COLORED QUARKLIKE SCALARS AS CONSTITUENTS OF NEW HADRONS*

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Dedicated to the memory of Professor Jan Rzewuski

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Colored quarklike scalars, y, appear both in the familiar supersymmetric model and in a recently proposed Dirac's square-root model based on (in general reducible) representations of the Dirac algebra, defined by means of Clifford algebras. Five types of new (generally unstable) hadrons, $q\bar{y}$, $y\bar{y}$, qqy, qyy, yyy, are briefly discussed. Two different flavor assignments for the scalars y are considered.

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It is a popular way to think of the hypothetic colored quarklike scalars as of squarks forming together with sleptons three families of the not-yet-observed supersymmetric partners of leptons and quarks.

Recently, however, another possibility was pointed out, basing on the conjecture [1, 2] that *all* kinds of fundamental spin-$^1/_2$and spin-0 matter particles existing in Nature *can be deduced* from the Dirac's square-root procedure, $\sqrt{p^2} \rightarrow \Gamma \cdot p$. In order to make our presentation fairly comprehensible let us first recapitulate briefly this alternative.

In fact, Dirac's square-root procedure leads to the sequence $N=1, 2, 3, \ldots$ of different (in general reducible) representations

$$\Gamma^\mu = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \gamma_i^\mu \tag{1}$$

of the Dirac algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}\,, \tag{2}$$

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defined by the sequence $N = 1, 2, 3, \ldots$ of Clifford algebras

$$\{\gamma_i^\mu, \gamma_j^\nu\} = 2\delta_{ij} g^{\mu\nu} \quad (i, j = 1, 2, \ldots, N). \tag{3}$$

Then, the sequence $N = 1, 2, 3, \ldots$ of Dirac-type equations

$$[\Gamma \cdot (p - gA) - M]\, \psi = 0\,, \tag{4}$$

if written down in terms of reduced forms

$$\Gamma^\mu = \gamma^\mu \otimes \underbrace{1 \otimes \cdots \otimes 1}_{(N-1)\text{ times}} \tag{5}$$

of the representations (1) (γ^μ and 1 are here the usual 4×4 Dirac matrices), provides us with the equations

$$[\gamma \cdot (p - g\, A) - M]_{\alpha_1 \beta_1}\, \psi_{\beta_1 \alpha_2 \ldots \alpha_N} = 0 \tag{6}$$

for the sequence $N = 1, 2, 3, \ldots$ of the wave functions (or fields) $\psi(x) = (\psi_{\alpha_1 \alpha_2 \ldots \alpha_N}(x))$. Here, $\alpha_1, \alpha_2, \ldots, \alpha_N$ are N bispinor indices of which only α_1 is affected by the gauge fields $A_\mu(x)$, while the rest of them, $(\alpha_2, \ldots, \alpha_N)$, are decoupled. For $N = 1$ Eq. (6) is obviously the usual Dirac equation, whereas for $N = 2$ it is known as the Dirac form [3] of the Kähler equation [4]. For $N = 3, 4, 5, \ldots$ we get new Dirac-type equations.

In our argument, $A_\mu(x)$ symbolize the standard-model gauge fields including the $SU(3) \otimes SU_L(2) \otimes U(1)$ coupling matrices λ's, τ's, Y and $\Gamma^5 = i\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3$. These imply for each solution to Eqs. (6) with N odd the familiar 16 standard-model particle states forming two left-handed weak isospin doublets or four right-handed weak-isospin singlets, half of them colorless leptonlike and half colored quarklike. For each solution to Eqs. (6) with N even, if then the coupling $g\Gamma \cdot A$ in Eqs. (4) *does not include* the chiral matrix Γ^5, one gets in place of 16 the familiar 8 standard-model particle states forming two weak-isospin doublets, one of them colorless leptonlike and one colored quarklike (*otherwise* this standard-model content were "chirally" doubled like for sleptons and squarks from each family in the supersymmetric model).

The algebraically composite representations (1), leading to the many-indexed wave functions $\psi_{\alpha_1 \alpha_2 \ldots \alpha_N}(x)$, may or *may not* be a signal of some, here neglected, *spatial* compositeness of particle states concentrated closely around their centres of mass $\vec{x}$ (*cf.* the second Ref. [1] for an argument against such a compositeness).

Under the assumption that the decoupled bispinor indices $\alpha_2, \ldots, \alpha_N$ describe the particle's *undistinguishable* degrees of freedom obeying the

Fermi statistics along with the Pauli exclusion principle, the wave functions $\psi_{\alpha_1\alpha_2\ldots\alpha_N}$ must be fully antisymmetric with respect to $\alpha_2,\ldots,\alpha_N$. Then, the sequence $N = 1,2,3,\ldots$ of equations (6) *ought to terminate* at $N = 5$.

Applying then the theory of relativity consequently to *all* bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N$ and making use of the probabilistic interpretation of wave functions $\psi_{\alpha_1\alpha_2\ldots\alpha_N}$, we are led to the conclusion [1, 2] that there are *three* (and only three) families $N = 1,3,5$ of leptons and quarks and *two* (and only two) families $N = 2,4$ of fundamental scalars. They correspond to

$$\begin{aligned}\psi^{(1)}_{\alpha_1} &\equiv \psi_{\alpha_1}, \\ \psi^{(3)}_{\alpha_1} &\equiv \tfrac{1}{4}\left(C^{-1}\gamma^5\right)_{\alpha_2\alpha_3}\psi_{\alpha_1\alpha_2\alpha_3}, \\ \psi^{(5)}_{\alpha_1} &\equiv \tfrac{1}{24}\varepsilon_{\alpha_2\alpha_3\alpha_4\alpha_5}\psi_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5},\end{aligned} \tag{7}$$

and

$$\begin{aligned}\psi^{(2)} &\equiv \left(C^{-1}\gamma^5\right)_{\alpha_1\alpha_2}\psi_{\alpha_1\alpha_2}, \\ \psi^{(4)} &\equiv \tfrac{1}{6}\left(C^{-1}\gamma^5\right)_{\alpha_1\alpha_2}\psi_{\alpha_1\beta_2\beta_3\beta_4}\varepsilon_{\alpha_2\beta_2\beta_3\beta_4},\end{aligned} \tag{8}$$

respectively, for any familiar standard-model signature (here, C is the usual 4×4 matrix of charge conjugation and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$).

In contrast to sleptons from three families in the supersymmetric model, the colorless leptonlike scalars from one or both families in our Dirac's square-root model might be Higgs bosons.

Squarks in the supersymmetric model, as belonging to three families, can appear in six strong flavors identical to these for quarks: $2I_3 = -1$ and 1, *strangeness* $= -1$ and *charm* $= 1$, *bottomness* $= -1$ and *topness* $= 1$ (I_3 is here the third component of the strong isospin $\vec{I}$). In contrast, the colored quarklike scalars in the Dirac' square-root model, belonging now to two families, could develop only four strong flavors: *e.g.* either *(i)* $2I_3 = -1$ and 1, *strangeness* $= -1$ and *charm* $= 1$, or *(ii)* *strangeness* $= -1$ and *charm* $= 1$, *bottomness* $= -1$ and *topness* $= 1$. In both models, the conventional QCD is extended by additionally including to the color SU(3) group a colored quarklike scalar sector (different in two models). The analogical extension takes place for the electroweak group $\mathrm{SU}_L(2)\otimes\mathrm{U}(1)$.

Unfortunately, the supersymmetry, if it appears, must be badly broken, so the direct experimental verification, whether in Nature there are six or only four scalar flavors, may be extremely difficult. The indirect verification might offer more optimistic chances, when some new hadrons interpreted as bound states involving colored quarklike scalars would be observed. In particular, observing a stable hadronic state of this kind would mean that such a state could not decay into ordinary hadrons (and, possibly, leptons or photons) plus a gaugino or higgsino (or a slepton). This would imply one of two things: either gauginos, higgsinos and sleptons are too heavy to allow for decays of the discovered new hadron, or the broken supersymmetry is not realized in Nature, the Dirac square-root being then left in the field.

In the present note, we discuss briefly new hypothetic hadrons built up of quarks q from their ground family, $q = (u, d)$, and colored quarklike scalars y (call them "yukawions") from their lowest-in-mass family. Not deciding whether yukawions are squarks or rather our quarklike scalars, we will assume *phenomenologically* that their lowest-in-mass family consists either *(i)* of the flavors $2I_3 = -1$ and 1: $y = (y_u, y_d)$, or *(ii)* of the flavors *strangeness* $= -1$ and *charm* $= 1$: $y = (y_c, y_s)$ (of course, the option of the flavors *bottomness* $= -1$ and *topness* $= 1$: $y = (y_t, y_b)$ may be discussed as well). Note that the case *(i)* may seem natural for squarks though it is then not necessarily true [5], while the case *(ii)* as well as the case *(i)* may be realized for our quarklike scalars (if yukawions are squarks, they produce doublets of mass eigenstates which then may be building blocks of new hadrons).

In both cases, quarklike scalars carry the baryon number $B = 1/3$, if for them the familiar charge formula

$$2Q = 2I_3 + \text{strangeness} + \text{charm} + \text{bottomness} + \text{topness} + B \tag{9}$$

holds, what will be conjectured here. Then, new (generally unstable) hadrons of five types can be formed: $q\bar{y}$, $y\bar{y}$ with $B = 0$ and $q\,q\,y$, $q\,y\,y$, $y\,y\,y$ with $B = 1$, all of these states being colorless. Of course, q's and y's obey the Fermi and Bose statistics, respectively.

In the case *(i)*, the lowest-in-mass versions of new hadrons should get the following $J^P(I)$-signatures:

$$\begin{aligned}
q\,\bar{y} &: \tfrac{1}{2}^+(0) \text{ or } \tfrac{1}{2}^+(1)\ (B = 0 : Q = 0 \text{ or } Q = 1, 0, -1)\,,\\
y\,\bar{y} &: 0^+(0) \text{ or } 0^+(1)\ (B = 0 : Q = 0 \text{ or } Q = 1, 0, -1)\,,\\
q\,q\,y &: 0^+(\tfrac{1}{2}) \text{ or } 1^+(\tfrac{1}{2}) \text{ or } 1^+(\tfrac{3}{2})(B = 1 : Q = 1, 0 \text{ or } Q = 1, 0 \text{ or } Q = 2, 1, 0, -1)\,,\\
q\,y\,y &: \tfrac{1}{2}^+(\tfrac{1}{2})\,(B = 1 : Q = 1, 0)\,,\\
y\,y\,y &: 0^-(\tfrac{3}{2})\,(B = 1 : Q = 2, 1, 0, -1)\,.
\end{aligned}$$

Among them, only the $1/2^+(1/2)$-hadron $q\,y\,y$ with $B = 1$ and $0^+(0)$- and $0^+(1)$-hadrons $y\,\bar{y}$ with $B = 0$ have familiar signatures, identical with those for the nucleon and for f_0 and a_0 mesons, respectively (it is interesting to note troubles with interpreting $f_0(975)$ and $a_0(980)$ mesons as the conventional pure $^3P_0$ states $q\bar{q}$ [6]). The other cannot be realized as quark states $q\bar{q}$ and qqq (or any purely quark states). Especially interesting are here the $0^+(1/2)$-hadron qqy with $B = 1$ (a "spin-0 baryon") and $(1/2)^+(0)$- and $1/2^+(1)$-hadrons $q\bar{y}$ with $B = 0$ ("spin-$^1/_2$mesons"); the $0^-(3/2)$-hadron yyy with $B = 1$ (another "spin-0 baryon") requires an antisymmetric spatial part of wave function constructed from three orbital angular momenta 1.

It may happen (for adequate masses of new hadrons) that within this hypothetic class of hadronic states only the $1/2^+(0)$- and $1/2^+(1)$-hadrons $q\bar{y}$ with $B = 0$ (and the lepton number $L = 0$) are stable in strong interactions. Of these two, the neutral component of the isotriplet, if it is lower in mass than the charged components, may be stable also in electroweak interactions, and so absolutely stable (the isoscalar is likely to include an admixture of hidden heavy flavors, first of all of strangeness — *cf.* the familiar case of η versus π^0).

In the case *(ii)*, the lowest-in mass versions of new hadrons should get different $J^P(I)$-signatures:

$$
\begin{aligned}
q\,\bar{y}_s\,, q\,\bar{y}_c &: \tfrac{1}{2}^+(\tfrac{1}{2})(B=0 : Q=1,0,\ Q=0,-1)\,,\\
y_s\,\bar{y}_s\,, y_c\,\bar{y}_c\,, y_s\,\bar{y}_c\,, y_c\,\bar{y}_s &: 0^+(0)(B=0 : Q=0\,, Q=0\,, Q=-1\,, Q=1)\,,\\
qqy_s\,, qqy_c &: 0^+(0) \text{ or } 1^+(1)(B=1 : Q=0, Q=1 \text{ or } Q=1,0,-1, Q=2,1,0),\\
q\,y_s\,y_c &: 1/2^+(\tfrac{1}{2})\,(B=1 : Q=1,0)\,,\\
q\,y_s\,y_s\,, q\,y_c\,y_c &: \tfrac{1}{2}^-(\tfrac{1}{2}) \text{ or } \tfrac{3}{2}^-(\tfrac{1}{2})(B=1 : Q=0,-1,\ Q=2,1)\,,\\
y_s\,y_s\,y_s\,, y_c\,y_c\,y_c &: 0^-(0)\,(B=1 : Q=-1,\ Q=2)\,,\\
y_s\,y_s\,y_c\,, y_s\,y_c\,y_c &: 1^-(0)\,(B=1 : Q=0,\ Q=1)\,.
\end{aligned}
$$

(In the case of the option of $y = (y_t,\ y_b)$, the labels c, s are replaced by t, b, respectively.) Now, no new hadrons have signatures identical to those for the nucleon and a_0 meson, in the first case because the quarklike scalars y_s and y_c are strange and charmed, respectively. But, the $0^+(0)$-hadrons $y_s\bar{y}_s$ and $y_c\bar{y}_c$ with $B = 0$ have the signature identical to that for f_0 meson (it is interesting to note that they contain hidden strangeness and charm, respectively).

In this case, it may happen (for adequate masses of new hadrons) that only the $1/2^+(1/2)$-hadrons $q\bar{y}_s$ and $q\bar{y}_c$ with $B = 0$ (and $L = 0$) are stable in strong interactions. Of these two, the lower-in-mass component of the strange isodoublet may be absolutely stable (the charmed isodoublet is likely to be considerably heavier and so to decay electroweakly into the strange isodoublet when $y_c \to y_s e^+\nu_e$ or $y_c \to y_s\mu^+\nu_\mu$). This time, in contrast to the case *(i)*, the stable new hadrons could not transit through virtual gluino exchange into Majorana fermionic states and thus strongly annihilate, when in particle-particle couples, into ordinary hadrons.

Let us denote by λ the $1/2^+(1/2)$-hadron $q\bar{y}_s$ with $B = 0$ and $Q = 1, 0$, the strange isodoublet whose lower-in-mass component is a candidate for an absolutely stable hadron in the case *(ii)*. This interesting strange fermion (*strangeness* = 1), although it is not a baryon ($B = 0$) but rather a strange "spin-$^1/_2$meson", may strongly interact with pions through the effective Yukawa coupling $g_{\lambda\lambda\pi}\bar{\lambda}\gamma^5\vec{\tau}\lambda \cdot \vec{\pi}$. Thus, λ ($\bar{\lambda}$) may form a lot of stable bound states with other λ's ($\bar{\lambda}$'s) and with nucleons.

Such strange hypernuclei of a new kind, if they existed, should be eventually observed, though they would not display characteristic decays, as the usual hypernuclei involving unstable Λ hyperons would do. Their baryon number would be contributed only by nucleons, while their mass might be characteristically dominated by λ's ($\bar{\lambda}$'s), in the case of heavy y_s leading to heavy λ. Note, however, that spectra of atoms formed around positively charged hypernuclei of this new kind would not differ qualitatively from those of the usual atoms with Z accidentally equal to the number of protons plus (minus) the number of charged λ's ($\bar{\lambda}$'s) involved in such hypernuclei. So, in astrophysical observations they might be taken for spectra of conventional atoms, generally with shifted number Z of protons in their nuclei, say, $Z + 1$ ($Z - 1$) in the case of one λ^+ (λ^-).

On the other hand, heavy λ's might be comparatively scarce in the universe and, simply for this reason, escape so far our observation, in contrast to much lighter nucleons.

In the case of the option of $1/2^+(1/2)$-hadron $q\bar{y}_t$ with $B = 0$ and $Q = 0, -1$ as a constituent of alternative new top hypernuclei, the discussion is *mutatis mutandis* analogical.

REFERENCES

[1] W. Królikowski, *Phys. Rev.* **D45**, 3222 (1992); *Clifford algebras and algebraic structure of fundamental fermions*, in *Spinors, Clifford Algebras and Quantum Deformations*, eds. Z Oziewicz et al., Kluwer Academic Publishers 1993, p. 171; *Acta Phys. Pol.* **B23**, 1245 (1992).

[2] W. Królikowski, *Phys. Rev.* **D46**, 5188 (1992); *Acta Phys. Pol.* **B24**, 1149 (1993); *Nuovo Cimento* **107A**, 69 (1994).

[3] . T. Banks, Y. Dothan, D. Horn, *Phys. Lett.* **B117**, 413 (1982).

[4] E. Kähler, *Rendiconti di Matematica* **21** , 425 (1962); *cf.* also D. Ivanenko, L. Landau, *Z. Phys.* **48**, 341 (1928).

[5] *Cf.* M. Drees, M.M. Nojiri, *Phys. Rev. Lett.* **72**, 2324 (1994) and references therein, where the lighter of two stop mass eigenstates is the lightest squark state (as it is recently argued on the ground of minimal SUSY standard model).

[6] For a discussion *cf.* M. Genovese, E. Predazzi, D.B. Lichtenberg, *Phenomenology of spin zero mesons and glueballs*, preprint IUHET 248/ DFTT 14/93 (April 1993); M Genovese, *Nuovo Cimento* **107 A**, 1249 (1994).