FROM UNPOLARIZED TO POLARIZED QUARK DISTRIBUTIONS IN THE NUCLEON*

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Starting from Martin, Roberts and Stirling fit for unpolarized deep inelastic structure functions and using experimental data on spin asymmetries we get a fit which provides polarized quark distributions. We analyze the behaviour of such functions near x equal to 1. The first moments of these distributions are also discussed. Our fit prefers combination of proton and neutron data versus proton-deuteron one.

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The interest in the spin structure of the nucleon has been renewed due to new experiments being made at CERN [1, 2] and SLAC [3]. The polarized deep inelastic asymmetries for deuteron and proton (Spin Muon Collaboration at CERN) as well as for neutron (at ^{3}He target in E142 experiment at SLAC) were precisely measured also in small x region. Together with an old SLAC [4] and EMC [5] data for proton one has considerable amount of information which can be used to study nucleon spin structure and particularly quark parton distributions.

The unpolarized quark distributions in the nucleon are known due to several fits [6, 7] to the experimental data. The one of the most recent

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fit is given by Martin, Roberts and Stirling (MRS) [6] who used existing experimental points in order to determine quark and gluon distributions.

In this paper we would like to give an example of determination of polarized quark parton distributions starting from the MRS fit and using data for proton, neutron and deuteron spin asymmetries. We shall concentrate not only on a controversial low x region but also on the behaviour of polarized parton distributions at $x \to 1$. The phenomenological analysis of CERN and SLAC data gives different results for $\Delta\Sigma$ (quark spin content of the proton) and Δs (strange quark polarization in the proton). Our results, as one will see later, prefers rather the numbers gotten from an analysis based on neutron E142 data.

Let us start with the formulas for unpolarized quark parton distributions (at $Q^2 = 4\,\mathrm{GeV}^2$) given by Martin, Roberts and Stirling. First of all we shall consider their fit called D'_- with very singular behaviour of "sea" distribution at small x values (which, however agrees with the results from HERA [8]). We have for the valence quarks distributions:

$$u_v(x) + d_v(x) = 1.422x^{-0.58}(1-x)^{3.92}(1 + 2.59\sqrt{x} + 4.21x)\,,$$
$$d_v(x) = 0.074x^{-0.76}(1-x)^{4.67}(1 + 28.7\sqrt{x} + 8.58x)\,, \qquad (1)$$

and for the "sea" ones:

$$2\bar{u}(x) = 0.4S(x) - \delta(x)\,,$$
$$2\bar{d}(x) = 0.4S(x) + \delta(x)\,,$$
$$2\bar{s}(x) = 0.2S(x)\,, \qquad (2)$$

where

$$S(x) = 0.083x^{-1.5}(1-x)^{7.4}(1 + 8.57\sqrt{x} + 15.8x)\,, \qquad (3)$$

and

$$\delta(x) = 0.164x^{-0.58}(1-x)^{7.4}. \qquad (4)$$

The most important features of this fit are that the valence "up" quark distribution dominates at $x \to 1$ and that the "sea" is not SU(2) symmetric (having Lipatov type behaviour [9] at small x). Because the unpolarized parton distributions are the sum of spin up and spin down distributions whereas the polarized ones are the difference of these functions our idea is just to split the numerical constants in formulas (1,3,4) in two parts in such a manner that we get positive defined distributions (which is not so easy to achieve). Our expressions for $\Delta q(x) = q^+(x) - q^-(x)$ ($q(x) = q^+(x) + q^-(x)$) are:

$$\Delta u_v(x) + \Delta d_v(x) = x^{-0.58}(1-x)^{3.92}(a_1 + a_2\sqrt{x} + a_3 x)\,,$$

$$\begin{aligned}\Delta d_v(x) &= x^{-0.76}(1-x)^{4.67}(b_1 + b_2\sqrt{x} + b_3 x)\,,\\ \Delta S(x) &= c x^{-0.50}(1-x)^{7.4}\,,\\ \Delta\delta(x) &= d x^{-0.58}(1-x)^{7.4}\,.\end{aligned} \tag{5}$$

We want to stress that we are using very simple way of parameterization of q^+ and q^- without introducing any additional powers of x or $(1-x)$, so only the coefficients for polarized quark distributions are fitted. In order to get the integrable function $\Delta S(x)$ we divide two first coefficients in Eq. (3) into equal parts.

We fit our formulas (with eight parameters) for spin asymmetries, given by:

$$\begin{aligned}A_1^p(x) &= \frac{4\Delta u_v(x) + \Delta d_v(x) + 2.2\Delta S(x) - 3\Delta\delta(x)}{4u_v(x) + d_v(x) + 2.2S(x) - 3\delta(x)}\,,\\ A_1^n(x) &= \frac{\Delta u_v(x) + 4\Delta d_v(x) + 2.2\Delta S(x) + 3\Delta\delta(x)}{u_v(x) + 4d_v(x) + 2.2S(x) + 3\delta(x)}\,,\end{aligned} \tag{6}$$

to the experimental data.

In this paper we assume that the spin asymmetries do not depend on Q^2 what is suggested by the experimental data [1, 3] and phenomenological analysis [10]. The unknown parameters in Eq. (5) are determined by making best fit to the measured spin asymmetries for proton (SLAC-Yale, EMC, SMC) and neutron (E142). We get the following values (taking care of positivity for quark distributions):

$$\begin{aligned}&a_1 = 0.874\,, \quad a_2 = 5.023\,, \quad a_3 = 12.73\,,\\ &b_1 = 0.074\,, \quad b_2 = 0.884\,, \quad b_3 = 0.649\,,\\ &c\ = -0.556\,, \quad d = -0.004\,.\end{aligned} \tag{7}$$

It is interesting to note that the fit shows no significant SU(2) symmetry breaking for "sea" polarization (d coefficient is close to zero). The χ^2 is 20.1 for 34 degrees of freedom for such a fit. In figures 1a, 1b and 2 the comparison of our fit with the experimental points for proton and neutron asymmetries is given. The *prediction* for the deuteron case (which adds 7.2 to χ^2 for 11 degrees of freedom) with A_1^d given by the following expression:

$$A_1^d(x) = \frac{5\Delta u_v(x) + 4.4\Delta S(x)}{5u_v(x) + 4.4S(x)}\left(1 - \frac{3}{2}p_{\rm D}\right), \tag{8}$$

($p_{\rm D}$ is D-state probability equal to 5.8%) is presented in figure 3. We see that in this case the calculated curve at small x lies above the experimental points. This is because we get positive A_1^d for all values of Bjorken variable,

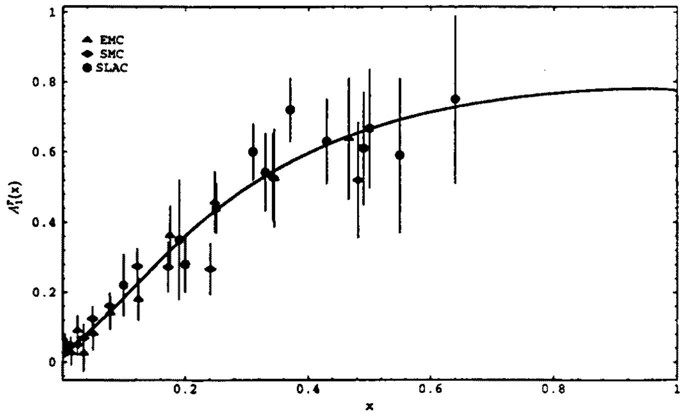

Fig. 1a. The comparison of spin asymmetry on protons (data points from SLAC-Yale, EMC and SMC experiments) with the curve gotten from our fit (Eqs (5), (6), (7)).

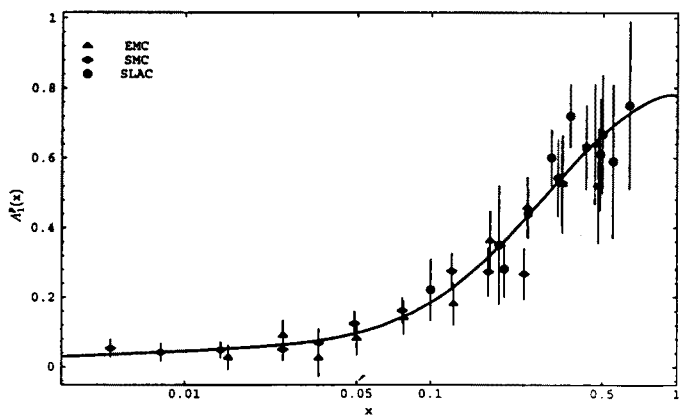

Fig. 1b. The same as in figure 1a but with x in logarithmic scale.

whereas the data are negative in small x region. Of course we can make a fit with inclusion of SMC deuteron data. Than we get $\chi^2 = 26.6$ (for 45 degrees of freedom) that is not much less than for proton+neutron case and predicted deuteron asymmetry ($\chi^2 = 20.1 + 7.2 = 27.3$). Making the fit to proton+deuteron asymmetries we get $\chi^2 = 22.2$ (37 degrees of freedom) which become 41.3 when one adds the result for A_1^n ($\chi^2 = 19.1$ for eight points). Hence, it is possible to get a satisfactory deuteron asymmetry using proton+neutron data and is not when one tries to get the neutron asymmetry from proton+deuteron data. This is our χ^2 argument why in our fit we use SLAC neutron data omitting CERN deuteron ones. As we will see below such preference is justified also when one analyzes an integrated quantities.

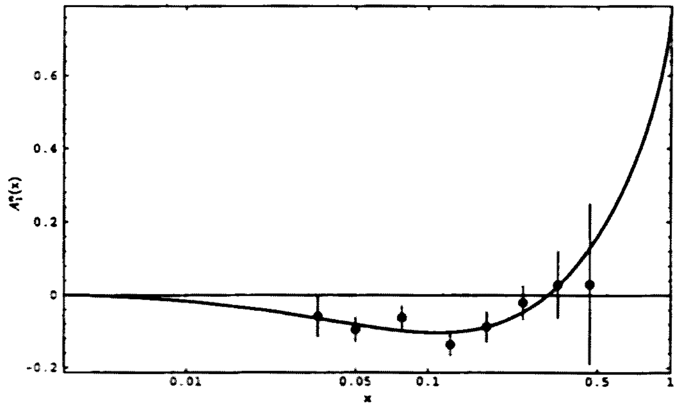

Fig. 2. The comparison of spin asymmetry on neutrons (SLAC E142 data) with the curve gotten from our fit (Eqs (5), (6), (7)).

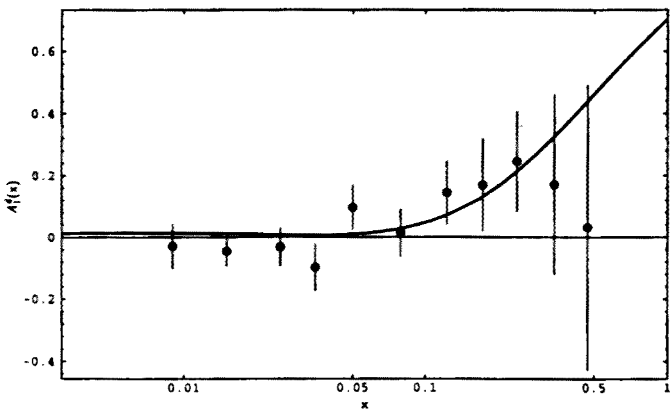

Fig. 3. Our prediction for deuteron asymmetry (NMC data).

We can use our fit to determine the first moments of parton distributions. We get *e.g.* for $I = \int_0^1 g_1(x, Q^2)\,dx$ at $Q^2 = 4$ GeV^2:

$$\begin{aligned} I^p &= \tfrac{4}{18}\Delta u + \tfrac{1}{18}\Delta d + \tfrac{1}{18}\Delta s = 0.178\,, \\ I^n &= \tfrac{1}{18}\Delta u + \tfrac{4}{18}\Delta d + \tfrac{1}{18}\Delta s = -0.027\,. \end{aligned} \tag{9}$$

Other combinations (singlet and octet ones) of quark polarizations are:

$$\begin{aligned} a_0 &= \Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.50\,, \\ a_3 &= \Delta u - \Delta d = 1.23\,, \\ a_8 &= \Delta u + \Delta d - 2\Delta s = 0.71\,, \end{aligned} \tag{10}$$

whereas $\Delta s = -0.07$.

We would like to stress that these results are obtained from the fit without any constraints (e.g. for integrated quantities), which is not a case in other fits (see *e.g.* Ref. [11]). We also do not use any information from neutron and hyperon β-decays. Hence, the value of a_3 should be compared to $g_A = 1.257 \pm 0.003$ [12] lowered by several percent due to QCD corrections (see *e.g.* Ref. [13]), *i.e.* to $a_3 \simeq 1.1$. The agreement is not perfect, although satisfactory. Also the value of I^p, when compared with the published number: $I^p = 0.136 \pm 0.011 \pm 0.011$ [2], is too high. This happens because the experimental groups use "Regge" type extrapolation in the $x \to 0$ region. For example, the latest SMC value for I^p is gotten assuming constant value for $g_1(x)$ in unmeasured small x region. When we modify our distributions in such a manner (only for x between 0 and 0.003) we get:

$$\begin{aligned} &I^p = 0.168\,, \quad I^n = -0.038\,, \quad \Delta\Sigma = 0.43\,, \\ &a_3 = 1.23\,, \quad a_8 = 0.62, \quad \Delta s = -0.06\,, \end{aligned} \tag{11}$$

the values which are not substantially different from those presented in Eqs (9), (10). The second reason for using this modification (for small x) is that MRS distribution for $u_v(x)$ is not positive defined for tiny x's ($x \sim 10^{-7}$) and hence such blunder is also present in our polarized distributions for $u_v^+(x)$ and $u_v^-(x)$.

If we fit the polarized parton distributions to all measured asymmetries (on proton, neutron and deuteron targets) we get *e.g.* $a_3 = 1.50$ which is approximately 40% to big. If one uses D_0' fit of Martin, Roberts and Stirling instead of D_-' (D_0' is not so divergent at $x \sim 0$) we get the similar values for integrals I^p and I^n, whereas $a_3 = 1.34$. Also the χ^2 is worse in this case. Hence, we prefer our distributions fitted to asymmetries measured on proton (SLAC–Yale, EMC, SMC) and neutron (SLAC E142) targets and having its roots in the D_-' fit.

Now, we would like to make some comments about the $x \to 1$ behaviour of valence quark distributions. Looking at the data points for proton spin asymmetry the value close to 1 at $x \sim 1$ is preferred, whereas for neutron and deuteron case values close to 0 seem to be natural (such observation is fragile due to the big experimental errors in this x region). In our approach we can give predictions for the behaviour of polarized quark distributions and spin asymmetries in the $x \to 1$ limit. Let us assume that valence u and d quark polarized distributions behave at $x \to 1$ as:

$$\begin{aligned} u_v^{\pm} &\to a_{\pm}(1-x)^p + \ldots\,, \\ d_v^{\pm} &\to b_{\pm}(1-x)^p + \ldots\,, \end{aligned} \tag{12}$$

where p is the smallest power of $(1-x)$ term. Than, we have in such a limit:

$$\frac{F_1^n}{F_1^p} \to \frac{a_+ - a_- + 4(b_+ - b_-)}{4(a_+ + a_-) + b_+ + b_-} \tag{13}$$

and:

$$\begin{aligned} A_1^p(x) &\to \frac{4(a_+ - a_-) + b_+ - b_-}{4(a_+ + a_-) + b_+ + b_-}, \\ A_1^n(x) &\to \frac{a_+ - a_- + 4(b_+ - b_-)}{a_+ + a_- + 4(b_+ + b_-)}. \end{aligned} \tag{14}$$

In the case of SU(6) symmetry the flavour-spin part of nucleon wave function gives: $a_+ : a_- : b_+ : b_- = 5:1:1:2$. Hence, one gets well known results: $F_1^n/F_1^p \to {}^1/_4$, whereas $A_1^p \to {}^5/_9$ and $A_1^n \to 0$. The counting rules for parton distributions at $x \to 1$ (see Ref. [14]) yield: $a_-/a_+ = b_-/b_+ = 0$, so one has $A_1^p, A_1^n \to 1$. Authors of Ref. [14] assume in addition: $a_+/b_+ = 5$ (as in the SU(6) symmetric case), hence they get $F_1^n/F_1^p \to {}^3/_7$. In the MRS fit one has $b_\pm/a_\pm = 0$ which leads to $F_1^n/F_1^p \to {}^1/_4$. For the spin asymmetries one gets $A_1^p = A_1^n \to (a_+ - a_-)/(a_+ + a_-) \leq 1$. In our case we have $A_1^p = A_1^n = A_1^d \cong 0.77$. The different limits (at $x \to 1$) for A_1^p and A_1^n one can get assuming that coefficients b are not negligible in comparison to a's (see Eq. (14)). But then it is impossible to get the value for F_1^n/F_1^p suggested by the experimental data, namely ${}^1/_4$ (see the figures in Ref. [11]).

So far we did not consider possible gluon contribution to polarized structure functions. We can, however, include explicit gluon terms into the asymmetries (in the way proposed in Ref. [15]), simply by substituting in our formulas (6) $\Delta q(x)$ by $\Delta q(x) - (\alpha_s/2\pi)\Delta G(x)$, where:

$$\Delta G(x) = f x^{-0.5}(1-x)^{5.3}. \tag{15}$$

The form of the x dependence comes from the gluon distribution in the unpolarized case and f is a new fitted parameter. Because of a new additional parameter f the χ^2 is better, but only slightly, (χ^2 per degree of freedom is, however, worse in this case) and the corresponding $\Delta G = \int_0^1 \Delta G(x)\,dx$ is huge (we get about 15 using $\alpha_s(4\text{GeV}^2) = 0.28$), much bigger than expected theoretically. In addition $a_8 = 1.9$, the figure which is excluded by the experiment. We conclude that gluon contributions do not lead to any improvement of the fit so we do not take them into account.

Starting from the MRS fit [6] to the unpolarized deep inelastic scattering data we have made a fit to proton and neutron spin asymmetries in order to obtain polarized quark parton distributions. We have got $\Delta u = 0.91$ ($\Delta u_v = 1.04$), $\Delta d = -0.33$ ($\Delta d_v = -0.18$) and $\Delta s = -0.07$ for integrated quantities. At $x \to 1$ the asymmetries for nucleons point towards the value equal to 0.77. With the improved behaviour at $x \to 0$ for the unpolarized parton distributions and consistent data for spin asymmetries (mainly for deuteron) our method of determination of quark polarized distributions looks promising and could be repeated when the new data on spin asymmetries will be available.

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