CURRENT ALGEBRA AND FLAVOUR SYMMETRY BREAKING*

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Quark-level symmetry properties of the pole model of nonleptonic baryon decays are discussed. The obtained expressions generalize current algebra prescriptions to the flavour-symmetry-breaking case. This generalization permits a natural explanation of a several-decade-old puzzle in nonleptonic hyperon decays and provides an understanding of differences between various contemporary models of nonleptonic decays of charmed baryons.

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1. Introduction

Over the last several years a lot of attention has been devoted to theoretical studies of hadrons containing heavy quarks (see Ref. [1]). These studies have been concerned mainly with the meson sector since data on heavy flavour baryons were quite scarce. Recently, however, more and more higher statistics data on charmed baryons and, in particular, on their nonleptonic decays have become available. The data do not allow a clear discrimination between different theoretical models for these decays as yet but such a possibility is already on the horizon. It is therefore of great interest to try to understand the theoretical origins of differences between predictions of the competing approaches. When experimental data become more complete, a clear understanding of their theoretical implications will be at hand.

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The main two theoretical schemes followed are:

(1) current algebra: a strictly quark-level approach in which hadrons appear in the initial and final states only [2], and

(2) pole model (in which amplitudes are constructed from hadron-level building blocks, with the quark model used to determine the structure of these blocks) [3]. Both schemes are usually supplemented with factorization contributions. Predictions of these two approaches differ considerably.

The standard pole model is simple and straightforward in principle but tedious in application: one has to consider many intermediate states and calculate all the relevant weak transitions and strong decay amplitudes involving these states. For parity violating amplitudes this means that calculations must involve negative-parity excited baryons for which sufficient experimental information is not available.

The strict quark-level approach based on current algebra (CA) seems to get rid of such uncertainties by using the closure property: the sum over many intermediate baryonic poles is replaced by the sum over three-quark states permitting a straightforward consideration of quark-level symmetries [2, 4]. As we shall see, the application of this simplifying method to the description of parity violating amplitudes requires, however, that flavour symmetry breaking is small when compared to the excitation energy of negative parity ($L = 1$) baryons. For charmed baryons, flavour symmetry breaking is measured by $\delta c \approx m_c - m_{u,d,s} \approx 1.1$ GeV, while the excitation energy between the $L = 1$ and $L = 0$ baryons is around $\Delta\omega \approx 470$ MeV. Clearly, we are far from the region where simple symmetry considerations inherent in current algebra are applicable. Even for hyperon decays, where the relevant numbers are $\delta s \approx 190$ MeV, $\Delta\omega \approx 570$ MeV, one may expect corrections to the CA predictions of order $\delta s/\Delta\omega \approx 30\%$ (see Ref. [5]).

Although flavour symmetry breaking is duly taken into account in the pole model [3, 5], the main drawback of that model is the lack of simplicity and clarity resulting from the need to perform explicit summation of contributions from several intermediate states. If it were possible to maintain flavour symmetry breaking in the intermediate states that is characteristic of the pole model, and — at the same time — to use closure property to replace the sum over all the intermediate hadron states by the much simpler quark-level prescriptions, the simplicity of such a scheme would allow a much better understanding of similarities and differences between the quark- and hadron-level approaches. Furthermore, in such a simple approach, it might be possible to see which assumptions of the model should be modified to yield explanation of the up-to-now not understood properties of weak nonleptonic decays of baryons. Such a simple scheme connecting the purely hadron-level pole model with the current algebra/quark-level approach has been recently developed [4, 6, 7].

2. Hyperon decays

Let us first turn our attention to parity violating amplitudes in hyperon nonleptonic decays. In the standard baryon pole model for these decays, one has to sum the contributions of all excited $\frac{1}{2}^-$ baryons B^* from the $(70, 1^-)$ multiplet of SU(6)⊗O(3) appearing in the intermediate states between the action of the weak Hamiltonian and the strong decay (see Fig. 1). For the

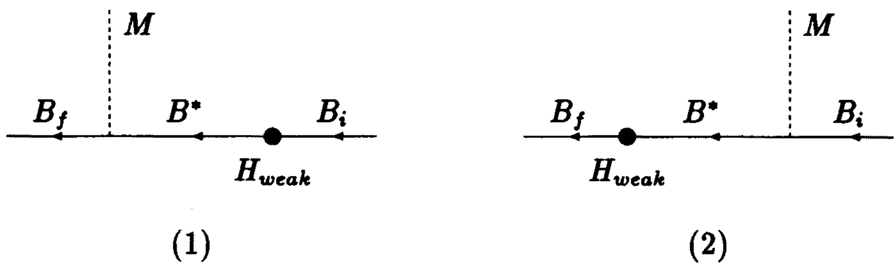

Fig. 1. Baryon-pole diagrams for weak decays.

sake of definiteness, let us consider the $\Sigma^+ \to p\pi^0$ process. Upon using the PCAC relation between the pion field and the divergence of axial current, the pole-model calculation of the S-wave $A(\Sigma^+ \to p\pi^0)$ parity violating amplitude involves consideration of the expressions

$$A_{(1)}(\Sigma^+ \to p\pi^0) = \sum_{N^*} \frac{\langle p|\partial_\mu A_\mu^{(0)}|N^*\rangle\langle N^*|H_{\text{weak}}^{\text{p.viol.}}|\Sigma^+\rangle}{\Delta\omega_{W_1}}, \tag{1}$$

and

$$A_{(2)}(\Sigma^+ \to p\pi^0) = \sum_{\Sigma^*} \frac{\langle p|H_{\text{weak}}^{\text{p.viol.}}|\Sigma^*\rangle\langle \Sigma^*|\partial_\mu A_\mu^{(0)}|\Sigma^+\rangle}{\Delta\omega_{W_2}} \tag{2}$$

corresponding in Fig. 1 to diagrams (1) and (2), respectively. [We are ignoring uninteresting common factors such as $1/f_\pi$, i, *etc.* here and in the following.] Energy denominators $\Delta\omega_{W_1}$, $\Delta\omega_{W_2}$ have subscripts W_1, W_2 since they correspond to energy difference "across" the weak interaction:

$$\Delta\omega_{W_1} = N^* - \Sigma\,, \tag{3}$$

$$\Delta\omega_{W_2} = \Sigma^* - p \tag{4}$$

(the symbol of a particle stands for its energy). Since the matrix elements of the spatial components (A_k) of the axial vector current between $\langle p|$ and $|N^*\rangle$ ($\langle\Sigma^*|$ and $|\Sigma^+\rangle$) vanish [5], we have

$$\begin{aligned}\frac{1}{i}\langle p|\partial_\mu A_\mu|N^*\rangle &= \Delta\omega_s\langle p|A_0|N^*\rangle\,,\\ \frac{1}{i}\langle\Sigma^*|\partial_\mu A_\mu|\Sigma^+\rangle &= -\Delta\omega_s\langle\Sigma^*|A_0|\Sigma^+\rangle\,,\end{aligned} \tag{5}$$

where the subscript 's' means that we are dealing with the baryon energy difference "across" the strong vertex.

When SU(3) is broken we have

$$\begin{aligned}\Delta\omega_{W_1} &\equiv N^* - \Sigma = \Delta\omega_s - \delta s\,,\\ \Delta\omega_{W_2} &\equiv \Sigma^* - p = \Delta\omega_s + \delta s\,,\end{aligned} \tag{6}$$

and we obtain:

$$\begin{aligned}A_{(1)}(\Sigma^+ \to p\pi^0) &= \frac{\Delta\omega_s}{\Delta\omega_s - \delta s}\sum_{N^*}\langle p|A_0|N^*\rangle\langle N^*|H_{\text{weak}}^{\text{p.viol.}}|\Sigma^+\rangle\\ A_{(2)}(\Sigma^+ \to p\pi^0) &= -\frac{\Delta\omega_s}{\Delta\omega_s + \delta s}\sum_{\Sigma^*}\langle p|H_{\text{weak}}^{\text{p.viol.}}|\Sigma^*\rangle\langle\Sigma^*|A_0|\Sigma^+\rangle\,.\end{aligned} \tag{7}$$

Having expressed energy denominators by factors that are identical for all diagrams of type (1) (all diagrams of type (2)) as given in the first (second) line of Eq. (7), one can now use closure to perform summation over intermediate states in Eq. (7).

In this way for the total amplitude $A(\Sigma \to p\pi^0)$ one obtains

$$\begin{aligned}A &= A_{(1)} + A_{(2)}\\ &= \tfrac{1}{1-x}\langle p|A^0 H_{\text{weak}}^{\text{p.viol.}}|\Sigma^+\rangle - \tfrac{1}{1+x}\langle p|H_{\text{weak}}^{\text{p.viol.}}A^0|\Sigma^+\rangle\,,\end{aligned} \tag{8}$$

where $x = \delta s/\Delta\omega_s \approx 1/3$.

If SU(3) symmetry breaking were negligible, we would put $x = 0$ in Eq. (8) to obtain the standard commutator of current algebra. Since x is significantly different from zero, substantial corrections to the CA results are expected.

Matrix element $\langle p|A^0 H_{\text{weak}}^{\text{p.viol.}}|\Sigma^+\rangle$, ($\langle p|H_{\text{weak}}^{\text{p.viol.}}A^0|\Sigma^+\rangle$) may be evaluated in the quark-diagram language by considering diagrams $b1, c1$ (respectively $b2, c2$) from Fig. 2 (see Ref. [8]). The relative size of reduced matrix elements b and c corresponding to diagrams ($b1$) and ($c1$) respectively (or to ($b2$) and ($c2$)) depends on dynamics. The c/b ratio may be estimated

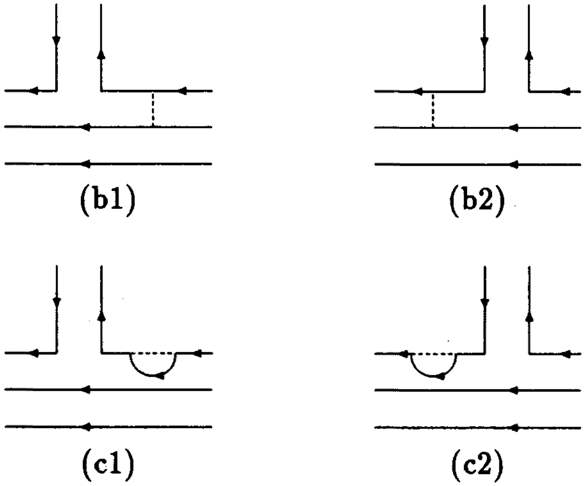

Fig. 2. Quark diagrams for weak decays.

by considering the limit of SU(3) symmetry ($x = 0$) when the right-hand side of Eq. (8) reduces to a commutator. By use of equality $[H^{\text{p.viol.}}_{\text{weak}}, A^0] = [H^{\text{p.cons.}}_{\text{weak}}, V^0]$, the parity violating amplitudes of Eq. (8) may be related to matrix element $\langle p|H^{\text{p.cons.}}_{\text{weak}}|\Sigma^+\rangle$ of the parity-*conserving* part of the weak Hamiltonian. The contribution of W-exchange diagrams (reduced matrix element b_0) and W-loop diagrams (reduced matrix element c_0) to the latter matrix element is known from experimental knowledge of parity conserving amplitudes (neglecting meson-pole or factorization contribution [10]), and we have in the SU(3)-limit $c/b = c_0/b_0 \equiv \frac{3}{2}(\frac{f_0}{d_0} + 1) = \frac{3}{2}(-1.8 + 1) = -1.2$, where f_0, d_0 are the familiar SU(3) invariant couplings parametrizing the $\langle B|H^{\text{p.cons.}}_{\text{weak}}|B'\rangle$ matrix element:

$$\langle B|H^{\text{p.cons.}}_{\text{weak}}|B'\rangle = f_0 Tr(S[B', B^\dagger]) + d_0 Tr(S\{B', B^\dagger\}) . \tag{9}$$

Thus, in the general SU(3) symmetry breaking case we obtain

$$A(\Sigma^+ \to p\pi^0) = \frac{1}{1-x}\left(-\frac{1}{6\sqrt{2}}c_0\right) - \frac{1}{1+x}\left(\frac{1}{2\sqrt{2}}b_0\right) . \tag{10}$$

Consequently, the contribution from c- (b-) type diagram is enhanced by factor $\frac{1}{1-x}$ (respectively suppressed by factor $\frac{1}{1+x}$). One calculates [6] that all the remaining measurable amplitudes are modified in the above way. Thus, the f/d ratio characterizing the parity violating amplitudes differs from the f_0/d_0 ratio relevant for parity conserving amplitudes:

$$\frac{f}{d} \equiv -1 + \frac{2}{3}\frac{c}{b} = -1 + \frac{2}{3}\frac{1+x}{1-x}\frac{c_0}{b_0}$$
$$\approx -1 + \frac{2}{3}\frac{\frac{4}{3}}{\frac{2}{3}}(-1.2) = -2.6\,. \tag{11}$$

Eq. (11) is in excellent agreement with experiment: $(f/d)_{\rm exp} \approx -2.5$. Our simple treatment of the contribution of intermediate $\frac{1}{2}^-$ states leads therefore to a very gratifying result: an explanation of the up-to-now not understood discrepancy between the f/d ratios for parity conserving and parity violating amplitudes [9, 10, 11]. In this explanation, deviations of these ratios from the CA (valence quark model) prediction of -1 are related, and the importance of the c-type diagrams describing the effects of sea quarks [12, 13] is demonstrated. Armed with the success of the explanation of this several-decade-old puzzle in hyperon nonleptonic decays we proceed with more confidence to the problem of charmed baryon decays.

3. Charmed baryon decays

As mentioned above, there are two main theoretical approaches to the problem of weak nonleptonic decays of charmed baryons. The first, based on quark diagrams and symmetry principles was originally attempted by Körner and collaborators [2]. The other, followed by Cheng and Tseng consists in carrying out explicit calculations in the framework of the pole model [3]. The two approaches differ in their predictions for both parity conserving and parity violating amplitudes. In Ref. [4] it was shown that for the parity conserving amplitudes the approach of Körner does not take into account the contribution from intermediate ground-state baryons, which — apart from meson pole or factorization contribution — constitutes the main contribution in the pole model. If propagation of ground-state baryons in the intermediate state is to be taken into account in the quark-diagram scheme, the spin-flavour factors corresponding to the quark-line diagrams $(b1), (c1)$ and $(b2), (c2)$ from Fig. 2 are to be subtracted due to the presence of energy denominators which are of opposite sign for diagrams of types (1) and (2). Similar effects are expected for parity violating amplitudes: the presence of energy denominators and the breaking of SU(4) symmetry will affect the way in which contributions from diagrams of types (1) and (2) are to be combined. For Cabibbo-allowed nonleptonic charmed-baryon decays there is no contribution from quark-level diagrams of type (c) (Fig. 2). The general structure of pole contributions from diagrams (1) and (2) in Fig. 1 is then

$$\frac{b_1}{B_s^* - B_c} + \frac{b_2}{B_c^* - B_s}\,, \tag{12}$$

where b_1, b_2 are spin-flavour factors that are most easily calculated by considering diagrams ($b1$) and ($b2$) of the quark-level approach (Fig. 2). In energy denominators of Eq. (12), $B_{c(s)}$ denotes energy of ground-state baryon containing one charmed quark (a strange quark in its place). Similar notation is adopted for $\frac{1}{2}^-$ excited baryons B^*_c, B^*_s. Energy denominators in Eq. (12) may be approximated by:

$$\begin{aligned} B^*_c - B_s &= \Delta\omega + \delta c \,, \\ B^*_s - B_c &= \Delta\omega - \delta c \,, \end{aligned} \tag{13}$$

where $\Delta\omega \approx 470$ MeV is the excitation energy of the $L = 1$ baryons with respect to the ground state, and $\delta c \approx m_c - m_{s,u,d} \approx 1.1$ GeV is the SU(4) breaking parameter. Eq. (13) may then be rewritten as

$$\frac{1}{\Delta\omega}\left\{(b_1 + b_2) + (b_1 - b_2)\frac{\delta c}{\Delta\omega}\right\}\frac{1}{1 - (\frac{\delta c}{\Delta\omega})^2}\,. \tag{14}$$

For $\delta c \to 0$ Eq. (14) gives standard symmetry structure of current algebra, *i.e.*

$$\frac{1}{\Delta\omega}(b_1 + b_2)\,. \tag{15}$$

On the other hand, in the SU(4)-breaking pole model one has

$$\begin{aligned} \frac{\delta c}{\Delta\omega} &\approx 2.2\,, \\ \frac{1}{1 - (\frac{\delta c}{\Delta\omega})^2} &\approx -0.2\,. \end{aligned} \tag{16}$$

Thus, in the pole model, SU(4)-symmetry breaking effects are so large that:

(1) the contribution of standard symmetry structure ($b_1 + b_2$, as in CA approach) is much smaller and of opposite sign than in CA,

(2) a different symmetry structure, corresponding to the *subtraction* of the relevant spin-flavour weights ($b_1 - b_2$) becomes dominant. For decays with pseudoscalar meson production, the symmetry structure of these terms is proportional to that of the factorization contribution. Consequently, the factorization approach often applied to the description of these decays does not test the genuine factorization contributions alone.

The origin of differences between the predictions of CA approach [2] and the full SU(4)-symmetry breaking pole model [3] is exhibited in Eq. (14) in a surprisingly simple and transparent way. The complicated and tedious calculations of Cheng have been reduced to the CA simplicity level, and yet flavour symmetry breaking has been maintained. The technique permits an easy discussion of the salient features of both CA and the pole model.

Calculations show [7] that the differences between CA and SU(4) -symmetry breaking pole model should be most clearly seen in the asymmetries of three two-body decays with vector meson emission (in these decays factorization contributions are absent):

$$\begin{aligned} \Xi_c^0 &\to \Sigma^+ \bar{K}^{*-} , \\ \Lambda_c^+ &\to \Xi^0 K^{*+} , \\ \Lambda_c^+ &\to \Sigma^+ \phi . \end{aligned} \tag{17}$$

For these decays the two approaches lead to opposite signs of asymmetries. These asymmetries are therefore singled out as the most important ones to be measured. Complete fits and predictions of the SU(4)-symmetry breaking model and their comparison with the results of other papers are given in [4, 7].

4. Outlook

In Cabibbo-allowed decays of charmed baryons, diagrams of type ($c1$), ($c2$) (Fig. 2) are not allowed. On the other hand, for Cabibbo-forbidden decays these diagrams should be considered. Successful explanation of the f/d ratios in hyperon decays proposed in Ref. [6] and discussed here indicates that the contribution of c-type diagrams is important. Consequently, additional deviations from simple-minded CA results are expected in Cabibbo-forbidden decays. The technique presented here permits the simplest and the least tedious way of obtaining the relevant predictions and pinpointing those decays the measurements of which are most interesting. Cabibbo-forbidden decays of charmed baryons should therefore be a good place to provide an independent test of the mechanism underlying the proposed explanation of deviations of the f/d ratios in hyperon decays from -1.

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