APPLICATION OF THE SOFT–GLUON APPROACH*

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The Mebarki–Abbes prescription within the soft-gluon approach is applied and approximate higher order corrections to the cross section of large P_{T} hadronic production of two photons collision are derived.

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1. Introduction

During the past few years, phenomenologists were very interested in the calculation of higher order corrections (HOC) in Perturbative Quantum Chromodynamics (PQCD), in studying particular various problems and ambiguities related to the choice of the renormalization and factorization scales [1–6]. In many cases the next-to-leading order terms with respect to the strong coupling constant α_s are comparable (or larger) to the supposed leading order ones. As a good example, the PQCD calculation up

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to the 3-loop order of the ratio $R_{e^+e^-}$ by the Russian group, has shown that the perturbative series diverge, in the sense that the α_s^2 order terms are bigger than the α_s order ones. This suggests that more tests of PQCD are needed. To do so, and to overcome the difficulties and problems of the HOC, one has to look for the simplest and fastest method to determine at least the magnitude and the sign of the corrections [5, 8].

Recently, we have shown [5] that in order to determine the dominant contribution (in hadronic processes) of HOC, one has to calculate the contribution coming from the Born and virtual diagrams. Following our prescription and rules (given in Rev. [5]), the Bremsstrahlung contribution is easily obtained, since its dominant part results from the soft and/or collinear configuration of the particles at the initial and/or final state.

The purpose of this work is to determine the contribution of HOC in the inclusive large $P_{\rm T}$ hadronic production from two photons collisions ($\gamma\gamma \rightarrow {\rm h}X$, where h is a hadron) by applying the Mebarki–Abbes prescription.

It is worth mentioning that the photon-photon reactions represent an important class of scattering processes which allow fundamental tests of QCD. In fact, they can provide important constraints on the validity of the standard model of strong interactions through the study of inclusive hadron production [9]. Exact higher order corrections to $\gamma\gamma$ collisions had already been considered in Refs [9] and [10]. In contrast (and it is the goal of this work) we determine the dominant part of this (HOC) by applying our prescription derived and described in Ref. [5].

In Section 2, we give a brief review of our prescription. In Section 3, we determine the full approximate corrections to the cross section of the process $\gamma\gamma \rightarrow hX$ and finally, in Section 4, we draw our conclusions.

2. Formalism and prescription

We have shown in [5] that for hadronic process of the form: $A + B \rightarrow C + D$, where A, B, C and D are hadrons (in general), the cross section can be written as [5, 11, 12]:

$$\begin{aligned}\sigma = \sum_{a,b,c,d} \int\!\!\int\!\!\int\!\!\int dx_a dx_b dx_c dx_d F_{a/A}(x_a, M) F_{b/B}(x_b, M) \\ \times D_{C/c}(x_c, M') D_{D/d}(x_d, M') \\ \times \left\{ \frac{\alpha_s(\mu)}{\pi} f_0 \delta\Big(1 + \frac{\hat{t} + \hat{u}}{\hat{s}}\Big) + \frac{\alpha_s^2(\mu)}{\pi} f\theta\Big(1 + \frac{\hat{t} + \hat{u}}{\hat{s}}\Big) \right\},\end{aligned} \tag{2.1}$$

where $F_{a/A}$, $F_{b/B}$ (respectively $D_{C/c}$, $D_{D/d}$) are structure (respectively fragmentation) functions; a, b, c, d are massless protons, f_0 the Born term and f the HOC, ($\hat{s}, \hat{t}$ and $\hat{u}$ are the usual Mandelstam variables at the partons sub process). Moreover, the dominant contribution of HOC "f" has the following form [5]:

$$\begin{aligned} &f_{\mathrm{sing}}(x_c, x_d, v, w) \\ &= \left[c_1 + \hat{c}_1 \ln\left(\frac{\hat{s}}{\mu^2}\right) + \tilde{c}_1 \ln\left(\frac{\hat{s}}{M^2}\right) + \tilde{\tilde{c}}_1 \ln\left(\frac{\hat{s}}{M'^2}\right)\right]\delta(1-w) \\ &+ \left[c_2 + \tilde{c}_2 \ln\left(\frac{\hat{s}}{M^2}\right) + \tilde{\tilde{c}}_2 \ln\left(\frac{\hat{s}}{M'^2}\right)\right]\frac{1}{(1-w)_+} + c_3\left[\frac{\ln(1-w)}{1-w}\right]_+ , \end{aligned} \tag{2.2}$$

where the dimensionless variables v and w are defined as:

$$v = 1 + \frac{\hat{t}}{\hat{s}}; \qquad w = -\frac{\hat{u}}{\hat{t} + \hat{s}}, \tag{2.3}$$

and μ, M and M' are the renormalization and factorization points of the structure and fragmentation functions, respectively. The distribution $\frac{1}{(1-w)_+}$ and $\left[\frac{\ln(1-w)}{1-w}\right]_+$ are defined as:

$$\int_{W_{\min}}^{1} dw \frac{g(w)}{(1-w)_+} = \int_{W_{\min}}^{1} dw \frac{g(w) - g(1)}{(1-w)} + \ln(1 - W_{\min}) g(1),$$

and

$$\int_{W_{\min}}^{1} dw\, g(w)\left[\frac{\ln(1-w)}{1-w}\right]_+ = \int_{W_{\min}}^{1} dw [g(w) - g(1)]\frac{\ln(1-w)}{(1-w)} + \frac{1}{2}\ln^2(1 - W_{\min}) g(1), \tag{2.4}$$

where $g(w)$ is some regular function. It is to be noted that f_{sing} is gauge invariant and receives contributions from soft and/or collinear gluon and/or quark Bremsstrahlung. Now, in order to determine the various terms $\hat{c}_1$, $\tilde{c}_1$, $\hat{c}_2$ *etc.*, it is shown that one has to use the following prescription [5]:

1. Calculate the Born term in $d = 4 - 2\varepsilon$ dimension.
2. Determine $\hat{c}_1$, $\tilde{c}_1$, $\tilde{\tilde{c}}_1$, $\tilde{c}_2$ and $\tilde{\tilde{c}}_2$ from the expression of the running coupling constant around the renormalization point, the factorization of the structure function at the scale M and the factorization of the fragmentation function at scale M' (see Eqs (4.1), (4.2) and (4.3) of Ref. [5]).
3. Write two and three bodies phase space in d dimension.
4. Calculate the virtual contribution and write it in the form:

$$F^{\varepsilon}\left[\frac{A'}{\varepsilon^2} + \frac{B'}{\varepsilon} + C'\right]\delta(1-w).$$

5. Set the Bremsstrahlung contribution in the form:

$$D^{\varepsilon}\left[\frac{A}{\varepsilon}+B+2C\right](1-w)^{-1-\alpha\varepsilon},$$

where

$$A=\alpha A'$$

and

$$B=-\alpha(\tilde{C}_1-B'-A'\ln f)-A\ln D\,.$$

6. Calculate the Bremsstrahlung contribution coming from soft and/or collinear configuration and which is proportional to the Born term times $\delta(1-w)$.
7. Add the singular parts $F_{ab}^{\rm sing}$ and $D_{ab}^{\rm sing}$ coming from the universal physical definition of the corrections [5, 13], and pick up the terms proportional to $\delta(1-w)$, $\frac{1}{(1-w)_+}$ and $\left[\frac{\ln(1-w)}{1-w}\right]_+$.
8. Determine C_1, C_2 and C_3.

3. Application to the inelastic process $\gamma\gamma\to h+X$

In what follows, we consider the sub process $\gamma\gamma\to q\bar{q}$ which contribute to the physical hadronic process $\gamma\gamma\to h+X$ (h hadron). We work in the Feynmann gauge, and to regulate the singularities, we use dimensional regularization with $d=4-2\varepsilon$. For the corrections, we use the universal definition [13]. Our results refer to the $\overline{MS}$ renormalization scheme. Of course, the divergent pieces appearing in the perturbative calculation are as usual eliminated with the help of the factorization theorem by the introduction of scale violating fragmentation function.

For the two photons collision of the hadronic inelastic process $\gamma\gamma\to h+X$ the expression (2.1) can be rewritten as:

$$\sigma=\sum_{c,d}\int dx_c D_{C/c}(x_c,M')\left\{f_0\delta\left(1+\frac{\hat{t}+\hat{u}}{\hat{s}}\right)+\frac{\alpha_s(\mu)}{\pi}f\theta\left(1+\frac{\hat{t}+\hat{u}}{\hat{s}}\right)\right\}. \tag{3.1}$$

Now, following our prescription, the Born term coming from the diagrams of Fig. 1 has an expression [9, 10]:

$$\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s},v)\Big|_{\rm Born}^{\gamma\gamma\to q\bar{q}}=\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{\hat{s}v(1-v)}\right)^{\varepsilon}\frac{1}{16\pi}(4\pi\alpha)^2e_q^4$$
$$\times(2-\varepsilon)(1-\varepsilon)\frac{1}{\hat{s}^2}\left[(1-\varepsilon)\left(\frac{v}{1-v}+\frac{1-v}{v}\right)-2\varepsilon\right]. \tag{3.2}$$

Fig. 1. Feynmann diagrams for the Born term of the subprocess $\gamma\gamma \to q\bar{q}$.

Notice that in this case, the Born term is not of order α_s. Thus, the term $\left(\text{proportional to } \ln\left(\frac{\hat{s}}{\mu^2}\right)\right)\hat{c}_1 = 0$. Moreover, for the process $\gamma\gamma \to h + X$ there are no structure functions, thus the terms $\Big($proportional to $\ln\left(\frac{\hat{s}}{M^2}\right)\Big)\tilde{c}_1$ and $\tilde{c}_2$ are equal to zero ($\tilde{c}_1 = \tilde{c}_2 = 0$).

In what follows and to keep our results transparent, we take as a common factor between the various terms: $\frac{2\alpha^2 e_q^4}{\hat{s}^2} C_F N_C$. Now, to determine $\tilde{\tilde{c}}_1$ and $\tilde{\tilde{c}}_2$ (terms proportional to $\ln\left(\frac{\hat{s}}{M'^2}\right)$ one uses Eq. (4) of Ref. [5] to get:

$$\tilde{\tilde{c}}_1 + \tilde{\tilde{c}}_2 \propto \frac{1}{2}\int_{X_C}^{1} \frac{dy}{y} p_{qq}\left(\frac{y}{X_C}\right)\hat{s}' \frac{d\sigma^{\text{Born}}}{dv}\left(\hat{s}', -\frac{\hat{u}'}{\hat{s}'}\right)\delta(\hat{s}' + \hat{u}' + \hat{t}'), \tag{3.3}$$

where

$$\frac{d\sigma^{\text{Born}}}{dv} = \hat{s}' \frac{d\hat{\sigma}}{d\hat{t}},$$
$$\hat{s}' = \hat{s}\,,\ \hat{t}' = x_c \frac{\hat{t}}{y}\,,\ \hat{u}' = x_c \frac{\hat{u}}{y}\,, \tag{3.4}$$

and

$$p_{qq}(z) = C_F\left[\frac{1 + z^2}{(1 - z)_+} + \frac{3}{2}\delta(1 - z)\right]. \tag{3.5}$$

After straightforward calculation and using the fact that,

$$\frac{1}{(v - vw)_+} = \frac{1}{v}\,\frac{1}{(1 - w)_+} + \frac{1}{v}\ln v\delta(1 - w) \tag{3.6}$$

one can write:

$$\tilde{\tilde{c}}_2 \propto \frac{1}{4v}(4\ln v + 3)\left(\frac{v}{1 - v} + \frac{1 - v}{v}\right), \tag{3.7}$$

and

$$\tilde{\tilde{c}}_2 \propto \frac{2}{v}\left(\frac{v}{1 - v} + \frac{1 - v}{v}\right). \tag{3.7}$$

Regarding the coefficients c_2 and c_3 which are proportional to $\frac{1}{(1-w)_+}$ and $\left[\frac{\ln(1-w)}{1-w}\right]_+$, respectively, they appear in the corrections of $f_{\rm sing}^{\rm Brems}$ (see Ref. [5]). They can be calculated easily and have the form:

$$c_2 \propto A \ln D + B\,,$$

and

$$c_3 \propto -\alpha A\,.$$

It is to be noted that the Block–Nordsieck mechanism guaranties the cancellation of the infrared singularities and therefore $A = \alpha A'$. Moreover, and as a consequence of the Slavnov–Taylor identities, the ultraviolet singularities cancel and lead to (see Eq. (4.10) of Ref. [5]):

$$-\frac{1}{\alpha}(A \ln D + b) - \tilde{c}_1 = -(A' \ln F + B')\,,$$

where F and D come from the two and three bodies phase space, respectively. In our case, $\alpha = 1$ and $\tilde{c}_1 = 0$, which implies that:

$$c_3 \propto -A'\,,$$

and

$$c_2 \propto A' \ln F + B\,.$$

Thus the terms c_2 and c_3 can be completely determined from the virtual correction $\frac{d\hat{\sigma}}{dv}\Big|_{\rm virt}^{\gamma\gamma\to q\bar{q}}$. We remind the reader that $\frac{d\hat{\sigma}}{dv}\Big|_{\rm virt}^{\gamma\gamma\to q\bar{q}}$ was calculated in Refs [9] and [10], and has as an expression (in $d = 4 - 2\varepsilon$):

$$\begin{aligned}
\frac{d\hat{\sigma}}{dv}(\hat{s},v)\Big|_{\rm virt}^{\gamma\gamma\to q\bar{q}} =& \frac{\alpha_s(\mu)}{2\pi}\frac{1}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\varepsilon}\left(\frac{4\pi\mu^2}{\hat{s}v(1-v)}\right)^{\varepsilon} \\
&\times \frac{2\pi\alpha^2}{\hat{s}}e_q^4 C_F N_C\left[\left(\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\right)\left(\frac{v}{1-v}+\frac{1-v}{v}\right)+\frac{4}{\varepsilon}\right] \\
&+ \frac{\alpha_s(\mu)}{2\pi}\frac{2\pi\alpha^2}{\hat{s}}e_q^4 C_F N_C\left[\left(2\frac{\pi^2}{3}-3+\ln^2 v+\ln^2(1-v)\right)\right. \\
&\times\left(\frac{v}{1-v}+\frac{1-v}{v}\right)+2+\left(2+3\frac{1-v}{v}\right)\ln v \\
&+\left(2+3\frac{v}{1-v}\right)\ln(1-v)+\left(2+\frac{v}{1-v}\right)\ln^2 v \\
&\left.+\left(2+\frac{1-v}{v}\right)\ln^2(1-v)\right].
\end{aligned} \tag{3.8}$$

Now, knowing that

$$F^{\varepsilon} = \left(\frac{4\pi\mu^2}{\hat{s}}\right)^{2\varepsilon} v^{-\varepsilon}(1-v)^{\varepsilon}$$

one obtains:

$$A' \propto -2\Big(\frac{v}{1-v} + \frac{1-v}{v}\Big), \qquad B' \propto \Big(\frac{v}{1-v} + \frac{1-v}{v}\Big) + 4$$

and consequently:

$$c_3 = 2\Big(\frac{v}{1-v} + \frac{1-v}{v}\Big)$$

and

$$c_2 = -2\Big(\frac{v}{1-v} + \frac{1-v}{v}\Big)\left[2\ln\left(\frac{4\pi\mu^2}{\hat{s}}\right) - \ln v(1-v)\right] + \Big(\frac{v}{1-v} + \frac{1-v}{v}\Big). \tag{3.9}$$

Now, for the calculation of the term c_1 (which is proportional to $\delta(1-w)$), one has to notice that it is common to virtual and Bremsstrahlung corrections. It is given by [5]:

$$c_1 = [A\ln^2 D + B\ln D + C] + [A'\ln^2 F + B'\ln F + C']. \tag{3.10}$$

For the sub process $\gamma\gamma \to q\bar{q}$ Eq. (2.14) can be simplified to the following,

$$c_1 = c_2 \ln v + c' - c\,,$$

where c_2 is given by Eq. (3.9) and,

$$\begin{aligned} c' = \Bigg[& 2\frac{\pi^2}{3} - 3 + \ln^2 v + \ln^2(1-v)\Big(\frac{v}{1-v} + \frac{1-v}{v}\Big) \\ & + 2 + \left(2 + 3\frac{1-v}{v}\right)\ln v + \left(2 + 3\frac{v}{1-v}\right)\ln(1-v) \\ & + \left(2 + \frac{v}{1-v}\right)\ln^2 v + \left(2 + \frac{1-v}{v}\right)\ln^2(1-v)\Bigg], \end{aligned} \tag{3.11}$$

and c is a term coming only from the two Bremsstrahlung diagrams of figure 2 and gives a contribution proportional to $\frac{1}{(1-w)_+}$ [5]. In fact, for the two diagrams shown in Fig. 2, one has to deal with two types of integrals I_1 and I_2 such that:

$$I_1 = (ps)_3 J_1\,,$$

and

$$I_2 = (ps)_3 J_2 \,,$$

where $(ps)_3$ is the 3-body phase space given by:

$$(ps)_3 = \frac{\hat{s}}{2^8\pi^4 \Gamma)1-2\varepsilon)}\left(\frac{4\pi}{\hat{s}}\right)^{2\varepsilon} \int dv \int dw\, v(1-v)^{-\varepsilon}(1-w)^{-\varepsilon} v^{-2\varepsilon} w^{-\varepsilon}$$
$$\times \int_0^\pi d\theta_2 \sin^{-2\varepsilon}\theta_2 \int_0^\pi d\theta_1 \sin^{1-2\varepsilon}\theta_1 \,, \qquad (3.12)$$

with θ_1 and θ_2 are defined in the c.m.s. of p_1 and p_2 (see Fig. 2), such that

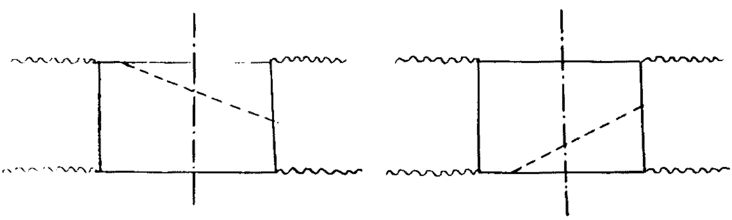


Fig. 2. The Bremsstrahlung Feynmann diagrams contributing to the term c of the subprocess $\gamma\gamma \to q\bar{q}$.

in "d" dimensions, one has:

$$p_1 = \frac{\hat{s}v}{2\sqrt{s_2}}(1,0,\ldots,0,\sin\psi,\cos\psi)\,,$$
$$p_2 = \frac{\hat{s}(1-vw)}{2\sqrt{s_2}}(1,0,\ldots,0,-\sin\psi,\cos\psi)\,,$$
$$p_3 = \frac{\hat{s}(1-v+vw)}{2\sqrt{2_2}}(1,0,\ldots,0,\sin\psi'',\cos\psi'')\,,$$
$$p_4 = \frac{\sqrt{s_2}}{2}(1,\ldots,\cos\theta_2\sin\theta_1,\cos\theta_1)\,,$$
$$k = \frac{\sqrt{s_2}}{2}(1,\ldots,-\cos\theta_2\sin\theta_1,-\cos\theta_1)\,, \qquad (3.13)$$

with

$$s_2 = \hat{s}v(1-w)\,,$$
$$\cos\psi = \sqrt{\frac{w(1-v)}{1-vw}}\,; \quad \sin\psi = \sqrt{\frac{1-w}{1-vw}}\,,$$
$$\cos\psi'' = \frac{1+v-vw}{1-v+vw}\cos\psi\,; \quad \sin\psi'' = -\frac{1-v-vw}{1-v+vw}\sin\psi\,,$$

J_1 and J_2 are given by:

$$J_1 = -2\pi \frac{vw}{\hat{s}(1-v+vw)(1-v)}(1-w^{-1}(4-2\varepsilon)(2-2\varepsilon)^2$$
$$\times \left\{ \frac{1}{v} + \frac{4}{v}\ln\left(\frac{v}{1-v}\right)\varepsilon + \frac{1}{v}\left[6\ln(1-v) + 4\ln\left(\frac{v}{1-v}+1-v+2v^2\right) + \frac{\pi^2}{2}\right]\varepsilon^2 \right\}, \tag{3.14}$$

and

$$J_2 = 2\pi \frac{1}{\hat{s}v(1-w)}(1-w)^{-1}(4-2\varepsilon)(2-2\varepsilon)^2\left[1-2(\ln 2)\varepsilon + \left(2\ln^2 2 + \frac{\pi^2}{6}\right)\varepsilon^2\right].$$

Now, using the expression:

$$(1-w)^{-1-\varepsilon} = -\frac{1}{\varepsilon}\delta(1-w) + \frac{1}{(1-w)_+} - \varepsilon\left[\frac{\ln(1-w)}{1-w}\right]_+ + 0(\varepsilon^2), \tag{3.15}$$

and taking only the contribution proportional to $\delta(1-w)$ we obtain:

$$c = -\left(4\ln 2 + 8\ln\left(\frac{v}{1-v}\right)\right)\frac{v}{(1-v)}, \tag{3.16}$$

therefore the expression of the term c_1 becomes:

$$c_1 = -2\left(\frac{v}{1-v} + \frac{1-v}{v}\right)\left[2\ln\left(\frac{4\pi\mu^2}{\hat{s}}\right) - \ln v(1-v) + 2\ln\mu - \frac{1}{2}\right]\ln v$$
$$+ 4\ln v\left[\left(\frac{2\pi^2}{3} - 3 + \ln^2 v + \ln^2(1-v)\right)\left(\frac{v}{1-v} + \frac{1-v}{v}\right) + 2\right.$$
$$+ (2 + 3\frac{1-v}{v}\ln v + \left(2 + 3\frac{v}{1-v}\right)\ln(1-v) + \left(2 + \frac{v}{1-v}\right)\ln^2 v$$
$$\left. + \left(2 + \frac{1-v}{v}\right)\ln^2(1-v)\right] + \left(4\ln 2 + 8\ln\left(\frac{v}{1-v}\right)\frac{v}{1-v}\right.. \tag{3.17}$$

4. Numerical results and conclusions

We first specify the impute partons fragmentation's functions entering in our numerical calculations for the physical process $\gamma\gamma \to h + X$ (in our case, we take h the pion π^+). The fragmentation of the quarks into pions is taken from Baier, Engels and Peterson [14] which is in a good agreement with the data of PEP [15] and EMC [16]. These are:

$$D_{\pi^+/u}(z, Q^2) = \frac{1+z}{2z}D(z, Q^2), \tag{4.1}$$

and

$$D_{\pi^+/d} = \frac{1-z}{z} D(z,Q), \qquad (4.2)$$

with ($Q_0 = 5$ GeV): $D(z,Q_0^2) = 0.5(1-z)$ (z is the momentum fraction of the quark into the pion) and from SU(3) symmetry, one can write:

$$D_{\pi^+/\bar{d}} = D_{\pi^+/u},$$

and

$$D_{\pi^+/d} = D_{\pi^+/\bar{u}} = D_{\pi^+/s} = D_{\pi^+/\bar{s}}. \qquad (4.3)$$

It is to be noted that the exponent in (4.2) is in accord with counting rules, and the normalization in agreement with $e^+e^- \to \pi + X$ data. The Q^2 dependence is determined from the Altarelli–Parisi evolution integro-differential equation. The fragmentation function is defined in the universal convention [16] (this is necessary when only leading logarithmic parametrization are available).

The results are presented at $\sqrt{s} = 63$ GeV and the QCD. Λ parameter equals 0.2 GeV. We restrict ourselves to the production of three flavours only. Moreover, the strong running coupling is calculated in the leading logarithmic approximation. To be more specific and to emphasize the effects of the renormalization and factorization points dependence, we have taken $Q^2 = \mu^2 = M'^2 = P_{\mathrm{T}}^2$ (P_{T} is the transverse momentum of the pion π^+). It is to be noted that the increasing Q^2 softens the fragmentation function. Thus, we expect that the scale violation in the quark fragmentation function reduces the spectrum when the larger factorization scale ($M'^2 = 2P_{\mathrm{T}}^2$) is used.

Fig. 3 displays the ratio $R \equiv \frac{\sigma_{\mathrm{tot}}}{\sigma_{\mathrm{Born}}}$ (k-factor); for a pseudo rapidity $y = 0$, $\sqrt{s} = 63$ GeV and $Q^2 = P_{\mathrm{T}}^2$ (dashed line) and $Q^2 = 2P_{\mathrm{T}}^2$ (dashed-dotted line). It should be noted that the greater instability in the predictions is obtained for the larger values of $x_{\mathrm{T}}\left(= \frac{2P_{\mathrm{T}}}{\sqrt{s}}\right)$, where the perturbative calculation is expected to be reliable. This suggests the necessity of including higher order corrections. Now, changing the scale Q^2 from P_{T}^2 to $2P_{\mathrm{T}}^2$ introduces in the ratio R, the scale dependent term associated to the quarks split function p_{qq} given by:

$$\frac{\alpha_s}{2\pi} \ln\left(\frac{2P_{\mathrm{T}}^2}{P_{\mathrm{T}}^2}\right) \frac{1}{\pi s x_{\mathrm{min}}} \left.\frac{d\sigma}{dv}\right|_{\gamma\gamma\to q\bar{q}} C_F \Bigg\{ 2\left[\ln(1-x_{\mathrm{min}}) + \frac{3}{4}\right] D_{\pi^+/q}(x_c)$$

$$+ \int\limits_{x_{\mathrm{min}}}^{1} dx \left[\frac{1+x^2}{x} D_{\pi^+/q}\left(\frac{x_c}{x}\right) - 2D_{\pi^+/q}(x_c)\right](1-x)\Bigg\}. \qquad (4.4)$$

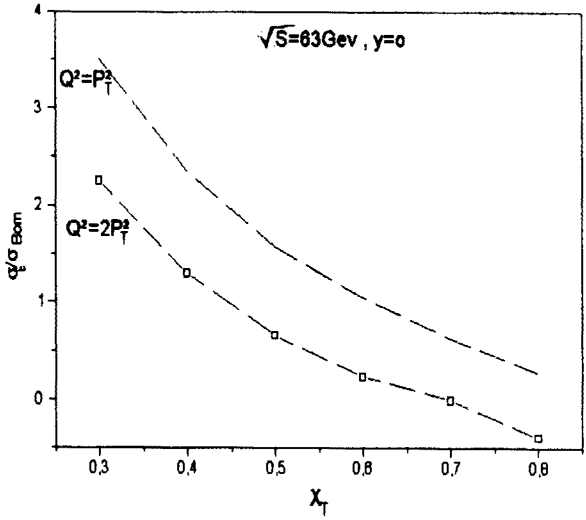
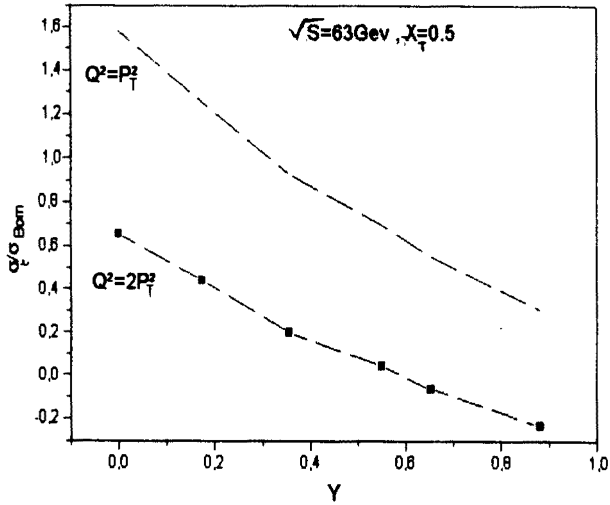


Fig. 3. The ratio $R \equiv \frac{\sigma_{\rm tot}}{\sigma_{\rm Born}}$ (k-factor) for the hadronic process $\gamma\gamma \rightarrow \pi^+ X$ as a function of $x_{\rm T}$ with $\sqrt{s} = 63$ GeV, a pseudo rapidity $y = 0$ and a — $Q^2 = P_{\rm T}^2$ (dashed line), b — $Q^2 = 2P_{\rm T}^2$ (dashed dotted line).

For $x_{\rm min}$ close to 1 (*i.e.* large $x_{\rm T}$) the first and second terms become large and negative, implying a decrease of the correction when the factorization scale increases. At small $x_{\rm T}$ the contribution of the previous terms becomes smaller and, therefore, the correction decreases (with the increase of Q^2) slowly in comparison with the first situation. This behaviour is, of course, necessary to compensate for the changes occurring in the Born term and, therefore, to stabilize the total (fully corrected) cross section $\sigma_{\rm tot}$.

Fig. 4 shows the rapidity dependence at $x_{\rm T} = 0.5$, $\sqrt{s} = 63$ GeV and with different choices of scales $Q^2 = P_{\rm T}^2$ (dashed line) and $Q^2 = 2P_{\rm T}^2$ (dashed-dotted line). Notice that for the choice $Q^2 = 2P_{\rm T}^2$, the ratio is very small and tends to become large and negative for smaller values of y (at the edge of phase space). However, for $Q^2 = P_{\rm T}^2$, and because of the important role played by the logarithmic term in Eq. (4.4), the ratio R is more important. It is to be noted that for both choices of scales, the behaviour of the correction at large $x_{\rm T}$ is affected showing a rapid growth with $x_{\rm T}$.

We conclude that with the soft-gluon approach and the prescription described previously, we have evaluated easily the next-to-leading order corrections without any use of complicated diagrams. More applications are under investigation.



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