THE SUPERSYMMETRY CONSIDERATIONS IN THE FIRST HALF OF THE *sd* SHELL*

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The search for the supersymmetry in light nuclei has been continued. It has been shown that the important ingredient of supersymmetry considerations in the first half of the *sd* shell has come from the assumption of two fermion+boson model for odd-odd nuclei instead of assuming bosons only. With this improvement the approximate supersymmetry has been visible for the considered supermultiplet $N = 5$.

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1. Introduction

The notion of supersymmetry which has come from elementary particle physics found a nice field of application in nuclear physics in the frame of the Interacting Boson Model (IBM) [1, 2]. In the first papers dealing with heavy nuclei [3, 4] there has been demonstrated a very distinct signature of supersymmetry. Although the possibility of introducing the supersymmetry to nuclear physics is from the very beginning only an approximation because bosons of the IBM are only approximate bosons being rather images of pairs of nucleons, these bosons together with unpaired nucleons form a basis for the supersymmetry application.

In recent years the supersymmetry has been extended also to light nuclei. The group theory calculation has become more involved as we need to treat the IBM with the full isospin formalism either in the frame of IBM(3) [5] or IBM(4) [6]. Two of the present authors [7, 8, 9] have been successful to obtain a visible supersymmetry behaviour for the light nuclei from the

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second half of *sd*-shell. However, there have been some problems to extend the supersymmetry to the first half *sd*-nuclei. We have found from our supersymmetry considerations that nuclei from the first half of the *sd*-shell behave differently from those of the second half. It is not the first observation. In the frame of the shell model, Elliott and Wilsdon had made the same remark in their paper [10]. It is only interesting to note that the interpretation of nuclear data in the frame of the shell model had been much better for the first half *sd*-nuclei contrary to the supersymmetry model. The second problem comes from the odd-odd nuclei which happened to be difficult to interpret both in the shell model as in the IBM [11].

2. Basic assumptions and Hamiltonian of the model

Because the IBM and the supersymmetry within the model is of phenomenological nature, we are free to look for the help from phenomenology. By inspection of experimental ground states of *sd*-odd-odd nuclei we have made an observation that, within the IBM model, the unpaired proton and neutron do not form a boson with $L = 0$ (or 2) but their angular momenta are parallel with the resulting $(J_{\mathrm{odd}})_{\max}$ for T_3 even (third component of the isospin) and $(J_{\mathrm{even}})_{\max}$ for T_3 odd due to antisymmetry rules. The observation leads to the assumption of bosons plus two unpaired nucleons for odd-odd nuclei instead of assuming bosons only. To check this assumption within the supersymmetry scheme we have taken the supermultiplet of *sd*-nuclei with $N = 5$ particles (bosons + nucleons) and have assumed that the unpaired nucleons are treated within the $j-j$ coupling and hence, they occupy the $j = 5/2$ level for $N = 5$. For the rest of nucleons on the *sd* shell we assume the IBM(3) version, i.e. bosons with angular momenta $L = 0$ or 2 and the isospin $T = 1$. If we would try to reformulate these assumptions within the shell model we would say that our supersymmetry model is parallel to the mixed $L-S$ and $j-j$ coupling. The reason for such a conclusion is coming from the $j-j$ coupling taken in our model for separate nucleons but bosons can be considered either as coupling of two nucleons in the $L-S$ or in the $j-j$ scheme.

The members of the $N = 5$ supermultiplet are the following:

even-even ${}^{26}_{12}\mathrm{Mg}_{14}$ and ${}^{26}_{14}\mathrm{Si}_{12}$ ($N = 5$ bosons above the double magic shell $A = 16$)

even-odd ${}^{25}_{12}\mathrm{Mg}_{13}$ and ${}^{25}_{13}\mathrm{Al}_{12}$ ($N = 4$ bosons + 1 nucleon)

odd-odd ${}^{24}_{11}\mathrm{Na}_{13}$ ($N = 3$ bosons + 2 nucleons)

The first condition of the supersymmetry behaviour of above nuclei in the supermultiplet $N = 5$ is the interpretation of their ground and excited states within *the same group theory Hamiltonian* coming from supersymmetry considerations and with *the same set of phenomenological parameters.* The simplest Hamiltonian is that one constructed with the help of Casimir invariants of the relevant group-chain. Because of the simple but strong assumptions, we do not expect an exact quantitative numbers but, at least, the firm qualitative results.

From the three IBM limits of the group-chains being denoted by SU(5) limit, SU(3) limit and SO(6) limit, the last two versions could be applied to the deformed nuclei of the *sd*-shell. After detailed inspection we have chosen the SO(6) limit in the IBM group-chain. The dimension of the boson space (the number of the single particle boson states) is, for the IBM (3), equal to 18 ($L = 0$; 2 and $T = 1$) and the fermion space is of the dimension 12 ($j = 5/2$ and $t = 1/2$). The group-chain under consideration reads

$$\mathrm{U}(18/12) \supset \mathrm{U}^B(18) \times \mathrm{U}^F(12)$$

and

$$\begin{aligned}\mathrm{U}^B(18) &\supset \mathrm{SU}^B(6)\times\mathrm{SU}^B_T(3) \supset \mathrm{SO}^B(6)\times\mathrm{SU}^B_T(2) \supset \mathrm{SO}^B(5) \\ &\qquad \times\mathrm{SU}^B_T(2) \supset \mathrm{SO}^B(3)\times\mathrm{SU}^B_T(2) \\ \mathrm{U}^F(12) &\supset \mathrm{SU}^F(6) \times \mathrm{SU}^F_T(2) \supset Sp^F(6) \\ &\qquad \times \mathrm{SU}^F_T(2) \supset \mathrm{SO}^F(3) \times \mathrm{SU}^F_T(2) \end{aligned} \tag{1}$$

and in the next step we form the boson-fermion groups for the total angular momentum and isospin

$$\mathrm{SO}^{BF}(3) \times \mathrm{SU}^{BF}_T(2)$$

To write the Hamiltonian and the energy formula we have made two assumptions.

1° We consider the dynamical Hamiltonian but only with such two-body interaction operators which can be transformed into the Casimir invariants of the group-chain (1).

2° For low energy levels (up to about 4 MeV) we have assumed that the nuclei of the $N = 5$ supermultiplet belong to the unique irreducible representations of the first groups of the chain (1) up to the SO(6) group and hence, the energies given by the eigenvalues of the Casimir operators of those groups give the constant contributions to all of the excited energies of a given nucleus and can be comprised by the constant part of the Hamiltonian.

Under these assumptions the constructed Hamiltonian reads

$$\begin{aligned} H = H_0 &+ A\, C_2[\mathrm{SO}^B(5)] + B\, C_2[\mathrm{SO}^B_L(3)] + \alpha\, C_2[\mathrm{SO}^F_{J_f}(3)] \\ &+ \beta\, C_2[\mathrm{SO}^{BF}_J(3)] + \gamma\, C_2[\mathrm{SU}^F_T(2)] + \delta\, C_2[\mathrm{SU}^B_T(2)] \\ &+ \varepsilon\, C_2[\mathrm{SU}^{BF}_T(2)]\,, \end{aligned} \tag{2}$$

where C_2 are the second order Casimir operators of the given groups, A, B, α, β, γ, δ and ε are phenomenological parameters and H_0 is a constant part which comprises the contributions of Casimir operators as well as the core contribution. We have considered the low energy states of a given nucleus for which $T = T_3$ and then, the last (ε) part of the Hamiltonian can also be put into H_0. Let us discuss also the δ-term of (2). For even-even nuclei $T_B = T$ and then, this term gives also a constant contribution to the energy. For even-odd nuclei within the $N = 5$ supermultiplet, there are 4 bosons and one nucleon. However, bosons are coupled to $T_B = 0; 2; 4$, but only $T_B = 0$ is allowed because $T = T_f = 1/2$. Hence, the δ-term vanishes. For odd-odd nuclei two nucleons are on the $j = 5/2$ level and due to the antisymmetry condition we get $T_f = 0$ for $J_f = 1; 3; 5$ and $T_f = 1$ for $J = 0; 2; 4$. The total isospin of an odd-odd partner of $N = 5$, *i.e.* for the ${}^{24}_{13}Na_{11}$ is equal to $T = 1$ hence, the three bosons (each of $T_B = 1$) can be coupled for the symmetric representation of $\mathrm{SU}^B_T(3)$ to $T_B = 1$ or 3. But $T_B = 1$ is only allowed and the δ-term in this case is also a constant term. Then the δ-term can be also taken into the H_0.

For odd-odd nuclei the α-term is very relevant depending strongly on the even-odd J_f (odd-even T_f). Hence we assume

$$\alpha = (-1)^{T_f}\alpha_0\,, \tag{3}$$

where T_f is equal either $T_f = 0$ or $T_f = 1$ for a pair of nucleons. The α-term need not to be taken into account for even-even and even-odd nuclei.

Hence, the general energy formula for the $N = 5$ supermultiplet reads

$$\begin{aligned} E = E_0 + A\,\tau(\tau+3) + B\,L(L+1) + (-1)^{T_f}\alpha_0\,J_f(J_f+1) \\ + \beta\,J(J+1) + \gamma\,T_f(T_f+1)\,. \end{aligned} \tag{4}$$

For even-even and even-odd nuclei the formula (4) is only the 3-parameter (A, B, β) formula. For odd-odd nuclei there are in (4) four but not five parameters because in the calculation of relative energies the α_0 and γ parameters enter the calculations only as a combination $11\alpha_0 - \gamma$. In Table I we give the adjusted parameters of our model

TABLE I

The Hamiltonian (2) parameters in MeV for the $N = 5$ supermultiplet.

A	B	β	$11\alpha_0 - \gamma$
0.19	0.12	0.08	0.82

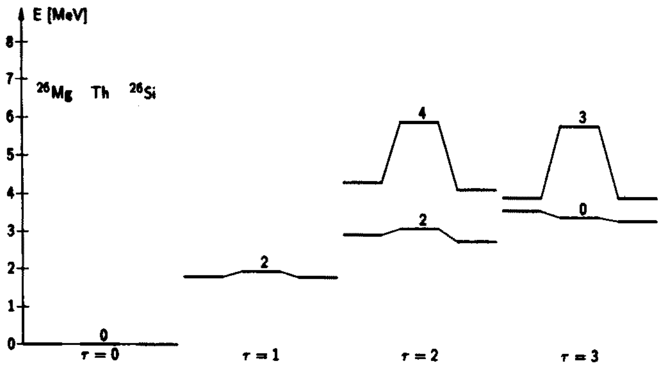

Fig. 1a. Comparison of theoretical and experimental energy levels for nuclei of the $N = 5$ supermultiplet from the first half of the *sd* shell. Levels are organized into the $SO(5)$ representations (τ). Theoretical levels are provided with angular momentum values. For each $SO(5)$ multiplet theoretical levels are placed in the middle while experimental ones for ^{26}Mg and ^{26}Si on the left and right side respectively.

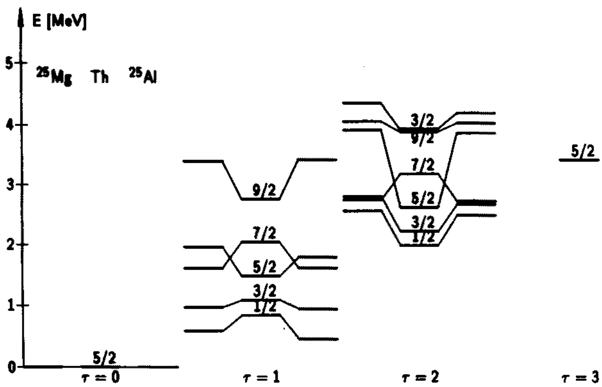

Fig. 1b. Experimental levels for ^{25}Mg (left columns), ^{25}Al (right columns) and theoretical ones (middle columns), see also caption to Fig. 1a.

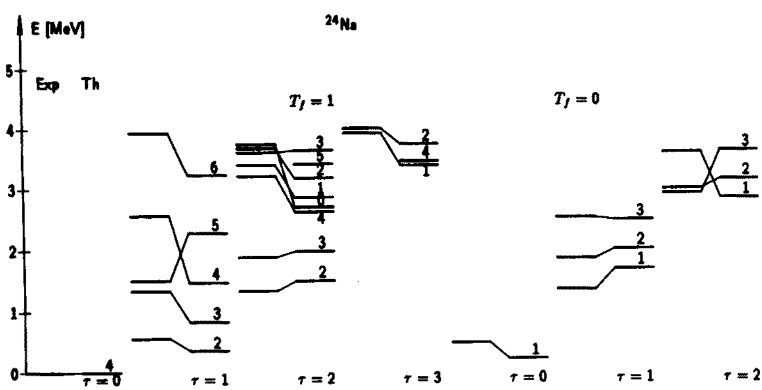

Fig. 1c. Experimental levels for ^{24}Na (left columns) and theoretical ones (right columns), see also caption to Fig. 1a.

The calculated spectra and comparison with experimental data [12] for nuclei of $N = 5$ are given in Fig. 1. We should stress that for all of nuclei there have been taken all of the experimental even-parity levels up to the considered energy (around 4 MeV) and also, all of the calculated energies are shown in Fig. 1. There are only two cases of the calculated energies which have no experimental partners. It is remarkable that the angular momenta are adjusted perfectly and the calculated energies with a quite reasonable accuracy describe the experimental levels. Having in mind that it has been done only with the four free parameter formula, we conclude that our supersymmetry model has found its approximate confirmation in the first half of the sd-shell for the $N = 5$ supermultiplet, at least, as the energy levels are taken into account.

The E(2) transitions

The electromagnetic transition calculation has an important value not only because it provides another set of numbers to compare with experiment but also and mostly because such calculations depend strongly on constructed state vectors and hence, the goodness of comparison with experimental data is a measure of the goodness of the model used in the calculations.

The electromagnetic transition operator for a system with bosons and fermions has to contain boson and fermion parts. Let us denote boson and fermion creation operators as b_{lt}^{+}; a_{jt}^{+} respectively, where $l(j)$ is for the angular momentum and t is the isospin number equal to 1 for bosons and 1/2 for fermions (we omit here the third components of those vectors). We consider the $E(2)$ transition operator under the following restrictions: *(i)* it is a scalar in the isospin space and *(ii)* it has the dynamical symmetry form due to the group SO(6), in other words it is constructed from the generators of the group SO(6) [8]. Hence, the $E(2)$ transition operator reads

$$T(E2) = q_B \left[(b_{01}^{+}\tilde{b}_{21})^{(20)} + (b_{21}^{+}\tilde{b}_{01})^{(20)} \right] + q_F (a_{\frac{5}{2}\frac{1}{2}}^{+}\tilde{a}_{\frac{5}{2}\frac{1}{2}})^{(20)}, \tag{5}$$

where the parentheses () mean the normal coupling with the help of Clebsch–Gordan coefficients in the angular momentum, spin and isospin spaces respectively, to the total LT for bosons and JT for fermions. The constructed $T(E2)$ operator is compact with the assumption of a dynamical symmetry group chain of the SO(6) type. The coefficients q_B and q_F play a role of effective charges.

The reduced transition probability between the initial $|i\rangle$ and final $|f\rangle$ state vectors reads

$$B(E2) = (2J_i + 1)^{-1} (\langle f \| T(E2) \| i \rangle)^2, \tag{6}$$

where (i) and (f) stand for relevant quantum numbers of the initial and final states and J_i is the angular momentum quantum number for the initial state. Because the tensorial characters of the $T(E2)$, in the relevant transformation groups, are known by construction, the calculation of the reduced matrix element (6) can be done purely in the group theory algebraic method [2, 8]. The selection rules following the construction (5) are: *(i)* the boson part of (5) gives non–zero transition probabilities only for $\tau_f = \tau_i \pm 1$ and *(ii)* the fermion part of (5) has non-vanishing matrix elements only between the states with the same boson quantum numbers τ, L, because fermions in the supersymmetry model are independent, by assumption, of bosons, and hence, the fermion operators are scalars in the boson space and vice versa.

The boson effective charge was fitted to experimental data of the nucleus ^{26}Mg as $q_B^2 = 4\mathrm{e}^2\mathrm{fm}^4$. We have taken somewhat arbitrarily the same value for $q_F^2 = 4\mathrm{e}^2\mathrm{fm}^4$. The experimental [12, 13] $B(E2)$ data are, in a quite wide extent, known for nuclei ^{26}Mg, ^{25}Mg and ^{25}Al from the supermultiplet $N = 5$. For those nuclei we give also in Table II the theoretical values of $B(E2)$. The study of Table II shows that the agreement with experimental data for the nucleus ^{26}Mg is rather good according to the usual standard of comparison. However, for odd nuclei ^{25}Mg and ^{25}Al theoretical numbers are quite apart from experimental values. We have found a probable explanation of such a deviation. Namely, in the work of Halse *et al.* [14] the even-even nuclei of the *sd*-shell were treated within the IBM but without assuming the dynamical symmetry. The results have shown that the structure of the spectrum changed significantly with the change of the boson number and the isospin of nuclei. For example, the ^{24}Mg has rather the rotational structure but the ^{26}Mg has shown the spectra of the SO(6) type. Our conclusion from the results is that, probably, for nuclei in between with $A = 25$ the dynamical symmetry is a mixture of the SU(3) and SO(6) symmetries. Then the Hamiltonian for the supermultiplet considered in our paper should be generalized to comprise the mixture of the symmetries. Our further effort is devoted to such a generalization of the Hamiltonian whose eigenproblem can be solved numerically in the constructed IBM basis [15]. We hope the extended calculation to improve not only the visible disagreement in $B(E2)$ for odd nuclei, but also other theoretical prediction in the supersymmetry model.

In the final conslusion we stress that the approximate supersymmetry in the first half of *sd*-nuclei can be seen only by the assumption that an odd-proton and an odd-neutron do not form a boson but they remain as a two-fermion state whose lower energy states are rather with maximum J but not with $J = 0$ or 2 as for bosons. The assumption is a crucial one for supersymmetry considerations in that region of nuclei.

TABLE II

$B(E2)$ transition probabilities (in e^2 fm^4)

	E_i	E_f	J_i	J_j	Exp [12,13]	Theory
^{26}Mg						
	1.81	0	2	0	59.9 ± 1.4	36.0
	2.94	0	2	0	1.8 ± 0.1	0
		1.81		2	27.0 ± 2.3	45.7
	3.59	1.81	0	2	4.8 ± 0.1	0
	4.32	1.81	4	2	20.6 ± 1.4	45.7
	4.33	0	2	0	1.1 ± 0.3	0
	4.35	2.94	3	2	41.2 ± 9.2	31.4
^{26}Si						
	1.8	0	2	0	68.6 ± 6.4	36.0
	3.33	1.8	0	2	45.8 ± 16.5	0
^{25}Mg						
	0.59	0	1/2	5/2	2.3 ± 0.04	25.6
	0.97	0	3/2	5/2	3.3 ± 0.1	25.6
		0.59		1/2	52.1 ± 4.3	1.1
	1.61	0	7/2	5/2	121.6 ± 17.4	25.6
	1.96	0	5/2	5/2	1.9 ± 0.9	25.6
		0.59		1/2	78.2 ± 30.4	0.4
		0.97		3/2	13.0 ± 5.6	1.1
	2.56	0	1/2	5/2	11.7 ± 1.3	0
	2.74	0	7/2	5/2	0.7 ± 0.1	0
		0.97		3/2	99.9 ± 8.7	6.8
	2.80	0	3/2	5/2	7.4 ± 2.2	0
	3.405	0	9/2	5/2	32.6 ± 2.2	25.6
		1.61		7/2	65.1 ± 5.2	0
	4.06	0	9/2	5/2	5.6 ± 0.4	0
		1.61		7/2	8.3 ± 0.9	11.6
^{25}Al						
	0.45	0	1/2	5/2	13.0 ± 0.4	25.6
	0.94	0	3/2	5/2	7.8 ± 2.2	25.6
	1.79	0	5/2	5/2	8.3 ± 1.3	25.6
		0.45		1/2	134.6 ± 17.4	0.4
		0.94		3/2	36.9 ± 5.2	1.1
	2.72	0.94	7/2	3/2	95.5 ± 26.1	6.8
	3.86	0.45	5/2	1/2	2.6 ± 0.9	8.2

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