signal of penguin contributions! In 1993 CLEO reported [90] $Br(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. In 1994 first measurement of the inclusive rate has been presented by CLEO [91]:

$$Br(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \tag{48}$$

where the first error is statistical and the second is systematic. This result agrees with (47) very well although the theoretical and experimental errors should be decreased in the future in order to reach a definite conclusion and to see whether some contributions beyond the standard model such as present in the Two-Higgs-Doublet Model (2HDM) or in the Minimal Supersymmetric Standard Model (MSSM) are required. In any case the agreement of the theory with data is consistent with the large QCD enhancement of $B \to X_s\gamma$. Without this enhancement the theoretical prediction would be at least by a factor of 2 below the data.

Fig. 4 presents the SM prediction for the inclusive $B \to X_s\gamma$ branching ratio including the uncertainties discussed in [87] together with the CLEO results represented by the shaded regions. We stress that the theoretical result (the error bars) has been obtained prior to the experimental result. Since the theoretical error is dominated by scale ambiguities a complete NLO analysis is very desirable. Such a complete next-to-leading calculation of $B \to X_s\gamma$ is described in [87] in general terms. As demonstrated formally there the cancellation of the dominant μ-dependence in the leading order can be achieved by calculating the relevant two-loop matrix element of the dominant four-quark operator Q_2. This matrix element is however renormalization-scheme dependent and moreover mixing with other operators takes place. This scheme dependence can only be cancelled by calculating $\hat{\gamma}^{(1)}$ in the same renormalization scheme. This however requires the three loop mixing mentioned above.

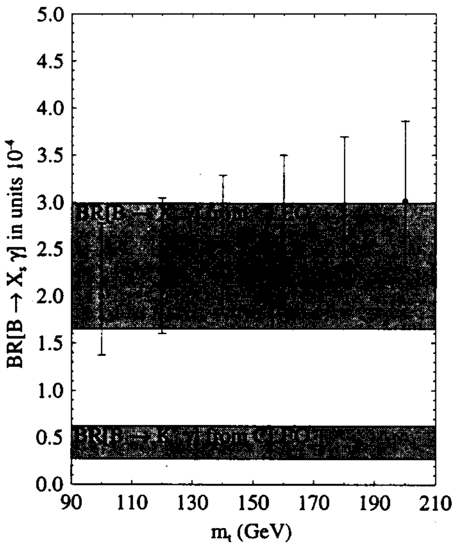

Fig. 4.

In this connections we would like to comment on an analysis of [92] in which the known two-loop mixing in the sector $(Q_1 \ldots Q_6)$ (see Table I) has been added to the leading order analysis of $B \to X_s\gamma$. Strong renormalization scheme dependence of the resulting branching ratio has been found, giving the branching ratio $(1.7\pm0.2)\cdot10^{-4}$ and $(2.3\pm0.3)\cdot10^{-4}$ at $\mu = 5$ GeV for HV and NDR schemes respectively. It has also been observed that whereas in the HV scheme the μ dependence has been weakened, it remained still strong in the NDR scheme. In our opinion the partial cancellation of the μ-dependence in the HV scheme is rather accidental and has nothing to do with the cancellation of the μ-dependence discussed above. The latter requires the evaluation of finite parts in two-loop matrix elements of the four-quark operators $(Q_1 \ldots Q_6)$. On the other hand the strong scheme dependence in the partial NLO analysis presented in [92] demonstrates very clearly the need for a full analysis. In view of this discussion we think that the decrease of the branching ratio for $B \to X_s\gamma$ relative to the LO prediction, found in [92] and given by $Br(B \to s\gamma) = (1.9 \pm 0.2 \pm 0.5) \cdot 10^{-4}$, is still premature and one should wait until the full NLO analysis has been done.

4.9. $B \to X_s e^+ e^-$ beyond leading logarithms

The effective Hamiltonian for $B \to X_s e^+ e^-$ at scales $\mu = O(m_b)$ is given by

$$\mathcal{H}_{\rm eff}(b \to s e^+ e^-) = \mathcal{H}_{\rm eff}(b \to s\gamma) - \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[C_{9V}(\mu) Q_{9V} + C_{10A}(M_W) Q_{10A}\right] , \quad (49)$$

where again we have neglected the term proportional to $V_{us}^* V_{ub}$ and Hamiltonian $\mathcal{H}_{\rm eff}(b \to s\gamma)$ is given in (45). In addition to the operators relevant for $B \to X_s \gamma$, there are two new operators Q_{9V} and Q_{10A} which appeared already in the decay $K_L \to \pi^0 e^+ e^-$ except for an appropriate change of quark flavours and the fact that now $\mu = \mathcal{O}(m_b)$ instead of $\mu = \mathcal{O}(1\ \text{GeV})$ should be considered. There is a large literature on this decay. In particular Hou *et al.* [93] stressed the strong dependence of $B \to X_s e^+ e^-$ on m_t. Further references to phenomenology can be found in [31]. Here we concentrate on QCD corrections.

The special feature of $C_{9V}(\mu)$ compared to the coefficients of the remaining operators contributing to $B \to X_s e^+ e^-$ is the large logarithm represented by $1/\alpha_s$ in P_0 in a formula like (40). Consequently the renormalization group improved perturbation theory for C_{9V} has the structure $\mathcal{O}(1/\alpha_s) + \mathcal{O}(1) + \mathcal{O}(\alpha_s) + \ldots$ whereas the corresponding series for the remaining coefficients is $\mathcal{O}(1) + \mathcal{O}(\alpha_s) + \ldots$. Therefore in order to find the next-to-leading $\mathcal{O}(1)$ term in the branching ratio for $B \to X_s e^+ e^-$, the full two-loop renormalization group analysis has to be performed in order to find C_{9V}, but the coefficients of the remaining operators should be taken in the leading logarithmic approximation. This is gratifying because, as we discussed above, the coefficients of the magnetic operators $Q_{7\gamma}$ and Q_{8G} are known only in the leading logarithmic approximation.

The coefficient $C_{9V}(\mu)$ has been calculated over the last years with increasing precision by several groups [94–96, 30] culminating in two complete next-to-leading QCD calculations [30, 31] which agree with each other. In particular in [31] the coefficient C_{9V} has been calculated in NDR and HV schemes. Calculating the matrix elements of the operators $(Q_1, \ldots Q_6)$ in the spectator model the scheme independence of the resulting physical amplitude has been demonstrated.

An extensive numerical analysis of the differential decay rate including NLO corrections has been presented in [31]. As an example we show in Fig. 5 the differential decay rate $R(\hat{s})$ divided by $\Gamma(B \to X_c e\bar{\nu})$ as a function of

$\hat{s} = (p_{e^+} + p_{e^-})^2/m_b^2$ for $m_t = 170$ GeV and $\Lambda_{\overline{MS}} = 225$ MeV. We observe that the QCD suppression in the leading order [94] is substantially weakened by the inclusion of NLO corrections. Similar result has been obtained by Misiak [30]. The $1/m_b^2$ corrections calculated in [97] enhance these results by roughly 10%.

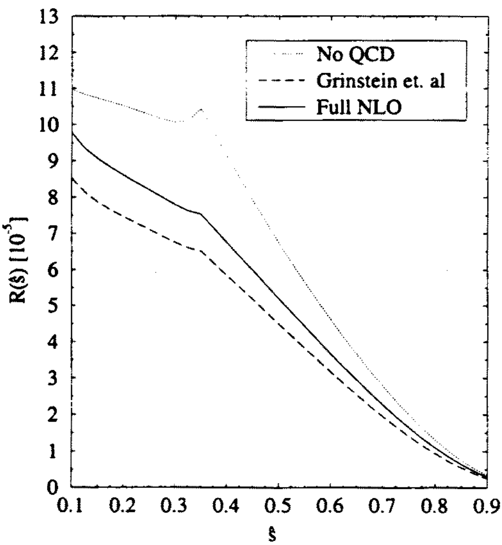

Fig. 5.

4.10. $K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \mu\bar{\mu}$, $B \to \mu\bar{\mu}$ and $B \to X_s\nu\bar{\nu}$

$K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ are the theoretically cleanest decays in the field of rare K-decays. Similarly $B \to \mu\bar{\mu}$ and $B \to X_s\nu\bar{\nu}$ are the theoretically cleanest decays in the field of rare B-decays. $K_L \to \pi^o\nu\bar{\nu}$, $B \to \mu\bar{\mu}$ and $B \to X_s\nu\bar{\nu}$ are dominated by short distance loop diagrams involving the top quark. $K^+ \to \pi^+\nu\bar{\nu}$ receives additional sizable contributions from internal charm exchanges. The decay $K_L \to \mu\bar{\mu}$ receives substantial long distance contributions and consequently suffers from large theoretical uncertainties. This is very unfortunate because this is the only rare Kaon decay which has already been measured. The most accurate is the measurement from Brookhaven [98]:

$$Br(K_L \to \bar{\mu}\mu) = (6.86 \pm 0.37) \cdot 10^{-9} \tag{50}$$

which is somewhat lower than the KEK-137 result: $(7.9 \pm 0.6 \pm 0.3) \cdot 10^{-9}$ [99]. For the short distance contribution I find using the formulae of [27]:

$$Br(K_L \to \bar{\mu}\mu)_{SD} = (1.5 \pm 0.8) \cdot 10^{-9}\,. \tag{51}$$

Details on this decay can be found in [98, 27]. More promising from theoretical point of view is the parity-violating asymmetry in $K^+ \to \pi^+\mu^+\mu^-$ [100, 28].

The next-to-leading QCD corrections to all these decays have been calculated in a series of papers by Buchalla and myself [25–28]. These calculations considerably reduced the theoretical uncertainties due to the choice of the renormalization scales present in the leading order expressions [101]. Since the relevant hadronic matrix elements of the weak currents entering $K \to \pi\nu\bar{\nu}$ can be measured in the leading decay $K^+ \to \pi^0 e^+ \nu$, the resulting theoretical expressions for Br($K_L \to \pi^o\nu\bar{\nu}$) and Br($K^+ \to \pi^+\nu\bar{\nu}$) are only functions of the CKM parameters, the QCD scale $\Lambda_{\overline{MS}}$ and the quark masses m_t and m_c. The long distance contributions to $K^+ \to \pi^+\nu\bar{\nu}$ have been considered in [102] and found to be very small: two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio. The long distance contributions to $K_L \to \pi^o\nu\bar{\nu}$ are negligible as well. Similar comments apply to $B \to \mu\bar{\mu}$ and $B \to X_s\nu\bar{\nu}$ except that $B \to \mu\bar{\mu}$ depends on the B-meson decay constant F_B which brings in the main theoretical uncertainty.

The explicit expressions for $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_L \to \pi^0\nu\bar{\nu})$ are given as follows

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = 4.64 \cdot 10^{-11}$$
$$\times \left[\left(\frac{\operatorname{Im}\lambda_t}{\lambda^5} X(x_t)\right)^2 + \left(\frac{\operatorname{Re}\lambda_c}{\lambda} P_0(K^+) + \frac{\operatorname{Re}\lambda_t}{\lambda^5} X(x_t)\right)^2\right], \quad (52)$$

$$Br(K_L \to \pi^0\nu\bar{\nu}) = 1.94 \cdot 10^{-10} \cdot \left(\frac{\operatorname{Im}\lambda_t}{\lambda^5} X(x_t)\right)^2, \quad (53)$$

Here

$$\operatorname{Im}\lambda_t = \eta A^2\lambda^5, \qquad \operatorname{Re}\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right) A^2\lambda^5(1-\bar{\varrho}), \quad (54)$$

and $\operatorname{Re}\lambda_c = -\lambda(1-\lambda^2/2)$. $X(x_t)$ is given to an excellent accuracy by

$$X(x_t) = 0.65 \cdot x_t^{0.575}, \quad (55)$$

where the NLO correction calculated in [26] is included if $m_t \equiv \bar{m}_t(m_t)$. Next $P_0(K^+) = 0.40 \pm 0.09$ [27, 113] is a function of m_c and $\Lambda_{\overline{MS}}$ and includes the residual uncertainty due to the renormalization scale μ. The absence of P_0 in (53) makes $K_L \to \pi^0\nu\bar{\nu}$ theoretically even cleaner than $K^+ \to \pi^+\nu\bar{\nu}$.

Similarly for $B_s \to \mu\bar{\mu}$ one has [26]

$$Br(B_s \to \mu\bar{\mu}) = 4.1 \cdot 10^{-9} \left[\frac{F_{B_s}}{230 \text{ MeV}}\right]^2 \left[\frac{\bar{m}_t(m_t)}{170 \text{ GeV}}\right]^{3.12} \left[\frac{|V_{ts}|}{0.040}\right]^2 \left[\frac{\tau_{B_s}}{1.6\text{ps}}\right]. \quad (56)$$

The impact of NLO calculations is best illustrated by giving the scale uncertainties in the leading order and after the inclusion of the next-to-leading corrections:

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) = (1.00 \pm 0.20) \cdot 10^{-10} \Rightarrow (1.00 \pm 0.05) \cdot 10^{-10}, \quad (57)$$

$$Br(K_L \to \pi^0 \nu\bar{\nu}) = (3.00 \pm 0.30) \cdot 10^{-11} \Rightarrow (3.00 \pm 0.04) \cdot 10^{-11}, \quad (58)$$

$$Br(B_s \to \mu\bar{\mu}) = (4.10 \pm 0.50) \cdot 10^{-9} \Rightarrow (4.10 \pm 0.05) \cdot 10^{-9}, \quad (59)$$

The reduction of the scale uncertainties is truly impressive.

The present experimental bound on $Br(K^+ \to \pi^+ \nu\bar{\nu})$ is $5.2 \cdot 10^{-9}$ [103]. An improvement by one order of magnitude is expected at AGS in Brookhaven for the coming years. The present upper bound on $Br(K_L \to \pi^0 \nu\bar{\nu})$ from Fermilab experiment E731 is 10^{-5}. FNAL-E799 expects to reach the accuracy $\mathcal{O}(10^{-8})$ and the future experiments at FNAL and KEK will hopefully be able to reach the standard model expectations. The latter are given for both decays at present as follows:

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) = (1.1 \pm 0.4) \cdot 10^{-10}, \; Br(K_L \to \pi^0 \nu\bar{\nu}) = (3.0 \pm 2.0) \cdot 10^{-11}. \quad (60)$$

5. Finalists

5.1. General remarks

From tree level K decays sensitive to V_{us} and tree level B decays sensitive to V_{cb} and V_{ub} we have [5]:

$$\lambda = 0.2205 \pm 0.0018, \qquad |V_{cb}| = 0.040 \pm 0.004 \;\Rightarrow\; A = 0.83 \pm 0.08, \quad (61)$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.08 \pm 0.03 \;\Rightarrow\; \sqrt{\varrho^2 + \eta^2} = 0.36 \pm 0.14, \quad (62)$$

where the error on $|V_{cb}|$ still remains a subject of intensive discussions [33, 104].

A large part in the errors quoted in (61) and (62) results from theoretical (hadronic) uncertainties discussed by Bigi and Mannel at this symposium. Consequently even if the data from CLEO II improves in the future, it is difficult to imagine at present that in the tree level B-decays a better accuracy than $\Delta |V_{cb}| = \pm 2 \cdot 10^{-3}$ and $\Delta |V_{ub}/V_{cb}| = \pm 0.01$ ($\Delta R_b = \pm 0.04$) could be achieved unless some dramatic improvements in the theory will take place.

The question then arises whether it is possible at all to determine the CKM parameters without any hadronic uncertainties. As demonstrated in

[105] this is indeed possible although it will require heroic experimental efforts. To this end one has to go to the loop induced decays or transitions which are fully governed by short distance physics and study simultaneously CP asymmetries in B-decays. In this manner clean and precise determinations of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|V_{td}|$, ϱ and η can be achieved. Since the relevant measurements will take place only in the next decade, what follows is really a 21-st century story.

It is known that many loop induced decays contain also hadronic uncertainties [106] related to long distance contributions or poorly known B_i factors. Examples are $B^0 - \bar{B}^0$ mixing, ε_K and ε'/ε discussed above. Let us in this connection recall the expectations from a "standard" analysis of the unitarity triangle (see Figs 1 and 2) which is based on ε_K, x_d giving the size of $B^0 - \bar{B}^0$ mixing, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ with the last two extracted from tree level decays. As a typical analysis [4] shows, even with optimistic assumptions about the theoretical and experimental errors it will be difficult to achieve the accuracy better than $\Delta\varrho = \pm 0.15$ and $\Delta\eta = \pm 0.05$ this way. More promising at least from the theoretical point of view are the following four:

- CP-Asymmetries in B^0-Decays
- $K_L \to \pi^0 \nu\bar{\nu}$
- $K^+ \to \pi^+ \nu\bar{\nu}$
- $(B_d^o - \bar{B}_d^o)/(B_s^o - \bar{B}_s^o)$

Let us summarize their main virtues one-by-one.

5.2. CP-asymmetries in B^0-decays

The CP-asymmetry in the decay $B_d^0 \to \psi K_S$ allows in the standard model a direct measurement of the angle β in the unitarity triangle without any theoretical uncertainties. This has been first pointed out by Bigi and Sanda [107], analyzed in detail already in [10] and during the past years discussed by many authors [108]. Similarly the decay $B_d^0 \to \pi^+\pi^-$ gives the angle α, although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions [109]. The determination of the angle γ from CP asymmetries in neutral B-decays is more difficult but not impossible [110]. Also charged B decays could be useful in this respect [111]. We have for instance

$$A_{\rm CP}(\psi K_S) = -\sin(2\beta)\frac{x_d}{1+x_d^2}, \qquad A_{\rm CP}(\pi^+\pi^-) = -\sin(2\alpha)\frac{x_d}{1+x_d^2}, \tag{63}$$

where we have neglected QCD penguins in $A_{\rm CP}(\pi^+\pi^-)$. Since in the usual unitarity triangle one side is known, it suffices to measure two angles to determine the triangle completely. This means that the measurements of $\sin 2\alpha$ and $\sin 2\beta$ can determine the parameters ϱ and η. The main virtues of this determination are as follows:

- No hadronic or $\Lambda_{\overline{MS}}$ uncertainties.
- No dependence on m_t and V_{cb} (or A).

As various analyses [4, 112, 58] of the unitarity triangle show, $\sin(2\beta)$ is expected to be large: $\sin(2\beta) \approx 0.6 \pm 0.2$. The predictions for $\sin(2\gamma)$ and $\sin(2\alpha)$ are very uncertain on the other hand.

5.3. $K_L \to \pi^o \nu\bar{\nu}$

As we have discussed above $K_L \to \pi^o \nu\bar{\nu}$ is the theoretically cleanest decay in the field of rare K-decays. Moreover it proceeds almost entirely through direct CP violation [114]. The next-to-leading QCD calculation [26] reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression to $\pm 1\%$. Since the long distance contributions to $K_L \to \pi^0 \nu\bar{\nu}$ are negligible, the resulting theoretical expression for Br($K_L \to \pi^0 \nu\bar{\nu}$) given by (see (53) and (55))

$$Br(K_L \to \pi^0 \nu\bar{\nu}) = 1.50 \cdot 10^{-5} \eta^2 \mid V_{cb} \mid^4 x_t^{1.15}, \tag{64}$$

is only a function of the CKM parameters and m_t. The main features of this decay are:

- No hadronic uncertainties
- $\Lambda_{\overline{MS}}$ and renormalization scale uncertainties at most $\pm 1\%$ [26].
- Strong dependence on m_t and V_{cb} (or A).

5.4. $K^+ \to \pi^+ \nu\bar{\nu}$

$K^+ \to \pi^+ \nu\bar{\nu}$ is CP conserving and receives contributions from both internal top and charm exchanges. The NLO corrections [27] to this decay reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression to $\pm 5\%$. $K^+ \to \pi^+ \nu\bar{\nu}$ is then the second best decay in the field of rare decays. Compared to $K_L \to \pi^0 \nu\bar{\nu}$ it receives additional uncertainties due to m_c and the related renormalization scale. Also its QCD scale dependence is stronger. The main features of this decay are:

- Hadronic uncertainties below 1% [102]
- $\Lambda_{\overline{MS}}$, m_c and renormalization scales uncertainties at most $\pm(5 - 10)\%$ [27].
- Strong dependence on m_t and V_{cb} (or A).

5.5. $(B_d^0 - \bar{B}_d^0)/(B_s^0 - \bar{B}_s^0)$

Measurement of $B_d^0 - \bar{B}_d^0$ mixing parametrized by x_d together with $B_s^0 - \bar{B}_s^0$ mixing parametrized by x_s allows to determine R_t:

$$R_t = \frac{1}{\sqrt{R_{ds}}}\sqrt{\frac{x_d}{x_s}}\frac{1}{\lambda}, \qquad R_{ds} = \frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{m_{B_d}}{m_{B_s}} \left[\frac{F_{B_d}\sqrt{B_{B_d}}}{F_{B_s}\sqrt{B_{B_s}}}\right]^2, \tag{65}$$

where R_{ds} summarizes SU(3) — flavour breaking effects. Note that m_t and V_{cb} dependences have been eliminated this way and R_{ds} contains much smaller theoretical uncertainties than the hadronic matrix elements in x_d and x_s separately. Provided x_d/x_s has been accurately measured a determination of R_t within $\pm 10\%$ should be possible. Indeed the most recent lattice result [50] gives $F_{B_d}/F_{B_s} = 1.22 \pm 0.04$. It would be useful to know B_{B_s}/B_{B_d} with similar precision. For $B_{B_s} = B_{B_d}$ I find $R_{ds} = 0.62 \pm 0.07$. Consequently rescaling the results of [4], obtained for $R_{ds} = 1$, the range $12 < x_s < 39$ follows. Such a large mixing will not be easy to measure. The main features of x_d/x_s are:

- No $\Lambda_{\overline{MS}}$, m_t and V_{cb} dependence.
- Hadronic uncertainty in SU(3) — flavour breaking effects of roughly $\pm 10\%$.

Because of the last feature, x_d/x_s cannot fully compete in the clean determination of CKM parameters with CP asymmetries in B-decays and with $K_L \to \pi^0\nu\bar{\nu}$. Although $K^+ \to \pi^+\nu\bar{\nu}$ has smaller hadronic uncertainties than x_d/x_s, its dependence on $\Lambda_{\overline{MS}}$ and m_c puts it in the same class as x_d/x_s [106].

5.6. $\sin(2\beta)$ from $K \to \pi\nu\bar{\nu}$

It has been pointed out in [115] that measurements of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_L \to \pi^0\nu\bar{\nu})$ could determine the unitarity triangle completely provided m_t and V_{cb} are known. In view of the strong dependence of these branching ratios on m_t and V_{cb} this determination is not precise however [113]. On the other hand it has been noticed [113] that the m_t and V_{cb} dependences drop out in the evaluation of $\sin(2\beta)$. Introducing the "reduced" branching ratios

$$B_+ = \frac{Br(K^+ \to \pi^+\nu\bar{\nu})}{4.64 \cdot 10^{-11}}, \qquad B_L = \frac{Br(K_L \to \pi^0\nu\bar{\nu})}{1.94 \cdot 10^{-10}}, \tag{66}$$

one finds

$$\sin(2\beta) = \frac{2r_s(B_+, B_L)}{1 + r_s^2(B_+, B_L)}, \tag{67}$$

where

$$r_s(B_+, B_L) = \frac{\sqrt{(B_+ - B_L)} - P_0(K^+)}{\sqrt{B_L}}, \tag{68}$$

so that $\sin(2\beta)$ does not depend on m_t and V_{cb}. Here $P_0(K^+) = 0.40 \pm 0.09$ [27, 113] is a function of m_c and $\Lambda_{\overline{MS}}$ and includes the residual uncertainty due to the renormalization scale μ. Consequently $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ offer a clean determination of $\sin(2\beta)$ which can be confronted with the one possible in $B^0 \to \psi K_S$ discussed above. Any difference in these two determinations would signal new physics. Choosing $Br(K^+ \to \pi^+\nu\bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ and $Br(K_L \to \pi^0\nu\bar{\nu}) = (2.5 \pm 0.25) \cdot 10^{-11}$, one finds [113]

$$\sin(2\beta) = 0.60 \pm 0.06 \pm 0.03 \pm 0.02\,, \tag{69}$$

where the first error is "experimental", the second represents the uncertainty in m_c and $\Lambda_{\overline{MS}}$ and the last is due to the residual renormalization scale uncertainties. This determination of $\sin(2\beta)$ is competitive with the one expected at the B-factories at the beginning of the next decade.

5.7. Precise determinations of the CKM matrix

Using the first two finalists and $\lambda = 0.2205 \pm 0.0018$ [116] it is possible to determine all the parameters of the CKM matrix without any hadronic uncertainties [105]. With $a \equiv \sin(2\alpha)$, $b \equiv \sin(2\beta)$ and $Br(K_L) \equiv Br(K_L \to \pi^0\nu\bar{\nu})$ one determines ϱ, η and $\mid V_{cb} \mid$ as follows [105]:

$$\bar{\varrho} = 1 - \bar{\eta} r_+(b)\,, \qquad \bar{\eta} = \frac{r_-(a) + r_+(b)}{1 + r_+^2(b)}\,, \tag{70}$$

$$\mid V_{cb} \mid = 0.039 \sqrt{\frac{0.39}{\eta}} \left[\frac{170 \text{ GeV}}{m_t}\right]^{0.575} \left[\frac{Br(K_L)}{3 \cdot 10^{-11}}\right]^{1/4}, \tag{71}$$

where

$$r_\pm(z) = \frac{1}{z}(1 \pm \sqrt{1 - z^2})\,, \qquad z = a, b\,. \tag{72}$$

We note that the weak dependence of $\mid V_{cb} \mid$ on $Br(K_L \to \pi^0\nu\bar{\nu})$ allows to achieve high accuracy for this CKM element even when $Br(K_L \to \pi^0\nu\bar{\nu})$ is not measured precisely.

As illustrative examples we consider in Table II three scenarios. The first four rows give the assumed input parameters and their experimental errors. The remaining rows give the results for selected parameters. Further results can be found in [105]. The accuracy in the scenario I should be achieved at B-factories, HERA-B, at FNAL and at KEK. Scenarios II and

III correspond potentially to B-physics at Fermilab during the Main Injector era and at LHC respectively. The experimental errors on $Br(K_L \to \pi^0 \nu\bar{\nu})$ to be achieved in the next 15 years are most probably unrealistic, but I show this exercise anyway in order to motivate this very challenging enterprise.

TABLE II

Determinations of various parameters in scenarios I-III

	Central	I	II	III
$\sin(2\alpha)$	0.40	± 0.08	± 0.04	± 0.02
$\sin(2\beta)$	0.70	± 0.06	± 0.02	± 0.01
m_t	170	± 5	± 3	± 3
$10^{11} Br(K_L)$	3	± 0.30	± 0.15	± 0.15
ϱ	0.072	± 0.040	± 0.016	± 0.008
η	0.389	± 0.044	± 0.016	± 0.008
$\mid V_{ub}/V_{cb} \mid$	0.087	± 0.010	± 0.003	± 0.002
$\mid V_{cb} \mid /10^{-3}$	39.2	± 3.9	± 1.7	± 1.3
$\mid V_{td} \mid /10^{-3}$	8.7	± 0.9	± 0.4	± 0.3
$\mid V_{cb} \mid /10^{-3}$	41.2	± 4.3	± 3.0	± 2.8
$\mid V_{td} \mid /10^{-3}$	9.1	± 0.9	± 0.6	± 0.6

Table II shows very clearly the potential of CP asymmetries in B-decays and of $K_L \to \pi^0 \nu\bar{\nu}$ in the determination of CKM parameters. It should be stressed that this high accuracy is not only achieved because of our assumptions about future experimental errors in the scenarios considered, but also because $\sin(2\alpha)$ is a very sensitive function of ϱ and η [4], $Br(K_L \to \pi^0 \nu\bar{\nu})$ depends strongly on $\mid V_{cb} \mid$ and most importantly because of the clean character of the quantities considered.

It is instructive to investigate whether the use of $K^+ \to \pi^+ \nu\bar{\nu}$ instead of $K_L \to \pi^0 \nu\bar{\nu}$ would also give interesting results for V_{cb} and V_{td}. After all $K^+ \to \pi^+ \nu\bar{\nu}$ will certainly be seen before $K_L \to \pi^0 \nu\bar{\nu}$. We again consider scenarios I-III with $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ for the scenario I and $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (1.0 \pm 0.05) \cdot 10^{-10}$ for scenarios II and III in place of $Br(K_L \to \pi^0 \nu\bar{\nu})$ with all other input parameters unchanged. An analytic formula for $\mid V_{cb} \mid$ can be found in [105]. The results for ϱ, η, and $\mid V_{ub}/V_{cb} \mid$ remain of course unchanged. In the last two rows of table 2 we show the results for $\mid V_{cb} \mid$ and $\mid V_{td} \mid$. We observe that due to the uncertainties present in the charm contribution to $K^+ \to \pi^+ \nu\bar{\nu}$, which was absent in $K_L \to \pi^0 \nu\bar{\nu}$, the determinations of $\mid V_{cb} \mid$ and $\mid V_{td} \mid$ are less accurate. If the uncertainties due to the charm mass and $\Lambda_{\overline{MS}}$ are removed one day this analysis will be improved [105].

An alternative strategy is to use the measured value of R_t instead of $\sin(2\alpha)$. Then (70) is replaced by

$$\bar{\varrho} = 1 - \bar{\eta} r_+(b), \qquad \bar{\eta} = \frac{R_t}{\sqrt{2}} \sqrt{b r_-(b)} \tag{73}$$

The result of this exercise is shown in Table III. Again the last two rows give the results when $K_L \to \pi^0 \nu\bar{\nu}$ is replaced by $K^+ \to \pi^+ \nu\bar{\nu}$. Although this determination of CKM parameters cannot fully compete with the previous one the consistency of both determinations will offer an important test of the standard model.

TABLE III

As in Table II but with $\sin(2\alpha)$ replaced by R_t.

	Central	I	II	III
R_t	1.00	± 0.10	± 0.05	± 0.03
$\sin(2\beta)$	0.70	± 0.06	± 0.02	± 0.01
m_t	170	± 5	± 3	± 3
$10^{11} Br(K_L)$	3	± 0.30	± 0.15	± 0.15
ϱ	0.076	± 0.111	± 0.053	± 0.031
η	0.388	± 0.079	± 0.033	± 0.019
$\mid V_{ub}/V_{cb} \mid$	0.087	± 0.014	± 0.005	± 0.003
$\mid V_{cb} \mid /10^{-3}$	39.3	± 5.7	± 2.6	± 1.8
$\mid V_{td} \mid /10^{-3}$	8.7	± 1.2	± 0.6	± 0.4
$\mid V_{cb} \mid /10^{-3}$	41.3	± 5.8	± 3.7	± 3.3
$\mid V_{td} \mid /10^{-3}$	9.1	± 1.3	± 0.8	± 0.7

Of particular interest will be the comparison of $\mid V_{cb} \mid$ determined as suggested here with the value of this CKM element extracted from tree level semi-leptonic B-decays. Since in contrast to $K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$, the tree-level decays are to an excellent approximation insensitive to any new physics contributions from very high energy scales, the comparison of these two determinations of $\mid V_{cb} \mid$ would be a good test of the standard model and of a possible physics beyond it.

Precise determinations of all CKM parameters without hadronic uncertainties along the lines presented here can only be realized if the measurements of CP asymmetries in B-decays and the measurements of $Br(K_L \to \pi^0 \nu\bar{\nu})$, $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and x_d/x_s can reach the desired accuracy. All efforts should be made to achieve this goal.

6. Final remarks

In this review we have discussed the most interesting quantities which when measured should have important impact on our understanding of the CP violation and of the quark mixing. We have discussed both CP violating and CP conserving loop induced decays because in the standard model CP violation and quark mixing are closely related.

In this review we have concentrated on rare decays and CP violation in the standard model. The structure of rare decays and of CP violation in extensions of the standard model may deviate from this picture. Consequently the situation in this field could turn out to be very different from the one presented here. However in order to distinguish the standard model predictions from the predictions of its extensions it is essential that the theoretical calculations reach acceptable precision. In this context we have emphasized the importance of the QCD calculations in rare and CP violating decays. During the recent years a considerable progress has been made in this field through the computation of NLO contributions to a large class of decays. This effort reduced considerably the theoretical uncertainties in the relevant formulae and thereby improved the determination of the CKM parameters to be achieved in future experiments. At the same time it should be stressed that whereas the theoretical status of QCD calculations for rare semileptonic decays like $K \to \pi\nu\bar{\nu}$, $B \to \mu\bar{\mu}$, $B \to X_s e^+e^-$ is fully satisfactory and the status of $B \to X_s\gamma$ should improve in the coming years, a lot remains to be done in a large class of non-leptonic decays or transitions where non-perturbative uncertainties remain sizable.

I would like to thank the organizers for inviting me to this symposium and for their great hospitality. The splendid birthday party "Chez Zalewskis" will never be forgotten. Finally I would like to thank all the members of the Munich-NLO-Club for the great time we had and still have together.

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