Vol. 26(1995) *ACTA PHYSICA POLONICA B* No 9

MAGNETIC UNIVERSES AND MAGNETIZED BLACK HOLES IN 5-DIMENSIONAL PROJECTIVE UNIFIED FIELD THEORY

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(Received March 16, 1995; revised version received July 12, 1995)

Exact solutions are found in 5-dimensional projective unified field theory (PUFT) for a Melvin-like magnetic universe with (and without) a non-rotating black hole on its axis of symmetry. These solutions and the motion of test bodies are compared with those in Einstein-Maxwell theory. It is shown that particles with positive scalaric mass (a new hypothetical characteristic of matter in PUFT) can accrete onto stars if magnetic fields are present in their surroundings.

PACS numbers: 04.20. Qr

1. Introduction

The 5-dimensional projective unified field theory (PUFT) developed by Schmutzer [1] is based on the postulated Einstein- like field equations

$$\overset{5}{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\overset{5}{R} = \kappa_0 \Theta_{\mu\nu}\,, \tag{1a}$$

with

$$\Theta^{\mu\nu}_{;\nu} = 0\,, \tag{1b}$$

* This work was supported by Fundamental Research Foundation of Belarus.

where κ_0 is Einstein's gravitational constant and $\Theta^{\mu\nu}$ is the energy projector of the non-geometrized matter ("substrate"). The cosmological term is omitted here.

Projection of (1) into the 4-dimensional space-time by a specific vectorial projection formalism leads to the following 4-dimensional field equations (Gauss system of units):

$$\overset{4}{R}_{mn} - \frac{1}{2} g_{mn} \overset{4}{R} = \kappa_0 \left(E_{mn} + \Sigma_{mn} + \Theta_{mn} \right), \tag{2}$$

generalized gravitational field equations;

$$H^{mn}{}_{;n} = \frac{4\pi}{c} j^m, \tag{3a}$$

$$B_{[mn,k]} = 0, \tag{3b}$$

$$H_{mn} = e^{3\sigma} B_{mn}, \tag{3c}$$

generalized electromagnetic field equations;

$$\sigma^{,k}{}_{;k} = \kappa_0 \left(\frac{2}{3} \vartheta + \frac{1}{8\pi} B_{jk} H^{jk} \right), \tag{4}$$

scalaric field equation. To characterize the scalar field predicted in PUFT as a new fundamental phenomenon in Nature we introduced the notion "scalarism" in analogy to electromagnetism (see [2]). In the equations (2)–(4) B_{jk} and H_{jk} are, respectively, electromagnetic field strength and induction tensors,

$$E_{mn} = \frac{1}{4\pi} \left(B_{mk} H^k{}_n + \frac{1}{4} g_{mn} B_{jk} H^{jk} \right) \tag{5}$$

is the electromagnetic energy tensor,

$$\Sigma_{mn} = -\frac{3}{2\kappa_0} \left(\sigma_{,m} \sigma_{,n} - \frac{1}{2} g_{mn} \sigma_{,k} \sigma^{,k} \right) \tag{6}$$

is the scalaric energy tensor, Θ_{mn} is the substrate energy tensor, j^m is the electric four-current density and

$$\vartheta = e^{-\sigma} \Theta^\mu{}_\mu - \frac{3}{2} \Theta^m{}_m \tag{7}$$

is the scalaric substrate density. The scalaric substrate density is responsible for a new attribute of matter — the so-called scalaric mass, which can be in principle independent of ordinary inertial mass.

Obviously electromagnetism can give birth to the hypothetical scalaric σ-field. Another source of the scalaric field could be the scalaric mass of matter (see eq. (4)). Exact solutions with electromagnetic fields in PUFT are therefore interesting because they can give important information about the scalaric field itself and the influence of the latter on the electromagnetic field.

In this paper we find in PUFT the exact static cylindrically symmetric solution of the 4-dimensional field equations (2)–(4) with magnetic field parallel to the axis of symmetry. A similar solution in Einstein–Maxwell theory is known as Melvin's magnetic universe [3,4]. Several aspects concerning geodesics and motion of test bodies will be represented. Furthermore, we find the exact solution for a magnetic universe which contains a non-rotating black hole on its axis of symmetry. Much attention has been attracted by such a model of black holes immersed in external magnetic fields which are homogenous at infinity ("magnetized" black holes, see [5, 6]). As is well-known, magnetized black holes could be an appropriate model for some stars with a magnetic field in their vicinity. This magnetic field can be created, for instance, by physical processes occurring in the plasma nearby the star — and not by the star itself. We have come to the conclusion that particles with non-zero positive scalaric mass can accrete onto magnetized black holes.

2. Melvin-like magnetic universe

For a Melvin-like magnetic universe (see, for example [7]) we start with the line element:

$$ds^2 = e^{2\lambda(\rho)}\left(d\rho^2 - dt^2\right) + e^{-2\psi(\rho)}d\varphi^2 + \rho^2 e^{2\psi(\rho)}dz^2 . \tag{8}$$

The field equations (2)–(4) lead to the following set of equations (prime means derivation with respect to ρ)

$$\lambda'' + 2(\psi')^2 + \frac{2\psi' - \lambda'}{\rho} = -e^{(2\psi+3\sigma)}(B_{12})^2 - \frac{3}{2}(\sigma')^2 , \tag{9}$$

$$\psi'' + \frac{\psi'}{\rho} = e^{(2\psi+3\sigma)}(B_{12})^2 , \tag{10}$$

$$\lambda'' + \frac{\lambda'}{\rho} = e^{(2\psi+3\sigma)}(B_{12})^2 , \tag{11}$$

$$\frac{d}{d\rho}\left(\rho e^{(2\psi+3\sigma)}B_{12}\right) = 0 , \tag{12}$$

$$\sigma'' + \frac{\sigma'}{\rho} = 2e^{(2\psi+3\sigma)}(B_{12})^2 . \tag{13}$$

The magnetic field strength tensor has only one non-zero component $B_{12} = -B_{21}$ $(x^1 = \rho, x^2 = \varphi, x^3 = z, x^4 = t;$ z is the axis of symmetry). Equation (12) gives

$$B_{21} = \frac{B}{\rho} e^{-(2\psi+3\sigma)}, \tag{14}$$

where B is an appropriate constant of integration. From equations (11) and (13), by means of (10), we obtain

$$\lambda - \psi = C \ln \rho\,, \tag{15}$$

$$\sigma - 2\psi = D \ln \rho \tag{16}$$

(D and C are constants of integration). Taking into account the relations (14)–(16) we find in place of (9) the following equation:

$$(\psi')^2 + \frac{3D}{4\rho}\psi' = \frac{C - (3/4)D^2}{4\rho^2} - \frac{B^2 e^{-8\psi}}{4\rho^{3D+2}}. \tag{17}$$

In an intermediate step we define

$$\chi = \ln \rho \qquad \text{and} \qquad \eta = 8\psi + 3D\chi. \tag{18}$$

Equation (17) leads to

$$\left(\frac{d\eta}{d\chi}\right)^2 = 16C - 3D^2 - 16B^2 e^{-\eta}. \tag{19}$$

As a consequence of (19) we have the inequality

$$\widetilde{C} \equiv C - \frac{3D^2}{16} > 0. \tag{20}$$

Integration of (19) gives

$$e^{\eta/2} = U_0 \exp\left(2\sqrt{\widetilde{C}}\chi\right) + \frac{m^2}{4U_0} \exp\left(-2\sqrt{\widetilde{C}}\chi\right), \tag{21}$$

where $m^2 = 16B^2/(16C - 3D^2) = B^2/\widetilde{C}$, and U_0 is an arbitrary constant depending on the unit of measurement of the magnetic strength. With the help of (21), after some calculations, we obtain

$$\sigma = \frac{1}{2} \ln\left[U_0 \rho^{\left(D/2+2\sqrt{\widetilde{C}}\right)} + \frac{m^2}{4U_0} \rho^{\left(D/2-2\sqrt{\widetilde{C}}\right)}\right], \tag{22}$$

$$B_{21} = \frac{B}{\rho}\left[U_0 \rho^{2\sqrt{\widetilde{C}}} + \frac{m^2}{4U_0} \rho^{(-2\sqrt{\widetilde{C}})}\right]^{-2}. \tag{23}$$

The physical magnetic field strength can be defined as

$$B_{\text{phys}} = \frac{B_{21}}{\sqrt{g_{11}g_{22}}} = B\left\{ U_0 \rho^{\left[(C+1)/2+2\sqrt{\widetilde{C}}\right]} + \frac{m^2}{4U_0}\rho^{\left[(C+1)/2-2\sqrt{\widetilde{C}}\right]} \right\}^{-2}. \tag{24}$$

Both σ and B_{phys} must remain finite when ρ tends to zero. Consequently from the relations (22) and (24) we obtain

$$C = 1\,, \quad D = 2\,, \quad \widetilde{C} = \tfrac{1}{4}\,, \quad m^2 = 4B^2\,. \tag{25}$$

Here we have chosen $U_0 = m^2/4 = B^2$. Thus our exact solution now reads:

$$e^{2\lambda} = \left(1 + B^2\rho^2\right)^{1/2}, \tag{26a}$$

$$e^{2\psi} = \rho^{-2}\left(1 + B^2\rho^2\right)^{1/2}, \tag{26b}$$

$$\sigma = \frac{1}{2} ln\left(1 + B^2\rho^2\right), \tag{26c}$$

$$B_{21} = B\rho\left(1 + B^2\rho^2\right)^{-2}, \tag{26d}$$

$$B_{\text{phys}} = B\left(1 + B^2\rho^2\right)^{-2}. \tag{26e}$$

The constant B has the physical meaning of the magnetic field strength on the axis of symmetry. B_{phys} decreases with distance from the axis of symmetry somewhat faster than it does in Einstein-Maxwell theory (for $B\rho \ll 1$) (see [3]):

$$\left(B_{\text{phys}}\right)_{E-M} = B\left(1 + B^2\rho^2/4\right)^{-2}.$$

3. Geodesics and motion of test bodies in magnetic universes

In PUFT the equation of motion of a point-like test body reads [1]:

$$m u^m{}_{;k} u^k = \frac{e}{c} B^m{}_n u^n - s_0 c^2 \left(\sigma^{,m} + \frac{1}{c^2}\frac{d\sigma}{d\tau} u^m \right), \tag{27}$$

where $u^m = dx^m/d\tau$ is the four-velocity, and m, e, and s_0 are, respectively, the mass, the electric charge and the scalaric mass of the test body. Naturally a neutral test body with zero scalaric mass will move along a time-like geodesics. It can be shown that in PUFT, as in General Theory of Relativity, light beams propagate along null geodesics. In Table I we indicate the main features of the motion of test bodies in magnetic universes in the Einstein–Maxwell theory [4] and in PUFT.

TABLE I

Motion of test bodies in magnetic universes. Here ρ_0 is the positive root of the cubic equation $\left(\frac{e}{m}\right)^2 - 2\rho\left(\frac{3}{4}B^2\rho^2 - 1\right) = 0$.

Forms of motion	Einstein–Maxwell theory	PUFT
Circular geodesics	$0 \leq \rho \leq \frac{2}{\sqrt{3}B}$ time-like $\rho = \frac{2}{\sqrt{3}B}$ light-like $\frac{2}{\sqrt{3}B} < \rho \leq \frac{2}{B}$ space-like $\rho > \frac{2}{B}$ no circular geodesics	$0 \leq \rho \leq \infty$ time-like
Geodesics paralell to the axis of symmetry	Null-geodesics are given by the equation $z = \pm t$	
Circular orbits of charged particles	$0 \leq \rho \leq \rho_0$	$0 \leq \rho \leq \infty$ (Under the condition) $s_0/m \geq -1/2$

For circular time-like orbits the periods of rotation as measured by the particle are equal to:

(a) circular geodesics in Einstein–Maxwell theory

$$T = \frac{2\pi\sqrt{2}}{B}\rho^{1/2}\left(1 - \frac{3}{4}B^2\rho^2\right)^{1/2}\left(1 + \frac{1}{4}B^2\rho^2\right)^{-1}, \tag{28}$$

in PUFT

$$T = \frac{2\pi\sqrt{2}}{B}\rho^{1/2}\left(1 + B^2\rho^2\right)^{-1/4}, \tag{29}$$

(b) circular orbits of charged particles in the Einstein–Maxwell theory

$$T = \frac{4\pi}{B}\rho\left(1 - \frac{3}{4}B^2\rho^2\right)\left(1 + \frac{1}{4}B^2\rho^2\right)^{-1}$$

$$\times\left\{\pm\frac{e}{m} + \left[\left(\frac{e}{m}\right)^2 + 2\rho\left(1 - \frac{3}{4}B^2\rho^2\right)\right]^{1/2}\right\}^{-1}, \tag{30}$$

in PUFT

$$T = \frac{4\pi}{B}\rho\left(1 + B^2\rho^2\right)^{1/2}$$

$$\left\{\pm\frac{e}{m} + \left[\left(\frac{e}{m}\right)^2 + 4\rho\left(1 + B^2\rho^2\right)^{9/2}\left(\gamma_0 + \frac{1}{2}\right)\right]^{1/2}\right\}^{-1}, \tag{31}$$

where $\gamma_0 = s_0/m$. Here and in Table I ρ denotes the radius of the circular orbit.

4. "Magnetized" black holes

In the general theory of relativity it is possible to obtain the exact solution describing magnetized black holes by applying Ernst's transformation to Melvin's magnetic universe [4, 7, 8]. The results are [5] ($x^1 = r, x^2 = \varphi, x^3 = \theta$, $x^4 = t$):

$$ds^2 = \Lambda^2\left[\left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + \frac{r^2 \sin^2\theta d\varphi^2}{\Lambda^4} - \left(1 - \frac{2M}{r}\right)dt^2\right], \tag{32}$$

$$B_{21} = Br\Lambda^{-2}\sin^2\theta, \qquad B_{23} = Br^2\Lambda^{-2}\sin\theta\cos\theta, \tag{33}$$

where M is the mass of the black hole,

$$\Lambda = 1 + \tfrac{1}{4}B^2 r^2 \sin^2\theta. \tag{34}$$

The expressions (32) and (33) coincide with Melvin's solution when $M \to 0$ and with the Schwarzschild solution when $B \to 0$. In PUFT similar methods which generate new exact solutions, as far as we know, have not been elaborated. But it can be shown that by analogy with (32) and (33) magnetized black holes are described by the exact solution

$$ds^2 = L^{1/2}\left[\left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + \frac{r^2\sin^2\theta d\varphi^2}{L} - \left(1 - \frac{2M}{r}\right)dt^2\right], \tag{35}$$

$$B_{21} = BrL^{-2}\sin^2\theta\,, \qquad B_{23} = Br^2L^{-2}\sin\theta\cos\theta\,, \tag{36}$$

$$\sigma = \tfrac{1}{2}\ln L, \tag{37}$$

where $L = 1 + B^2r^2\sin^2\theta$. The black hole has no scalaric mass. Therefore, the scalaric σ-field (37) was generated exclusively by the magnetic field (36), which is, as expected, homogenous and parallel to the axis of symmetry at infinity.

The solution (35)–(37) is not asymptotically flat. When $B \to 0$ it goes into the solution for an ordinary black hole without scalaric mass. (This is the case when the Schwarzschild solutions in general relativity and in PUFT are the same. For more details see [1].) When $M \to 0$ it coincides with the Melvin-like solution found in Section 2. An event horizon exists at $r = 2M$, which is not singular.

According to (37) the scalaric σ-field diverges logarithmically at infinity. (An analogous behaviour at infinity has the electromagnetic potential generated by an infinitely long charged string.) But the divergence of σ-field appearing in the equation of motion of a point-like test body (27) remains finite. Of course for realistic objects magnetic field falls to zero as one moves far away enough from the center, and the scalaric field goes to a constant. More details on the motion of test bodies in the metric (35) and in the fields (36), (37) can be worked out using the Hamilton–Jacobi equation. (The Hamilton–Jacobi equation in PUFT can be obtained on non-trivial assumptions.) Our investigations show that a particle in an external scalaric field has an effective potential energy proportional to its scalaric mass and to the value of the scalaric field (see also [1]). Using the fact that around a magnetized black hole the scalaric field increases monotonously with the distance from the center (see formula (37)), we conclude that particles with positive scalaric mass will accrete onto the black hole. Scalaric mass of matter plays an exclusive role in PUFT. Therefore, if one can observe any tracks of scalar mass in stars (with magnetic fields in vicinity) it would be highly interesting.

It should be taken into account that in PUFT and in Einstein–Maxwell theory the approximate solutions of magnetized black holes up to the second order of B coincide with each other.

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