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# POSSIBLE DEFORMATION OF TIME RUN INDUCED BY CHANGE OF PARTICLE NUMBER*

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A report is presented on some developments of the author's recent conjecture that a change of the overall particle number in a localized physical process induces in its neighbourhood a small deformation of the time run. This is a hypothetic quantum effect caused by a thermodynamic-type mechanism not present in the Einsteinian classical theory of gravitation, but natural if the familiar analogy between the thermal equilibrium and the unitary quantum time evolution is accepted as a physically profound correspondence.

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## 1. Introduction

We describe in this paper some developments of the recent conjecture [1] that a change of the overall particle number in a localized physical process induces in its proximity a small deformation of the time run. It is a hypothetic quantum effect caused by a thermodynamic-type mechanism not present in the Einsteinian classical theory of gravitation, but natural if the familiar analogy [2] between the thermal equilibrium and the unitary quantum time evolution is assumed to be a physically profound correspondence.

In order to express more precisely our idea let us consider first a classical particle moving in an interval of time along the trajectory

$$\vec{r} = \vec{r}(t)\,. \tag{1}$$

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Then, we can speak of the particle's density and current given by the distributions

$$\rho(\vec{r},t) = \delta^3\left(\vec{r} - \vec{r}(t)\right), \quad \vec{j}(\vec{r},t) = \dot{\vec{r}}(t)\delta^3\left(\vec{r} - \vec{r}(t)\right). \tag{2}$$

It is easy to see that they satisfy the local conservation equation (continuity equation):

$$\frac{\partial\rho(\vec{r},t)}{\partial t} + \mathrm{div}\vec{j}(\vec{r},t) = 0\,. \tag{3}$$

Now, let us ascribe to the classical particle the ability of appearing or disappearing at a moment of time $t = t_{\mathrm{S}}$ and point of space $\vec{r} = \vec{r}_{\mathrm{S}}$. Then, we may write

$$\rho(\vec{r},t) = \theta\left[\pm(t - t_{\mathrm{S}})\right]\delta^3\left(\vec{r} - \vec{r}(t)\right), \; \vec{j}(\vec{r},t) = \theta\left[\pm(t - t_{\mathrm{S}})\right]\dot{\vec{r}}(t)\delta^3\left(\vec{r} - \vec{r}(t)\right) \tag{4}$$

in the first or second situation, respectively. Here, $\theta(t) = 1,\ 0$ for $t > 0$, $< 0$, while $\vec{r}(t_{\mathrm{S}}) = \vec{r}_{\mathrm{S}}$. In this case, the continuity equation (3) is not valid, being modified by a source term:

$$\frac{\partial\rho(\vec{r},t)}{\partial t} + \mathrm{div}\vec{j}(\vec{r},t) = \pm\delta^3\left(\vec{r} - \vec{r}_{\mathrm{S}})\right)\delta(t - t_{\mathrm{S}})\,, \tag{5}$$

respectively, because of $d\theta(\pm t)/dt = \pm\delta(t)$. Hence, for a fixed spatial region $V$ including the point $\vec{r}_{\mathrm{S}}$

$$\frac{d}{dt}\int_V d^3\vec{r}\,\rho(\vec{r},t) = -\int_{\partial V} d^2\vec{\sigma}\cdot\vec{j}(\vec{r},t) \pm \delta(t - t_{\mathrm{S}})\,, \tag{6}$$

respectively. When $V$ becomes the whole space, the surface integral, as containing $d^2\vec{\sigma} \equiv \vec{n}(\vec{r})d^2\sigma \to (\vec{r}/r)r^2 d\Omega$, vanishes since at $t$ fixed $\delta^3\left(\vec{r} - \vec{r}(t)\right) \to \delta^3(\vec{r})$ with $r \to \infty$.

If in a physical process $N_{\mathrm{S}}$ classical particles disappear at a moment $t_{\mathrm{S}}$ and point $\vec{r}_{\mathrm{S}}$, and $N'_{\mathrm{S}}$ classical particles appear at the same moment and same point, then Eq. (5) is generalized to the form:

$$\frac{\partial\rho(\vec{r},t)}{\partial t} + \mathrm{div}\vec{j}(\vec{r},t) = (N'_{\mathrm{S}} - N_{\mathrm{S}})\,\delta^3\left(\vec{r} - \vec{r}_{\mathrm{S}}\right)\delta(t - t_{\mathrm{S}})\,. \tag{7}$$

Thus, only when $N'_{\mathrm{S}} = N_{\mathrm{S}}$ the overall particle number

$$N(t) = \int d^3\vec{r}\rho(\vec{r},t) \tag{8}$$

is conserved. In general, Eq. (7) implies

$$\frac{dN(t)}{dt} = (N'_{\mathrm{S}} - N_{\mathrm{S}})\,\delta(t - t_{\mathrm{S}})\,. \tag{9}$$

In the case of the current modified as in Eq. (4), the particle's velocity along the trajectory is replaced by

$$\theta\,[\pm(t - t_{\mathrm{S}})]\,\dot{\vec{r}}(t)\,, \tag{10}$$

so that its acceleration becomes

$$\theta\,[\pm(t - t_{\mathrm{S}})]\,\ddot{\vec{r}}(t) \pm \delta(t - t_{\mathrm{S}})\dot{\vec{r}}(t)\,, \tag{11}$$

making the Newton equation $m\ddot{\vec{r}} = \vec{F}$ to be modified into the form:

$$\theta\,[\pm(t - t_{\mathrm{S}})]\left[m\ddot{\vec{r}}(t) - \vec{F}\right] = \mp\delta(t - t_{\mathrm{S}})m\dot{\vec{r}}(t)\,. \tag{12}$$

Then, also the d'Alembert principle $(\vec{F} - m\ddot{\vec{r}}) \cdot \delta\vec{r} = 0$ takes the modified form:

$$\theta\,[\pm(t - t_{\mathrm{S}})]\left[\vec{F} - m\ddot{\vec{r}}(t)\right] \cdot \delta\vec{r} = \pm\delta(t - t_{\mathrm{S}})\,m\,\dot{\vec{r}}(t) \cdot \delta\vec{r}\,. \tag{13}$$

It means that the classical dynamical equilibrium expressed traditionally [3] by the d'Alembert principle is *violated* at the moment $t = t_{\mathrm{S}}$ when the particle appears or disappears. Notice, however, that this event has *no consequences at all* for the dynamical equilibrium in the interval $t > t_{\mathrm{S}}$ or $t < t_{\mathrm{S}}$ where Eqs. (12) and (13) reduce to the conventional forms $m\ddot{\vec{r}} = \vec{F}$ and $(\vec{F} - m\ddot{\vec{r}}) \cdot \delta\vec{r} = 0$. The fundamental reason (or assumption) leading to such a conclusion is that in the motion of a classical particle the sharp moment $t_{\mathrm{S}}$ of time transition is *always* well defined (as a limiting concept), *even* on the energy shell.

When passing to a quantum particle, the situation changes drastically due to Heisenberg's uncertainty relation for time and energy. This relation, when applied to the moment of time transition, makes such a moment uncertain.

From the conceptional viewpoint, the phenomenon of changing the particle number is perfectly described by the procedure of field quantization. However, our classical argument presented above suggests that in processes, where the overall number of particles changes, the quantum dynamical equilibrium expressed (in the Schrödinger picture) by the unitarity state equation

$$i\hbar\frac{d\Psi(t)}{dt} = H\Psi(t)\ ,\ \ H^{\dagger} = H \tag{14}$$

may be in principle *violated.* Accordingly, it was conjectured recently [1] that this equation, always accepted in the quantum theory of closed systems [4] and then implying the unitary time evolution, undergoes a slight *modification* in processes with a change of the overall particle number. Thus, in such processes tiny *departures* appear for physical systems from their quantum dynamical equilibrium. Such departures should manifest themselves as small nonunitary *deviations* from their unitary time evolution characteristic for this equilibrium. So, the last can be conveniently interpreted as a thermodynamic-type equilibrium with the physical spacetime treated as a "minimal" unavoidable surroundings of all so-called closed systems which, therefore, are always "minimally" open. If the conjectured departures from the quantum dynamical equilibrium really appear, this interpretation is not only a way of speaking.

As is well known, there is a far-going formal analogy [2] between temperature of an open system persisting in a thermal equilibrium with a thermostat (or heat reservoir), and time that parametrizes quantum evolution of a so-called closed system:

$$kT \leftrightarrow \frac{\hbar}{it}\,. \tag{15}$$

If taken earnestly, this analogy can strengthen and make more specific the thermodynamic-type interpretation of quantum dynamical equilibrium as a *temporal equilibrium* of physical systems with the physical spacetime playing here the role of a *chronostat* (or *energy-width reservoir*). By definition, this guarantees equal run of time at all space points (in a Minkowski frame). Of course, such a thermodynamic-type interpretation has a phenomenological character: it abstracts from the physical nature of spacetime, and does it in an analogical way as the thermodynamics abstracts from the physical nature of the body playing the role of a thermostat. This causes, by definition, equal distribution of temperature. Hence the term "chronostat". The term "energy width" is also of a thermodynamic origin.

In fact, our thermodynamic-type interpretation of time evolution implies, due to Eq. (15), the following *extension* of the first law of thermodynamics:

$$dU = \delta W + \delta Q - i\delta\Gamma\,. \tag{16}$$

This includes an imaginary term $-i\delta\Gamma$, where $\Gamma$ is a new thermodynamic-type quantity providing us with an analogue of heat $Q$ when $-i\hbar/t$ takes over the role of $kT$. We call it *energy width* transferred to the system from the physical spacetime as from its surroundings. Consequently, the internal energy $U$ of the system is generally complex. In the temporal equilibrium $\delta\Gamma \equiv 0$.

## 2. Equation for time deformation

In analogy with the thermal-nonequilibrium temperature field $T+\delta T(\vec{r},t)$ we can speak of the temporal-nonequilibrium time field $t+\delta t(\vec{r},t)$. Here, the identity $\delta t(\vec{r},t)\equiv 0$ characterizes the temporal equilibrium much like the identity $\delta T(\vec{r},t)\equiv 0$ specifies the thermal equilibrium. Then, the temperature — time analogy (15), when assumed to be a physically profound correspondence, suggests for the *inverse–time–deformation field*

$$\varphi(\vec{r},t)\equiv\frac{1}{t+\delta t(\vec{r},t)}-\frac{1}{t} \tag{17}$$

a conductivity equation of the form:

$$\left(\triangle-\frac{1}{\lambda c}\frac{\partial}{\partial t}\right)\varphi(\vec{r},t)=0\,. \tag{18}$$

This is an analogue of the familiar conductivity equation for the temperature-deformation field $\delta T(\vec{r},t)$. Here, $\lambda>0$ is an unknown length-dimensional conductivity constant (in the vacuum). Obviously, $\varphi(\vec{r},t)\equiv 0$ in the temporal equilibrium.

The conductivity equation (18) can be considered as a nonrelativistic approximation for a tachyonic-type (and so *ultraluminar*) Klein–Gordon equation of the form:

$$\left(\triangle-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{1}{4\lambda^2}\right)\chi(\vec{r},t)=0\,. \tag{19}$$

In fact, the substitution

$$\chi(\vec{r},t)\equiv\varphi(\vec{r},t)\exp\left(\frac{c\,t}{2\lambda}\right) \tag{20}$$

gives

$$\begin{aligned}\left(\triangle-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{1}{4\lambda^2}\right)\chi &\equiv \exp\left(\frac{ct}{2\lambda}\right)\left(\triangle-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{1}{\lambda c}\frac{\partial}{\partial t}\right)\varphi\\ &\simeq \exp\left(\frac{ct}{2\lambda}\right)\left(\triangle-\frac{1}{\lambda c}\frac{\partial}{\partial t}\right)\varphi\end{aligned} \tag{21}$$

if $|(1/\lambda c)\partial\varphi/\partial t|\gg|(1/c^2)\partial^2\varphi/\partial t^2|$. Thus, in this approximation, $\varphi(\vec{r},t)$ appearing in Eq. (20) can be identified with the field $\varphi(\vec{r},t)$ satisfying the conductivity equation (18).

Making use of the Tomonaga–Schwinger spacelike hypersurface $\sigma$ taking over covariantly the role of flat time $t$ [5], one may introduce by means of its unit normal field $n(x) = (n_\mu(x))$ $\big(n(x)^2 = 1\big)$ the inverse–time–deformation field (17) in a *covariant* way (in the sense of special relativity):

$$\varphi[\sigma](x) \equiv \frac{c}{n(x)\cdot[x+\delta x(x)]} - \frac{c}{n(x)\cdot x}\,, \tag{22}$$

where $x = (x^\mu) = (ct, \vec{r})$ and $\delta x(x) = (\delta x^\mu(x))$. Then, the field $\chi(x) \equiv \chi(\vec{r}, t)$ satisfying the relativistic equation (19) is *assumed* to be identical with

$$\chi[\sigma](x) \equiv \varphi[\sigma](x)\exp\left[\frac{n(x)\cdot x}{2\lambda}\right]. \tag{23}$$

Thus, $\chi[\sigma](x) \equiv \chi(x)$ is independent of $\sigma$, what implies

$$\varphi[\sigma](x)\exp\left[\frac{n(x)\cdot x}{2\lambda}\right] \equiv \varphi(\vec{r}, t)\exp\left(\frac{ct}{2\lambda}\right). \tag{24}$$

due to Eqs. (20) and (23). For the special choice $n(x) = (1,0,0,0)$ one gets $\varphi[\sigma](x) = \varphi(\vec{r}, t)$ as given in Eq. (17). In general, the *coordinate* spacelike hypersurface $\sigma$ may be arbitrarily deformed and so must not be mistaken for the *dynamical* hypersurface $t+\delta t(\vec{r}, t)$ or its covariant version $n(x)\cdot[x+\delta x(x)]$ (the field $\chi(x)$ describes only an arbitrary timelike projection $n(x)\cdot\delta x(x)$ of $\delta x(x)$ in terms of $n(x)\cdot x$ and $x$ but, by construction, does it in the same way for all $n(x)$). A formal hydrodynamic analogue of the field $c\,n(x)$ is the field of four-velocity $u(x) = (u_\mu(x))$ normalized as $u(x)^2 = c^2$ [6]. However, this is related to a coninuous matter medium. Then, $u(x)/c = (1,0,0,0)$ in the so-called comoving frame of reference.

Since it was conjectured that departures from temporal equilibrium appear in processes where the overall particle number changes, it is natural to assume that the field equation (19) takes in the presence of matter sources the following inhomogeneous form:

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{1}{4\lambda^2}\right)\chi(\vec{r}, t) = -4\pi\, g\lambda\left[\frac{\partial\rho(\vec{r}, t)}{\partial t} + \mathrm{div}\vec{\jmath}(\vec{r}, t)\right]. \tag{25}$$

Here, $g > 0$ is an unknown dimensionless coupling constant and $(c\rho, \vec{\jmath}) \equiv (j^\mu)$ denotes the matter four-current. In the case of quantum particles, this can be defined as

$$j^\mu(x) \equiv \langle\Psi(t)|J^\mu(\vec{r})|\Psi(t)\rangle_{\mathrm{av}}\,, \tag{26}$$

where $\langle\,\rangle_{\mathrm{av}}$ stands for the spin-averaged expectation value (in the state $\Psi(t)$) for the operator $J^\mu(\vec{r})$ of overall particle-number four-current (here, $\Psi(t)$ and $J^\mu(\vec{r})$ are taken, for instance, in the Schrödinger picture). Since the overall particle number is no constant of motion, the continuity equation $\partial\rho/\partial t + \mathrm{div}\vec{j} = 0$ does not hold in general, and so matter sources in Eq. (25) are generally nonzero. They vanish in states $\Psi(t)$ where the overall particle number does not change, what allows for the temporal equilibrium: $\chi(\vec{r},t) \equiv 0$. We are aware of the infrared problem which exists, when photons are included in the matter sources (*i.e.*, in $J^{\mu}$ and so $j^{\mu}$), as they should be from the standpoint of our conjecture.

In the nonrelativistic approximation, Eq. (25) reduces to the following inhomogeneous form corresponding to the conductivity equation (18):

$$\left(\triangle - \frac{1}{\lambda c}\frac{\partial}{\partial t}\right)\varphi(\vec{r},t) = -4\pi g\lambda\left[\frac{\partial\rho(\vec{r},t)}{\partial t} + \mathrm{div}\vec{j}(\vec{r},t)\right]\exp\left(-\frac{ct}{2\lambda}\right), \quad (27)$$

when Eqs. (20). and (21) are invoked.

The Feynman-type propagator for the field $\chi(x)$ satisfies the equation

$$\left(\Box + \frac{1}{4\lambda^2}\right)\Delta_F(x-x') = -4\pi\delta^4(x-x'), \quad (28)$$

with $\Box = \triangle - (1/c^2)\,\partial^2/\partial t^2$, and is specified by the formula

$$\begin{aligned}\Delta_F(x-x') =& -4\pi\int\frac{d^4k}{(2\pi)^4}\frac{\exp[-ik\cdot(x-x')]}{k^2+\frac{1}{4\lambda^2}+i\varepsilon}\\ =& 4\pi i\theta(x_0-x_0')\int\frac{d^3\vec{k}}{(2\pi)^3}\exp[i\vec{k}\cdot(\vec{r}-\vec{r}\,')]\\ &\times\left\{\theta\left(\vec{k}^2-\frac{1}{4\lambda^2}\right)\frac{\exp[-ik_0(x_0-x_0')]}{2k_0}\right.\\ &\left.+\,i\theta\left(\frac{1}{4\lambda^2}-\vec{k}^2\right)\frac{\exp[-|k_0|(x_0-x_0')]}{2|k_0|}\right\}_{k_0=\sqrt{\vec{k}^2-1/4\lambda^2}}\\ &+(x_o\leftrightarrow x_0')\,. \end{aligned} \quad (29)$$

Here, with $k \equiv |\vec{k}|$ one may write

$$\begin{aligned}\int\frac{d^3\vec{k}}{(2\pi)^3}\exp[i\vec{k}\cdot(\vec{r}-\vec{r}\,')]\{\ldots\} &= 2\int_0^\infty\frac{kdk}{(2\pi)^2}\frac{\sin(k|\vec{r}-\vec{r}\,'|)}{|\vec{r}-\vec{r}\,'|}\{\ldots\}\\ &= \int_{-\infty}^{\infty}\frac{kdk}{(2\pi)^2}\frac{\sin(k|\vec{r}-\vec{r}\,'|)}{|\vec{r}-\vec{r}\,'|}\{\ldots\}, \end{aligned} \quad (30)$$

as $\{\ldots\}$ depends on $k^2$.

The general solution to the field equation (25) can be presented in the form:

$$\chi(x) = \chi^{(0)}(x) + g\lambda \int d^4x' \Delta_F(x - x')\partial'_\mu j^\mu(x'), \tag{31}$$

where $\partial_\mu = \partial/\partial x^\mu$ and

$$\left(\Box + \frac{1}{4\lambda^2}\right)\chi^{(0)}(x) = 0. \tag{32}$$

As is seen from Eq. (29), the propagator $\Delta_F(x - x')$ contains both the ultraluminar time-oscillating part with

$$k_0 = |k_0| \ , \ \frac{v}{c} \equiv \frac{|\vec{k}|}{k_0} > 1$$

and the time-damped part with

$$k_0 = i|k_0| \ , \ \frac{v}{c} \equiv \frac{|\vec{k}|}{k_0} = -i\frac{|\vec{k}|}{|k_0|}.$$

For $\lambda \to \infty$ the first part becomes luminar, $v/c \to 1 + 0$, while the second vanishes. If $\partial_\mu j^\mu(x) = c\delta^4(x - x_S)$, Eq. (31) gives

$$\chi(x) = \chi^{(0)}(x) + g\lambda\, c\, \Delta_F(x - x_S). \tag{33}$$

The static solution to Eq. (25) is provided by the formula:

$$\chi(\vec{r}) = g\lambda \int d^3\vec{r}\,' \frac{\cos(|\vec{r} - \vec{r}\,'|/2\lambda)}{|\vec{r} - \vec{r}\,'|} \mathrm{div}'\vec{j}(\vec{r}\,'). \tag{34}$$

Then, from Eq. (20)

$$\varphi(\vec{r}, t) = \chi(\vec{r}) \exp\left(-\frac{ct}{2\lambda}\right), \ \varphi(\vec{r}, 0) = \chi(\vec{r}). \tag{35}$$

If $\mathrm{div}\vec{j}(\vec{r}) = (1/\tau)\delta^3(\vec{r} - \vec{r}_S)$ where $\tau > 0$ is a time-dimensional constant, Eq. (34) gives

$$\chi(\vec{r}) = \frac{g\lambda}{\tau} \frac{\cos(|\vec{r} - \vec{r}_S|/2\lambda)}{|\vec{r} - \vec{r}_S|}. \tag{36}$$

Here, the constant $1/\tau$ may be interpreted as the overall number of particles produced per unit of time by a spherically symmetric stationary source.

If the source is a pointlike target in an accelerator during its stationary run, then one may use the mathematical model, where

$$\mathrm{div}\vec{\jmath}(\vec{r}) = \frac{1}{\tau}\left[\delta^3(\vec{r}-\vec{r}_{\mathrm{S}}) + \frac{\cos(|\vec{r}-\vec{r}_{\mathrm{S}}|/2\lambda)}{|\vec{r}-\vec{r}_{\mathrm{S}}|^3}\sum_{l\,m_l} c_{l\,m_l}\frac{l(l+1)}{4\pi}Y_{l\,m_l}(\vartheta,\varphi)\right] \tag{37}$$

with $\sum_{l\,m_l} c_{l\,m_l} Y_{l\,m_l}(0,0) = 1$ and $\vec{r}-\vec{r}_{\mathrm{S}} = (|\vec{r}-\vec{r}_{\mathrm{S}}|,\vartheta,\varphi)$. This gives

$$\chi(\vec{r}) = \frac{g\lambda}{\tau}\frac{\cos(|\vec{r}-\vec{r}_{\mathrm{S}}|/2\lambda)}{|\vec{r}-\vec{r}_{\mathrm{S}}|}\sum_{l\,m_l} c_{l\,m_l}Y_{l\,m_l}(\vartheta,\varphi)\,, \tag{38}$$

as then one can check directly that

$$\left(\triangle + \frac{1}{4\lambda^2}\right)\,\chi(\vec{r}) = -4\pi\,g\lambda\,\mathrm{div}\vec{\jmath}(\vec{r})\,. \tag{39}$$

Also in this case $1/\tau$ is the overall number of particles produced per unit of time, since

$$\int d^2\vec{\sigma}\cdot\vec{\jmath}(\vec{r}) = \int d^3\vec{r}\,\mathrm{div}\vec{\jmath}(\vec{r}) = \tfrac{1}{\tau} \tag{40}$$

in virtue of

$$l(l+1)\int_0^{\pi}\int_0^{2\pi} \sin\vartheta\,d\vartheta\,d\varphi\,Y_{l\,m_l}(\vartheta,\varphi) = 0\,. \tag{41}$$

## 3. Equation for quantum time evolution

In analogy with the familiar law for heat (valid when $\delta W = 0$)

$$\delta Q \propto \int d^3\vec{r}\rho(\vec{r})k\,d[T+\delta T(\vec{r})]\,, \tag{42}$$

we may put for the energy width

$$-i\delta\Gamma \propto \int d^3\vec{r}\rho(\vec{r})(-i\hbar)d\left[\frac{1}{t+\delta t(\vec{r})}\right]\,, \tag{43}$$

if we take earnestly the temperature — time correspondence $kT \leftrightarrow -i\hbar/t$. In consequence, we decide to assume the formula

$$\Gamma(t) \equiv g\hbar\int d^3\vec{r}\,\rho(\vec{r},t)\,\chi(\vec{r},t) \tag{44}$$

or its covariant version

$$\Gamma[\sigma] \equiv g\hbar \int_\sigma d^3\sigma_\mu j^\mu(x)\,\chi(x)\,, \tag{45}$$

where $d^3\sigma_\mu = n^\mu(x)d^3\sigma$ and $d^3\sigma_0 = (1/c)d^3\vec{r}$, while $g > 0$ is the coupling constant introduced in Eq. (25). At this point we use (in the present paper) the function $\chi(\vec{r},t)$ rather than $\varphi(\vec{r},t) = \chi(\vec{r},t)\exp(-ct/2\lambda)$, since — for the former — static solutions $\chi(\vec{r})$ exist as *e.g.* that given in Eq. (38).

Then, consistently with our extended first law of thermodynamics (16), it is natural to introduce (in the Schrödinger picture) the temporal-nonequilibrium quantum state equation in the form:

$$i\hbar\frac{d\Psi(t)}{dt} = [H - i\mathbf{1}\Gamma(t)]\,\Psi(t)\,,\; H^\dagger = H\,, \tag{46}$$

where $\mathbf{1}$ stands for the unit operator. Of course, in the temporal equilibrium $\chi(\vec{r},t) \equiv 0$ and thus $\Gamma(t) \equiv 0$, what reduces in this case Eq. (46) to the conventional state equation (14). In general, however, the nonunitarity state equation (46) includes small nonunitary deviations from the conventional temporal-equilibrium time evolution (the coupling constant $g > 0$, appearing both in Eq. (25) and (44) is expected to be small enough, probably extremely small, in order not to contradict the heritage of the conventional quantum theory). Note that eliminating $\chi(\vec{r},t)$ from Eq. (44) by means of Eq. (25) (which gives Eq. (31), where now $\chi^{(0)}(x) = 0$) we obtain

$$\Gamma(t) = g^2\lambda \int d^3\vec{r} \int d^4x' \rho(x)\Delta_F(x-x')\partial'_\mu j^\mu(x')\,. \tag{47}$$

Concluding, we have constructed a mixed set of two coupled equations. On one side: Eq. (25) for the parameter-valued field $\chi(\vec{r},t)$ describing the hypothetic departures of time from its conventional flat run $t$ (induced by changes of the overall particle number). On the other side: Eq. (46) for the quantum state vector $\Psi(t)$ including deviations from its conventional unitary time evolution (introduced there by the former time-run departures). It is important to note that these time-run departures from its conventional flatness have physical causes different from those active in the Einsteinian classical gravitation theory, *viz.* possible nonzero divergence of matter current $\partial_\mu j^\mu(x)$ in the former case *versus* generic nonzero matter energy-momentum tensor $T^{\mu\nu}(x)$ in the latter. *A priori*, both effects are not connected and so may have different orders of magnitude.

Strictly speaking, the set of Eqs. (25) and (46) is nonlinear and nonlocal with respect to the state vector $\Psi(t)$, so it slightly violates the superposition principle, fundamental in the conventional quantum theory (valid in the temporal equilibrium). However, this set becomes linear and local in the approximation, where in Eq. (26) defining $j^{\mu}(\vec{r},t)$ the state vector $\Psi(t)$ is replaced in the zero order by $\Psi^{(0)}(t)$ satisfying the temporal-equilibrium state equation (14). This gives $j^{(0)\mu}(\vec{r},t)$ and then, in the first order, Eqs. (25), (44) and (46) lead to

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{1}{4\lambda^2}\right)\chi^{(1)}(\vec{r},t) = -4\pi g\lambda\left[\frac{\partial\rho^{(0)}(\vec{r},t)}{\partial t} + \mathrm{div}\vec{\jmath}^{(0)}(\vec{r},t)\right], \tag{48}$$

$$\Gamma^{(1)}(t) \equiv g\hbar\int d^3\vec{r}\,\rho^{(0)}(\vec{r},t)\chi^{(1)}(\vec{r},t) = O(g^2)\,, \tag{49}$$

and

$$i\hbar\frac{d\Psi^{(1)}(t)}{dt} = \left[H - i1\Gamma^{(1)}(t)\right]\Psi^{(1)}(t)\,. \tag{50}$$

The last of these equations implies

$$\Psi^{(1)}(t) = \Psi^{(0)}(t)\exp\left[-\frac{1}{\hbar}\int_{t_0}^{t}dt'\,\Gamma^{(1)}(t')\right]\,, \tag{51}$$

where

$$\Psi^{(0)}(t) = \exp\left[-\tfrac{i}{\hbar}H(t-t_0)\right]\Psi^{(0)}(t_0)\,, \tag{52}$$

with $\Psi^{(0)}(t_0) \equiv \Psi_H$ being the exact state vector in the Heisenberg picture, if the Schrödinger and Heisenberg pictures coincide at $t = t_0$ .

We can see that the norm of the state vector $\Psi^{(1)}(t)$, given by

$$\langle\Psi^{(1)}(t)|\Psi^{(1)}(t)\rangle = \langle\Psi_H|\Psi_H\rangle\exp\left[-\frac{2}{\hbar}\int_{t_0}^{t}dt'\Gamma^{(1)}(t')\right]\,, \tag{53}$$

changes slightly in time in the interval, where $\chi^{(1)}(\vec{r},t) \neq 0$ and so, due to Eq. (49), $\Gamma^{(1)}(t) \neq 0$. This is originated by the fact that the physical system is always open to the physical spacetime as to its surroundings and, when $\partial\rho^{(0)}/\partial t + \mathrm{div}\vec{\jmath}^{(0)} \neq 0$, the temporal equilibrium between both is perturbed: $\chi^{(1)}(\vec{r},t) \neq 0$ and thus, in consequence of Eqs. (17) and (20), $\delta t^{(1)}(\vec{r},t) \neq 0$. The nonunitarity behaviour (51) (and so (53) ) of the state vector $\Psi^{(1)}(t)$ should manifest itself among others [1] in slightly violating the optical theorem for the $S$ matrix [7].

In a future fully dynamical quantum theory — which, hopefully, would describe together with our matter system also the physical spcetime in a quantal way — the Hilbert space of the matter system would be only a subspace of a whole Hilbert space. Then, the state vector of the matter system could be interpreted as a projection of the whole state vector onto this Hilbert subspace and so, according to the general formalism [8], should evolve in time nonunitarily, displaying a half-width dependent on the matter-system state. Obviously, a proper version of the quantum gravity would be included in this fully dynamical quantum theory. The theory described in the present paper may be considered as a thermodynamical-type approximation to such a future theory, working in the experimental situation, where the physical spacetime can be treated essentially as a chronostat, but small deviations from the corresponding temporal equilibrium may be allowed.

## 4. Hydrogen gas in proximity of a big accelerator

Consider a sample of hydrogen atoms situated in the proximity of a pointlike target $\vec{r}_S$ of a big accelerator producing $1/\tau$ particles of all sorts per unit of time. Then, during its stationary run it excites the inverse–time–deformation field

$$\chi^{ex}(\vec{r}) = \frac{g\lambda}{\tau}\frac{\cos(|\vec{r}-\vec{r}_S|/2\lambda)}{|\vec{r}-\vec{r}_S|}\sum_{l\, m_l} c_{l\, m_l} Y_{l\, m_l}(\vartheta,\varphi)\,, \tag{54}$$

playing the role of an external field for the sample of hydrogen atoms. In Eq. (54) the mathematical model described by the formulae (37) and (38) is used.

This external inverse–time–deformation field modifies the temporal-equilibrium wave function

$$\psi^{(0)}(\vec{r}_e,\vec{r}_p)\exp\left[-\tfrac{i}{\hbar}\,E\,(t-t_0)\right] \tag{55}$$

of any hydrogen atom of the sample, leading in the first order to the modified function as given in Eq. (51)

$$\psi^{(0)}(\vec{r}_e,\vec{r}_p)\exp\left[-\frac{i}{\hbar}\,E\,(t-t_0)\right]\exp\left[-\frac{1}{\hbar}\int_{t_0}^{t} dt'\,\Gamma^{(1)}(t')\right]. \tag{56}$$

Here, the stationary run of the accelerator is switched on at the moment $t_0$ and still lasts at the later moment $t$. In our case

$$\begin{aligned}\rho^{(0)}(\vec{r}) &= \int_V d^3\vec{r}_e\int_V d^3\vec{r}_p|\psi^{(0)}(\vec{r}_e,\vec{r}_p)|^2\left[\delta^3(\vec{r}-\vec{r}_e)+\delta^3(\vec{r}-\vec{r}_p)\right]\\ &= \frac{2}{V}\ \text{ for }\ \vec{r}\in V\,,\end{aligned} \tag{57}$$

where

$$\psi^{(0)}(\vec{r}_e, \vec{r}_p) = \psi^{(0)}(\vec{r}_e - \vec{r}_p) \frac{1}{\sqrt{V}} \exp\left(\frac{i}{\hbar} \vec{P} \cdot \vec{R}\right) \tag{58}$$

with $V$ being the volume of the sample. Thus, from Eq. (49)

$$\Gamma^{(1)} = \frac{2g\hbar}{V} \int_V d^3\vec{r}\, \chi^{\text{ex}}(\vec{r}) \simeq \frac{2g^2\lambda\hbar}{\tau} \frac{\cos(d/2\lambda)}{d} . \tag{59}$$

Here, $d = \langle |\vec{r} - \vec{r}_S| \rangle$ denotes the average distance of the sample from the target, while the configuration of the sample is such that the average angles vanish, $\langle \vartheta \rangle = 0$ and $\langle \varphi \rangle = 0$. Then, Eqs. (56) and (57) give

$$\rho^{(1)}(\vec{r}, t) = \frac{2}{V} \exp\left[-\frac{2}{\hbar}\Gamma^{(1)}(t - t_0)\right] \quad \text{for } \vec{r} \in V . \tag{60}$$

Of course, $\rho^{(1)}(\vec{r}, t_0) = \rho^{(0)}(\vec{r})$.

On the ground of Eq. (60) we can conclude that in consequence of the varying norm of modified wave function (56) the average number of hydrogen atoms in the sample slightly *decreases in time* during the stationary run of the accelerator. In fact, we get

$$\rho^{(0)}_{\text{sample}}(\vec{r}) = N_H \tfrac{1}{2} \rho^{(0)} = \frac{N_H}{V} \quad \text{for } \vec{r} \in V , \tag{61}$$

$$\Gamma^{(1)}_{\text{sample}} = N_H \tfrac{1}{2} \Gamma^{(1)} = \frac{N_H g\hbar}{V} \int_V d^3r\, \chi^{\text{ex}}(\vec{r}) , \tag{62}$$

and

$$\rho^{(1)}_{\text{sample}}(\vec{r}, t) = \frac{N_H}{V} \exp\left[-\frac{2}{\hbar}\Gamma^{(1)}_{\text{sample}}(t - t_0)\right] = \frac{N^{(1)}(t)}{V} \quad \text{for } \vec{r} \in V , \tag{63}$$

where $N_H = 2.69 \times 10^{19} V/\text{cm}^3$ (in normal conditions) is the Loschmidt number multiplied by $V$. Hence,

$$N^{(1)}(t) = N_H \exp\left[-\frac{2}{\hbar}\Gamma^{(1)}_{\text{sample}}(t - t_0)\right] . \tag{64}$$

In order to find an estimation for the exponent in Eq. (64),

$$p^{(1)} \equiv \frac{2}{\hbar}\Gamma^{(1)}_{\text{sample}}(t - t_0) \simeq \frac{2g^2\lambda N_H}{\tau} \frac{\cos(d/2\lambda)}{d}(t - t_0) , \tag{65}$$

take (somewhat like for the Tevatron) the luminosity $\sim 10^{31}\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$, the $\bar{p}p$ total cross-section $\sim$ 100 mb and the average particle multiplicity $\sim$ 100, what leads to $1/\tau \sim 10^8\,\mathrm{sec}^{-1}$ for the particle-production rate. Then, expecting that $\lambda \gg d$ and taking for example $V/d \sim 100\ \mathrm{cm}^2$, one obtains

$$p^{(1)} \sim 10^{28} g^2 \frac{\lambda}{d} \frac{V}{\mathrm{cm}^3} \frac{t - t_0}{\mathrm{sec}} \sim 10^{30} g^2 \frac{\lambda}{\mathrm{cm}} \frac{t - t_0}{\mathrm{sec}} . \tag{66}$$

Unfortunately, the constants $g > 0$ and $\lambda > 0$ are completely unknown from the very beginning (only, $g \ll 1$ and $\lambda \gg d$ were expected).

Among *a priori* various options for the order of magnitude of the length-dimensional conductivity constant $\lambda$ there is one extreme, where $\lambda$ ranges to the present cosmological scale: $\lambda \sim c \times$ age of universe $\sim 10^{28}$cm (here, the age of the universe is put equal to $1.5 \times 10^{10}$yr). Of course, in this option the "negative mass square" $-1/4\lambda^2$ can be neglected on the lhs of the time-deformation field equation (25) in all present experiments, except possibly cosmological observations. On the other hand, the coefficient $\lambda$ on its rhs strengthens (at $g$ fixed) the action of matter sources inducing time deformations. Then, $\exp(-ct/2\lambda) \sim 1$ in Eqs. (20) and (27). This is true even if the physical time $t$ is counted from the Big Bang as from its *natural* beginning (in such a case, the analogy between the absolute temperature $T \geq 0$ and the inverse of *cosmological time* $t \geq 0$ really appeals to our imagination). For such reckoning of time one gets $t =$ age of universe and so $\exp(-ct/2\lambda) \simeq 0.6065$ when $\lambda \simeq c \times$ age of universe.

In the case of the extreme cosmological option where $\lambda \sim 10^{28}$ cm, Eq. (66) gives

$$p^{(1)} \sim 10^{58} g^2 \frac{t - t_0}{\mathrm{sec}} . \tag{67}$$

Hence, *e.g.* for the run-time $t - t_0 = 1$ month $\sim 10^6$ sec one gets

$$p^{(1)} \sim 10^{64} g^2 . \tag{68}$$

Then, if *e.g.* $g^2 \sim 10^{-66}$ to $10^{-68}$, the decrement factor in Eq. (64) is

$$\exp(-p^{(1)}) \sim 1 - (0.01 \ \text{ to } \ 0.0001) , \tag{69}$$

depending on the value ascribed to $g^2$.

In contrast, if $\lambda \ll c \times$ age of universe (though $\lambda \gg d$), then in Eqs. (20) and (27) $\exp(-ct/2\lambda) \ll 1$ for the cosmological time $t =$ age of universe, what implies a practically vanishing $\varphi(\vec{r}, t)$ and also such $\delta t(\vec{r}, t)$.

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