SEMILEPTONIC DECAYS OF HEAVY MESONS FROM AN MIT BAG MODEL*,**

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Using the (modified) MIT bag model we have calculated formfactors and decay widths for pseudoscalar–pseudoscalar and pseudoscalar–vector semileptonic decays of heavy mesons. A comparison with the experimental results and predictions of other models is also given.

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1. Introductory remarks

According to the bag paradigm hadrons are bubbles in the superdielectric medium (dielectric constant ~ 0) of the QCD vacuum. Inside the bubbles there are quarks, which are the fundamental constituents of particles. The MIT bag model [1, 2, 3] in the cavity approximation treats quarks as the free Dirac particles contained in a spherical bag of radius R. The quarks satisfy the Dirac equation inside the bag:

$$(\boldsymbol{\alpha p} + \beta m + V(r))\psi = E\psi , \tag{1}$$

where in the standard MIT bag $V(r < R) = 0$ and in the Coulomb-like version [4]

$$V(r < R) = -b\left(\frac{1}{r} - \frac{1}{R}\right) . \tag{2}$$

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The constant b is related to the strong coupling constant α_s. The solution of equation (1) has the form:

$$\psi_q^0(\boldsymbol{r},t) = \begin{bmatrix} iF(r)U \\ G(r)\boldsymbol{\sigma}\cdot\boldsymbol{r}U \end{bmatrix}, \tag{3}$$

where $F(r)$, $G(r)$ are proportional to the spherical Bessel functions in the case of the standard MIT bag model and to confluent hipergeometric functions in the Coulomb-like bag model. U are Pauli spinors. The momentum of the quarks is determined by the boundary condition:

$$i\gamma_\mu n^\mu \psi(R,t) = \psi(R,t), \tag{4}$$

where n^μ is a unit vector normal to the surface of the bag. The radius of the bag depends (in a rather complicated manner) on several quantities such as the kinetic energy of quarks, volume of the bag and also on chromoelectric and chromomagnetic energy of the interactions between quarks. Particularly interesting is the influence of the chromomagnetic energy on the bag radius in the case of the mesons. For the bags containing a heavy quark and a light antiquark the chromomagnetic energy is negligible for very large mass of the heavy quark. Due to this fact there is no difference between the pseudoscalar and the vector bag radii. The opposite statement is true in the case of the bags containing only light quarks. The pseudoscalar bag radius R_0 is smaller then the radius R_1 of the vector bag[1].

These facts have an important consequence for the semileptonic decay processes. In the case of the heavy quark decaying into another heavy one, the pseudoscalar–pseudoscalar and pseudoscalar–vector transitions are described by the same universal formfactor called the Isgur–Wise function [6]. In the case of the heavy quark decays into a light quark the formfactors in the point of zero momentum transfer are described by two different factors: $\mathcal{N}_1$ for $0^- \to 0^-$ transitions and $\mathcal{N}_2$ for $0^- \to 1^-$ transitions [7], where

$$\mathcal{N} = \langle \Phi_q | \Phi_Q \rangle \langle \Phi_{\bar{q}'} | \Phi_{\bar{q}} \rangle . \tag{5}$$

The first overlap in $\mathcal{N}$ is between the heavy parent quark and the light daughter quark. The second bracket is the overlap between the spectator antiquarks. This result is also true for the heavy to heavy transitions, but in this case $\mathcal{N}_1 \approx \mathcal{N}_2$.

[1] $R_0 \approx 3.4\ \mathrm{GeV}^{-1}$ and $R_1 \approx 4.4\ \mathrm{GeV}^{-1}$ [5].

2. Isgur–Wise function calculation

The Isgur–Wise function ξ can be written in the form [8]:

$$\xi(\omega) = \sqrt{\frac{2}{\omega+1}}\langle \Phi_{\bar{q}'}|\Phi_{\bar{q}}\rangle,$$

where ω is the product of four-velocities of mesons taking part in the decay. Choosing for calculations the modified Breit frame where the velocities of the initial and final meson are equal and opposite, the overlap between the spectators of the process in the MIT bag model can be written as the following integral [9, 10]:

$$\langle \Phi_{\bar{q}'}|\Phi_{\bar{q}}\rangle = \int\limits_{CB} \Phi^{\dagger}(L^{-1}_{-\boldsymbol{v}}(0,\vec{\boldsymbol{x}}))S^{\dagger}(\boldsymbol{v})S(-\boldsymbol{v})\Phi(L^{-1}_{-\boldsymbol{v}}(0,\vec{\boldsymbol{x}}))d^3\boldsymbol{x}\,, \tag{7}$$

where Φ is the wave function of the light spectator antiquark in the rest frame of the meson. Due to the heavy quark symmetries this wave function is the same for both mesons. $L^{-1}_{\pm\boldsymbol{v}}$ are Lorentz transformations connecting the rest frames of the particles with the modified Breit frame and $S(\pm\boldsymbol{v})$ are the boost matrices. Assuming that these matrices are hermitian one finds that $S(\boldsymbol{v})S(-\boldsymbol{v}) = 1$. The integration is performed over the bag contracted along the direction of motion by the factor R/γ. After a simple calculation we arrive at the formula:

$$\xi(\omega) = \left(\frac{2}{\omega+1}\right) \int\limits_{K(0,R)} d^3x\rho(r)j_0\left(2E_q\sqrt{\frac{\omega-1}{\omega+1}}r\right), \tag{8}$$

where $\rho(r)$ is the light quark density in the meson calculated from the wave function (3). E_q is the energy of the light quark. The integration is performed over the spherical bag from the rest frame of the meson.

TABLE I

Comparison of the numerical results for ρ^2 parameter with other models and with experimental data.

Model	ρ^2_B	$\rho^2_{B_s}$
MIT1 [9]	1.239	1.625
MIT2 [10]	0.98	1.135
Lattice UKQCD [12]	$0.9^{+2}_{-2}{}^{+4}_{-2}$	$1.2^{+2}_{-2}{}^{+2}_{-1}$
Lattice BSS [13]	—	1.24(26)(26)
Relat. quark model [14]	1.40	1.64
Exp. [15]	$1.01 \pm 0.15 \pm 0.09$	—

From formula (8) we can evaluate the slope parameter ρ^2:

$$\rho^2 = \tfrac{1}{2} + \tfrac{1}{3}E_q^2\langle r^2\rangle\,, \tag{9}$$

from which we find immediately the lower bound on the slope:

$$\rho^2 \geq \tfrac{1}{2}\,, \tag{10}$$

which is stronger then the well known Bjorken limit. This knew limit appears as a consequence of the trivial kinematical factor from equation (6) and of relativistic corrections coming from contraction of the bag. The numerical values of the slope parameters for the B and B_s semileptonic decays are collected in Table I and compared there with other models and with experimental results[2]. According to the MIT bag model the slope parameter is an increasing function of the mass of the light spectator antiquark, in agreement with lattice calculations, but in contrary to the predictions of paper [16].

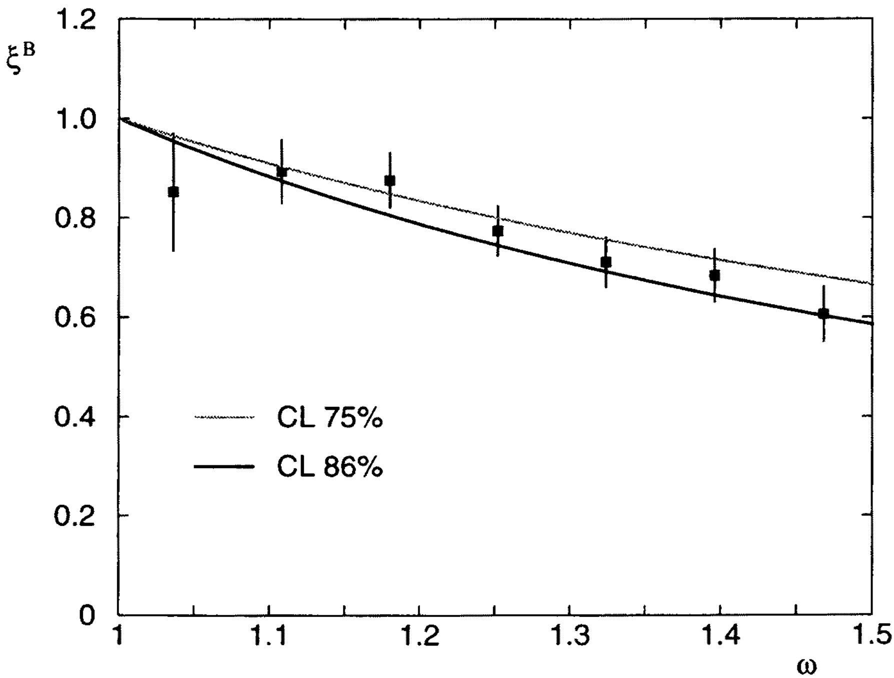

Fig. 1. Comparison of the Isgur–Wise function from the MIT bag model (black curve) and Coulomb-like bag model (gray curve) with the CLEO II [18] data.

[2] Masses of the light quarks are: $m_u = m_d = 0$ and $m_s = 0.291$ MeV [11].

Instead of using the rather cumbersome formula (8), we approximate our numerical result by the function obtained in paper [17]. The deviation of this approximation from the exact result is less then one per cent, in the range $1 \leq \omega \leq 3$ for both $\overline{B} \to D^{(*)} l\overline{\nu}$:

$$\xi^{(B)}(\omega) = \left(\frac{2}{\omega+1}\right)^{2+\frac{0.6}{\omega}} \quad [9], \quad \xi^{(B)}(\omega) = \left(\frac{2}{\omega+1}\right)^{1.52+\frac{0.45}{\omega}} \quad [10],$$

and for $\overline{B}_s \to D_s^{(*)} l\overline{\nu}$:

$$\xi^{B_s}(\omega) = \left(\frac{2}{\omega+1}\right)^{2.7+\frac{0.6}{\omega}} \quad [9], \quad \xi^{B_s}(\omega) = \left(\frac{2}{\omega+1}\right)^{1.67+\frac{0.59}{\omega}} \quad [10].$$

The agreement of these results with the CLEOII data is very good as shown in the Fig. 1.

3. Heavy to light decays

In the case of pseudoscalar–pseudoscalar transitions the matrix elements of the hadronic currents can be described in terms of two invariant formfactors:

$$\langle P(p')|V_\mu|H(p)\rangle = \frac{1}{2\sqrt{mm'}} \left(f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu\right) , \quad (11)$$

where P stands for a light pseudoscalar (π, K, η) and H for a heavy one (D, B, D_s, B_s), m', m are the masses of the P and H mesons respectively, p' and p are their four - momenta. In the case of the pseudoscalar–vector transitions we have four independent formfactors [19]:

$$\langle V(\varepsilon,p')|V_\mu|H(p)\rangle = \frac{i}{2\sqrt{mm'}} \frac{g(q^2)}{m+m'} \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}(p+p')^\rho(p-p')^\sigma , \quad (12)$$

$$\langle V(\varepsilon,p')|A_\mu|H(p)\rangle = \frac{1}{2\sqrt{mm'}} \Big[(m+m')f(q^2)\varepsilon^*_\mu + \frac{a_+(q^2)}{m+m'}(\varepsilon^*\cdot p)(p+p')_\mu + \frac{a_-(q^2)}{m+m'}(\varepsilon^*\cdot p)(p-p')_\mu\Big] , \quad (13)$$

where $V(\varepsilon, p')$ describes the vector particle (ρ, K^*, ϕ) with momentum p' and polarization ε_μ. For vanishing electron mass the formfactors a_-, f_-

do not contribute to the decay probabilities. The matrix elements of the hadronic current in the bag model can be found in [20]:

$$\langle X(p')|J_\mu|H(p)\rangle$$
$$= \langle X(p')| \int\limits_{\mathrm{Bag}} d^3\boldsymbol{r} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \bar{\psi}_q(\boldsymbol{r},t)\gamma_\mu(1-\gamma_5)\psi_Q(\boldsymbol{r},t)|H(p)\rangle|_{t=0} \cdot \langle\psi_{\bar{q}'}|\psi_{\bar{q}}\rangle|_{t=0}\,, \tag{14}$$

where ψ_q and ψ_Q are spinors describing respectively the final light quark and the initial heavy quark. The bracket at the end of the formula is the overlap function of the spectator antiquarks in the parent ($\bar{q}$) and the daughter ($\bar{q}'$) particle. This overlap corresponds to the Isgur–Wise function in the case of the heavy to heavy meson transitions. The vector $\boldsymbol{k} = \boldsymbol{p} - \boldsymbol{p}'$ is the momentum transfer in the process. The integral is over the whole space filled by the (anti)quark fields at the moment of the decay. In the case of the bag model this is the intersection of the bags of the parent and daughter particles. The moment of the decay is arbitrary and is here chosen to be zero. The integral must be performed in some well defined reference frame and in principle should give us the full dependence of the formfactors in terms of the momentum transfer $\boldsymbol{k}$. Unfortunately, the prescription is not covariant because we do not know the boost operators acting on the quark fields. Such operators are necessary, because the spinors ψ are known only in the reference frame in which the bag is at rest. Generally in the case of the decay it is not possible to find a frame where both bags are at rest. For this reason only the calculation for small values of $\boldsymbol{k}$ is reliable. Expanding the right hand side of equation (14) in powers of $|\boldsymbol{k}|$ and comparing with (11)–(13), one can find the formfactors in the point of maximum four - momentum transfer q^2_{max}, where $\boldsymbol{k} = 0$. The details of these calculations can be found in papers [7, 19]. Performing the calculation in the modified Breit frame in the infinite heavy quark mass limit we get [7]:

$$f_+(q^2_{\mathrm{max}}) = \frac{m+m'}{2\sqrt{mm'}}\mathcal{N}_0 \tag{15}$$

for pseudoscalar–pseudoscalar transitions and

$$-a_+(q^2_{\mathrm{max}}) = g(q^2_{\mathrm{max}}) = \frac{m+m'}{2\sqrt{mm'}}\mathcal{N}_1\,, \tag{16}$$

$$f(q^2_{\mathrm{max}}) = \frac{2\sqrt{mm'}}{m+m'}\mathcal{N}_1\,, \tag{17}$$

for pseudoscalar–vector transitions, where $\mathcal{N}$ is given by equation (5). This result is very easily understandable. In the point of zero momentum transfer the heavy quark at rest decays into the light quark at rest — this process is described by the first overlap in $\mathcal{N}$. In the same time the spectator antiquark in the heavy meson changes into the antiquark in the light meson — this process is described by the second overlap in $\mathcal{N}$. Other factors in (15)–(17) are of purely kinematical origin. The spectator overlap corresponds in the case of heavy to heavy transitions to the Isgur–Wise function and can be calculated as described in the previous section[3]. The heavy to light overlap can be calculated as follows. The heavy quark occupies the center of the bag so the overlap has to be proportional to the light quark wave function at the origin. For dimensional reasons we have to introduce an additional constant μ with the dimension of mass to the power 3/2. This constant depends on some feature of the light quark. Taking all this into account we get:

$$\mathcal{N} = \text{const.}\frac{F_q^*(0)}{\mu^{3/2}}\langle\psi_{\bar{q}'}|\psi_{\bar{q}}\rangle\,, \tag{18}$$

where F_q is the upper component of the wave function (3). This function feels the difference of radii between the pseudoscalar and the vector bags due to normalization of the whole wave function to unity inside the bag. Therefore, we used in equations (15) – (17) two different symbols: $\mathcal{N}_1$ and $\mathcal{N}_2$ that distinguish between pseudoscalar–pseudoscalar and pseudoscalar–vector transitions.

The strategy of our calculation is standard. At first we calculated the values of the formfactors in the point of maximum four-momentum transfer $q^2_{\rm max}$ and then we assume the pole dependence of the formfactor:

$$f(q^2) = \frac{f(0)}{1 - q^2/m^{*2}}\,, \tag{19}$$

where the m^* are: 2.01 GeV for g, f_+ and 2.42 GeV [21] for f, a_+ for $c \to q$ $(q = u, d)$ transitions, and 2.11 GeV for g, f_+ and 2.53 GeV [21] for f, a_+ for $c \to s$. The assumption about pole dominance for estimation of the formfactors is frequently used in phenomenological computations and for several of the D decays it was also confirmed by lattice calculations (*e.g.* [22]). For pseudoscalar–pseudoscalar transitions this assumption has also some theoretical justification [23]. Using this assumption we know the formfactors up to the unknown factor const. $\mu^{-3/2}$, but we have already got the possibility to calculate quantities, for which this factor cancels. These quantities are: the polarization factors Γ_L/Γ_T and the ratios of the

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[3] This overlap can be calculated even in the full kinematical region.

decay widths between pseudoscalar–vector and pseudoscalar–pseudoscalar transitions. For example for:

$$R_{DK}^{DK^*} \equiv \frac{\Gamma(D \to K^* e\nu_e)}{\Gamma(D \to K e\nu_e)} . \tag{20}$$

The bag model has some parameters that were adopted from paper [5], in which they had been fitted to the spectroscopy of the light and heavy particles. Using this parameters we get [7] $R_{DK}^{DK^*} = 0.6$ and for decay $D \to K^*$ $\Gamma_L/\Gamma_T = 1.06$ to compare with experimental results [24]: $R_{DK}^{DK^*} = 0.57 \pm 0.08$ and $\Gamma_L/\Gamma_T = 1.15 \pm 0.17$. The agreement is very good. It is important to stress that if we assume that $\mathcal{N}_0 \approx \mathcal{N}_1$, as it is in the case of the heavy to heavy transitions, then we get the $R_{DK}^{DK^*}$ parameter close to unity. More numerical predictions can be found in paper [7].

4. Conclusion

The analysis of the semileptonic decays within the framework of the MIT bag model give us a very good predictions for the Isgur–Wise function. For small recoils we have also found a new lower bound for the slope parameter of this function. It results from the relativistic corrections to the moving bag. In the case of the heavy to light decays we explain the suppression of pseudoscalar–vector transition $D \to K^*$ relative to the pseudoscalar–pseudoscalar transition $D \to K$ as an effect of the colour magnetic interaction between the produced light quark and the antiquark spectator. We have also found the relations between various formfactors in the point of zero momentum transfer.

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