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FAST ROTATION OF NEUTRON STARS AND EQUATION OF STATE OF DENSE MATTER*, **

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Radio pulsars — rapidly rotating, magnetized neutron stars — are the fastest stellar rotators in the Universe. The structure of rotating neutron stars is determined by the equation of state of matter at supranuclear densities and the angular frequency of rotation. Results of calculations of rapidly rotating neutron star models, performed for a broad set of equations of state of dense matter, are reviewed. The upper limit to rotation frequency of stable rotation results from the appearance of instabilities in rapidly rotating neutron stars. Maximum rotation frequency depends on the equation of state of dense matter, but some general empirical relations between the maximally rotating and static neutron star models are shown to exist.

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1. Introduction

Neutron stars are the fastest stellar rotators in the Universe. These compact objects have typical mass $M \sim M_\odot$ (mass of the Sun $M_\odot = 1.989 \times 10^{33}$ g) and are expected to have radii $R \sim 10$ km, so that their mean density $\rho \sim 10^{15}$ g cm^{-3} (significantly higher than the saturation density of nuclear matter, $\rho_0 = 2.7 \times 10^{14}$ g cm^{-3}, which corresponds to nucleon density $n_0 = 0.16$ fm^{-3}). Rapidly rotating, magnetized neutron stars are observed as the radio pulsars. Radio pulsars were discovered in

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1967, and nearly $\sim$ 600 of them are known at present. The minimum observed pulsation period (equal to the period of rotation of magnetized neutron star) is 1.56 ms.

The fastest radio pulsars form a very special group of *millisecond pulsars*, characterized by the pulsation period shorter than 10 ms. The rotation of millisecond pulsars is extremely stable, the slowing down being typically $\sim 10^{-9}$ s per year. First millisecond pulsar was discovered in 1983, and their number grows rapidly due to the progress in observational techniques and data analysis.

In view of the very high stability of pulsar period (see above), their rotation should be to a very good approximation uniform (rigid body rotation), because any substantial amount of differential rotation would lead to dissipation of rotational energy due to shear viscosity of dense matter. In order to get a feeling about the orders of magnitude involved, it may be useful to make simple estimates, assuming $M = 1\ M_{\odot}$, $R = 10$ km, and $P = 1$ ms. The kinetic rotational energy can be estimated as $E_{\rm rot} \sim 10^{52}$ erg (some 10% of the typical binding energy of neutron star), and the angular momentum $J \sim 10^{49}$ g cm^2 s^{-1}. In view of the fact, that neutron star contains typically some 10^{58} nucleons, we come to the estimate of some $10^{18}\ \hbar$ of the orbital angular momentum per nucleon.

Massive neutron star, rotating at $P = 1$ ms, is a relativistic object, both because of high speed (velocity at the equator $v_{\rm equator} \sim 0.2c$) and significant space curvature (stellar radius is of the order of $\sim$ 3 Schwarzschild radii: $2GM/Rc^2 \simeq 0.3$). In view of this, theoretical treatment of rapidly rotating neutron stars is much more complicated than the description of the non-rotating ones (see Section 3).

The plan of the paper if as follows. In Section 2 problems related to the equation of state of dense matter, and calculation of the non-rotating neutron star models, are briefly discussed. Calculations of rapidly rotating neutron star models are described in Section 3. The problem of the maximum rotation frequency of neutron stars is discussed in Section 4, where some empirical formulae, based on results of exact calculations, are presented. Finally, Section 5 contains conclusions.

2. Equation of state of dense matter and static neutron star models

The structure of static (non-rotating) neutron stars (NS) is determined by the equation of state (EOS) of dense matter. Except for a very short period after the NS birth, thermal effects on the EOS are negligible. EOS is thus determined by the functional dependence of pressure and energy density on the baryon density of matter: $p = p(n_{\rm b})$, $\epsilon = \epsilon(n_{\rm b})$ (notice that ϵ includes also the rest energy of matter constituents). Matter density $\rho = \epsilon/c^2$. Eliminating n_b, we get $p = p(\rho)$.

It should be stressed, that the actual form of the EOS at $\rho > 2 - 3\rho_0$ (which is crucial for the structure of massive neutron stars) is largely unknown. This is due to the lack of knowledge of strong interaction between matter constituents, and to the difficulties in solving the many-body problem for the multicomponent dense mixture of baryons and leptons. Theoretically calculated EOS of dense matter diverge at $\rho > 2 - 3\rho_0$ (see Fig. 1) — this reflects the uncertainty in our knowledge of the EOS of dense matter. Generally, EOS can be classified according to their stiffness: stiffer EOS corresponds to higher p at the same value of ρ.

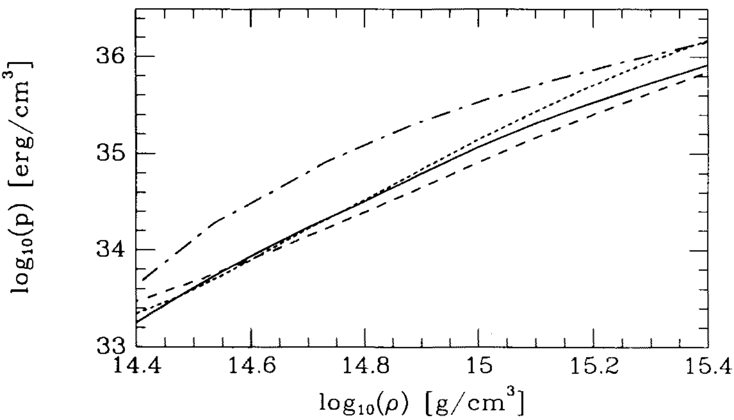

Fig. 1. Relation between pressure and density for several equations of state of dense matter. The equations of state are labeled according to Table I: solid line — 11; dash-dotted line — 4; short dashes — 10; long dashes — 2.

Hydrostatic equilibrium of a static (non-rotating) NS corresponds to an exact balance of gravity and pressure forces, at each point of the star. Solution of general relativistic equations of hydrostatic equilibrium combined with EOS of matter, leads then to a one-parameter family of spherically symmetric NS configurations, parametrized by, *e.g.*, central density, ρ_c (see, *e.g.*, [1]). Such a family of equilibrium models forms a set $\{\mathcal{M}^{\rm stat}\}$. Global parameters of NS, such as total (gravitational) mass M, and stellar radius, R, are then determined as $M = M(\rho_c)$, $R = R(\rho_c)$. These relations depend on the EOS of dense matter. Some examples of the $M - \rho_c$ curves are shown in Fig. 2.

It should be stressed, that under standard assumptions concerning thermodynamics of dense matter (full thermodynamic equilibrium), configurations to the right of the maxima of the $M - \rho_c$ curves are unstable with respect to small radial perturbations (see below), and therefore do not exist in the

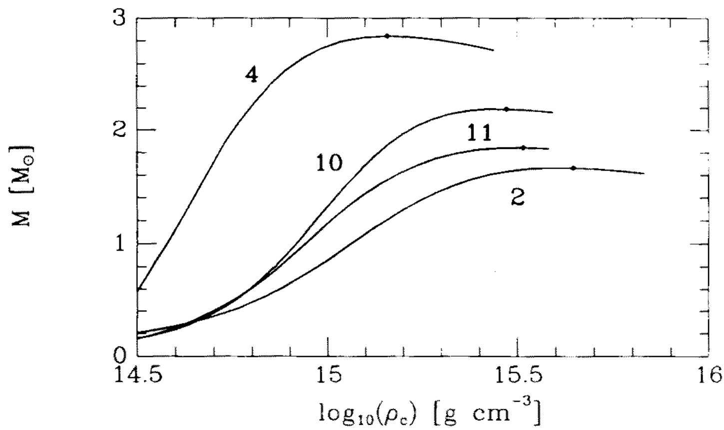

Fig. 2. Relation between mass and central density for non-rotating neutron star models, calculated using four equations of state from Table I (we use labels from the first column of Table I). Configuration to the right of the maxima, indicated by filled circles, are unstable with respect to small radial perturbations.

Universe. Both the existence of the maximum mass within the set $\{\mathcal{M}^{\rm stat}\}$ (which corresponds to the maximum central density of the stable equilibrium model, $\rho_{\rm c,max}$, and maximum baryon number, A, of stable neutron star, $A = A_{\rm max}$), as well as the instability with respect to radial perturbations are consequences of *general relativity*. Under standard assumptions (complete thermodynamic equilibrium, $T = 0$) the static criterion of the stability of configurations belonging to $\{\mathcal{M}^{\rm stat}\}$ is $\partial M/\partial\rho_{\rm c} > 0$; it is satisfied for $\rho_{\rm c} < \rho_{\rm c,max}$. Configurations with $\rho_{\rm c} > \rho_{\rm c,max}$ are unstable with respect to small radial perturbations.

TABLE I

Equations of state

Label	Model of EOS	Reference
1	model II	Diaz Alonso 1985 [2]
2	neutron matter	Pandharipande 1970 [3]
3	model IH	Bethe & Johnson 1974 [4]
4	model "0.17"	Haensel *et al.* 1981 [5]
5	"case 1"	Glendenning 1985 [6]
6	"case 2"	Glendenning 1985 [6]
7	"case 3"	Glendenning 1985 [6]
8[a]	FP(UV14+TNI)	Friedman & Pandharipande 1981 [7]
9[a]	AV14 +UVII	Wiringa *et al.* 1988 [8]
10[a]	UV14 +UVII	Wiringa *et al.* 1988 [8]
11	UV14 +TNI	Wiringa *et al.* 1988 [8]
12	$\Lambda^{00}_{\rm Bonn}$+HV	Weber *et al.* 1991 [9]
13	ideal Fermi gas of neutrons	

[a] Non-causal within central cores of massive neutron star models.

TABLE II

Static neutron star models with maximum allowable mass, Ω_{max} and P_{min}

EOS–label	M_{max}(stat) ($M_\odot$)	R_{max}(stat) (km)	Ω_{max} (10^4 s^{-1})	P_{min} (ms)
13	0.7199	10.403	0.6405	0.98
6	1.777	11.29	0.8289	0.76
4	2.827	13.68	0.8526	0.74
5	1.803	11.15	0.8653	0.73
7	1.964	11.30	0.8848	0.71
12	1.967	10.97	0.9293	0.68
1	1.928	10.93	0.9331	0.68
3	1.850	9.915	1.067	0.67
11	1.836	9.5885	1.1425	0.59
8[a]	1.960	9.410	1.238	0.55
10[a]	2.187	9.903	1.244	0.51
2	1.6574	8.537	1.286	0.49
9[a]	2.1235	9.455	1.318	0.48

[a] EOS is non-causal within neutron star models.

At given central density, the mass of NS increases with the stiffness of the EOS. The maximum allowable mass, M_{max}, depends thus on the EOS of dense matter. The value of M_{max} increases with the stiffness of EOS and combined with the value of the radius of the maximum mass configuration, R_{max}, constitutes an important characteristics of EOS. The values of M_{max}(stat), R_{max}(stat), calculated for a very broad set of EOS of dense matter (the list of EOS, together with references, is given in Table I), are displayed in Table II (we add label "stat" to stress that we deal with static NS models).

Static ($\Omega = 0$), spherically symmetric NS models are a very good approximation to slowly rotating neutron stars. Rotation may be considered as slow, if the velocity at the equator is much smaller than the velocity of light ($\Omega R \ll c$) and if the centrifugal force is negligible compared to gravity pull ($\Omega^2 R \ll GM/R^2$). Let us mention, that even the most rapid of the observed radio pulsars, PSR 1937+214 (period $P = 1.56$ ms), can be treated within the slow rotation approximation, provided it is reasonably massive ($M > 1\ M_\odot$), in view of $\Omega R = 0.13 \cdot (R/10\text{ km}) \cdot c$. Within the slow rotation approximation, the effects of rotation can be calculated using perturbational approach, and keeping only lowest order terms in the small expansion parameter, $\Omega R/c$; the effect on the bulk neutron star parameters is then quadratic in $\Omega R/c$. However, such an approach cannot be used for a precise determination of the maximum frequency of rotation of neutron stars, where the effects of rotation are very important (see [10] for a discussion of this point).

3. Rapidly rotating neutron star models

One can point out two astrophysical scenarios, which could produce rapidly rotating neutron stars. The first scenario involves collapse, while the second one — accretion of matter onto neutron star surface. Neutron star is born as an outcome of gravitational collapse of degenerate core of a massive, evolved star (*e.g.*, a read giant or a red supergiant), or in a gravitational collapse of a mass accreting white dwarf. In both case, initial (pre-collapse) configuration is expected to rotate (albeit slowly). Rapid contraction, on a timescale of a fraction of a second, combined with (at least partial) conservation of the angular momentum of collapsing core, leads to formation of a rotating neutron star, with Ω much larger than that of the pre-collapse configuration (example: the Crab pulsar, formed in the SN 1054, which has at present $P = 33$ ms). In the second scenario, neutron star in a close binary system accretes matter from its evolved companion star. As accreted matter has non-zero angular momentum with respect to neutron star (actually, it is expected that accretion takes place via an accretion disk), accreting neutron star increases not only its mass, M, but also its angular momentum, J (and therefore angular frequency, Ω). It is currently believed, that the most rapidly rotating neutron stars — the millisecond pulsars — were formed in this way, by spin-up via accretion of old neutron stars in a sufficiently long lived close binary system.

A natural question arises: what is the maximum frequency of stable rotation of neutron star? For the reasons, discussed in the Introduction, one can restrict to the case of rigid rotation. Clearly, for the rotation to be stable, the value of Ω cannot be larger the "mass shedding limit", $\Omega_{\rm ms}$, such that for $\Omega = \Omega_{\rm ms}$ the gravity pull at the equator is exactly balanced by the centrifugal force. For $\Omega < \Omega_{\rm ms}$ gravity pull within the star can be balanced by the combined pressure and centrifugal forces, and stationary equilibrium can be obtained. For $\Omega > \Omega_{\rm ms}$ mass will be shed from the equator of neutron star, and therefore $\Omega_{\rm ms}$ constitutes an absolute upper bound on Ω of stationary rotation.

The stationary configuration of uniformly rotating neutron star should be stable. The potentially most important instabilities in rapidly rotating neutron stars are those induced by the axi-symmetric perturbations (axi-symmetric (AS) instability), and the non-axisymmetric ones, related to the gravitational radiation back-reaction on oscillating neutron stars (gravitational radiation reaction (GRR) instability) (see, *e.g.*, [11]). Notice, that both AS and GRR instabilities have no counterpart in Newtonian theory of gravitation. Let us mention also, that AS instability of *rotating* neutron star corresponds, in the $\Omega = 0$ limit, to the $M_{\rm max}$ instability with respect to the small radial perturbations, mentioned in Section 3.

In Newtonian theory of rapidly rotating self-gravitating fluid stellar mass bodies, classical "shape instabilities" (Maclaurin spheroid $\longrightarrow$ Jacobi ellipsoid, Maclaurin spheroid $\longrightarrow$ Dedekind ellipsoid) set in at sufficiently high values of $E_{\rm rot}/|E_{\rm grav}|$ (see, *e.g.*, [1], Ch. 7). It is interesting to notice, that for realistic models of massive, rapidly rotating neutron stars these instabilities do not appear for $\Omega < \Omega_{\rm ms}$. This is due both to the importance of relativistic effects (which make gravity stronger compared to the Newtonian case), and to finite compressibility of dense neutron star matter. Notice also, that both Jacobi and Dedekind modes would be unstable due to gravitational radiation.

The GRR instabilities are quite efficiently damped by viscosity of neutron star matter (see [12, 13] and references therein), so that the practical limitation on Ω comes from the mass shedding criterion and the AS stability condition.

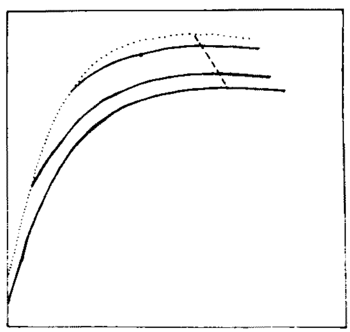

Fig. 3. Schematic representation of two stability limits relevant for stationary rotating neutron star models. Solid curves represent relation between mass and central density for stationary configurations at fixed angular momentum of neutron star. Lowest solid curve corresponds to non-rotating models ($J = 0$). Angular momentum increases upwards. Dotted curve corresponds to the mass shedding limit. Dashed line connects mass maxima: configurations to the right of this line are unstable with respect to small axi-symmetric perturbations.

For a given EOS of dense matter, one can calculate possible uniformly rotating, stationary NS models. They will form a two parameter family of models, which can be parametrized, *e.g.*, by the values of central density, ρ_c, and angular frequency, Ω, of rigid rotation: $\mathcal{M}^{\rm rot}(\rho_c, \Omega)$. Configurations $\{\mathcal{M}^{\rm rot}\}$ form surfaces $J(\rho_c, \Omega)$, $M(\rho_c, \Omega)$, $A(\rho_c, \Omega)$, where A is the total baryon number of neutron star. On these surfaces, one may construct lines, which correspond to neutron star sequences with constant J or A.

A schematic representation of the J = const. lines in the $M - \rho_c$ plane is shown in Fig. 3. At fixed J, mass shedding limit determines lower bound on ρ_c and M for stably rotating configurations. The ridge of a maximum M at fixed J marks the onset of the AS instability with respect to collapse [14]. Models stable against collapse correspond to $(\partial M/\partial\rho_c)_J > 0$ (notice that for $J = 0$ one recovers the static stability criterion for non-rotating models, Section 3). Among all stable models $\{\mathcal{M}^{\rm rot}(\rho_c, \Omega)\}$, one can pick up that with maximum value of Ω, $\Omega_{\rm max}$, and that with maximum mass, $M_{\rm max}(\rm rot)$. Numerical calculations for realistic EOS show, that rotating configurations with $\Omega_{\rm max}$ and $M_{\rm max}(\rm rot)$ are very close to each other; they were even believed to *coincide* for quite a long time [11]. Only recently, the application of very precise numerical techniques for the determination of maximally rotating configuration ($\Omega_{\rm max}$) and that with maximum mass ($M_{\rm max}(\rm rot)$) showed, that they do not coincide [15,16]

In practice, the precise determination of $\Omega_{\rm max}$(EOS), for realistic EOS, turns out to be difficult. At $\Omega \sim \Omega_{\rm ms}$ deviations from spherical symmetry are significant (typically, $R_{\rm equator} \simeq 1.5R_{\rm pole}$). The formulation of the problem within general relativity turns out to be quite difficult (dragging of inertial frames by the rotating fluid, conditions at infinity, *etc.*). Uniformly rotating configurations are axially symmetric, and the numerical calculations can be reduced to solving a 2-D system of nonlinear, coupled partial differential equations, combined with appropriate boundary conditions at the star center, stellar surface, and at infinity. It should be stressed, that the calculation of $\Omega_{\rm max}$ is additionally difficult, because it requires a very precise determination of the surface of the star at the brink of the mass shedding instability. Because of these difficulties, precise and reliable calculations of $\Omega_{\rm max}$ for the broad class of realistic EOS of dense matter have become feasible only quite recently [18, 19, 20].

Numerical calculations show, that $M_{\rm max}(\rm rot) \simeq 1.2 M_{\rm max}(\rm stat)$ (*i.e.*, rotation increases maximum allowable mass by about 20%). Configurations with $\Omega_{\rm max}$ (or $M_{\rm max}(\rm rot)$) are quite flattened spheroids: $R_{\rm equator} \simeq 1.5R_{\rm pole}$; their equatorial radius turns out to be typically some 30% larger than that for $\mathcal{M}^{\rm max}(\rm stat)$ [19, 20]; see also Section 4).

In Table II we show the values of $\Omega_{\rm max}$, calculated in [20, 22] for a broad set of twelve realistic EOS of dense matter, listed in Table I, supplemented with a schematic free Fermi gas model of neutron matter. Within this broad set of EOS, the minimum value of the rotation period is $P_{\rm min} = 2\pi/\Omega_{\rm max} = 0.48$ ms. It increases to $P_{\rm min} = 0.49$ ms, if non-causal EOS are excluded.

4. The empirical formula for the maximum rotation frequency

Let us estimate first the maximum rotation frequency for a simpler case of Newtonian gravitation, and neglecting the rotational deformation of the star. The mass shedding limit, which represents the upper bound to Ω, is determined from the condition of the exact balance of gravitational pull and centrifugal force at the equator of rotating star, of mass M and radius R. Within our approximations, this leads to

$$\Omega_{\rm max}^{\rm Newtonian} = 1.15 \times 10^4 \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10\ {\rm km}}\right)^{-3/2} \ {\rm s}^{-1} . \qquad (4.1)$$

Exact calculations of the neutron star models with realistic EOS show, that at $\Omega \sim \Omega_{\rm max}$ neutron star is significantly flattened by the effect of rotation, so that $R_{\rm equator} \sim 1.5 R_{\rm pole}$. In exact calculations, the stationary equilibrium configuration at $\Omega \sim \Omega_{\rm max}$ results from a delicate interplay of effects of rapid rotation, and those of (strong) gravitation. Notice, that neutron star model should be stable with respect to the axi-symmetric perturbations (see Section 3), and the axi-symmetric instability is a *general relativistic* effect. It seems thus difficult to expect the existence of a simple *universal* (i.e., independent of the EOS of dense matter) relation between the mass and radius of non-rotating model with a maximum allowable mass, $\mathcal{M}_{\rm max}^{\rm stat}$(EOS), and the value of $\Omega_{\rm max}$(EOS), which is a maximum value of Ω for the stable rotating models $\mathcal{M}^{\rm rot}$(EOS). Let us remind, that maximum mass allowed for $\mathcal{M}^{\rm rot}$ is some 20% higher, while the corresponding equatorial radius of the configuration with maximum allowable mass is more than 30% larger, than those for the non-rotating models $\mathcal{M}^{\rm stat}$ (see Section 3).

In view of this, it was quite surprising to find, that a simple *empirical* relation

$$\Omega_{\rm max} = C \left(\frac{M_{\rm max}({\rm stat})}{M_\odot}\right)^{1/2} \left(\frac{R_{\rm max}({\rm stat})}{10\ {\rm km}}\right)^{-3/2} , \qquad (4.2)$$

after a suitable choice of C (independent of EOS !), holds with a remarkable precision for realistic EOS of dense matter [21, 11, 22]. The form of Eq. (4.2) is identical to that of Eq. (4.1). However, the numerical value of C should be chosen by fitting the formula (4.2) to the values of $\Omega_{\rm max}$, obtained in exact calculations for a possibly broad set of EOS of dense matter. In what follows, we will use numerical results, presented in Table II, and taken from [22]. If we restrict ourselves to EOS, which are causal within neutron star models (speed of sound $v_{\rm sound} = ({\rm d}p/{\rm d}\rho)^{1/2} < c$), we get $C_{\rm causal} = 0.77 \times 10^4\ {\rm s}^{-1}$, the relative accuracy of the fit to the exact results of Table II being better than 5% ! This fit is visualized in Fig. 4. Non-causal EOS should be

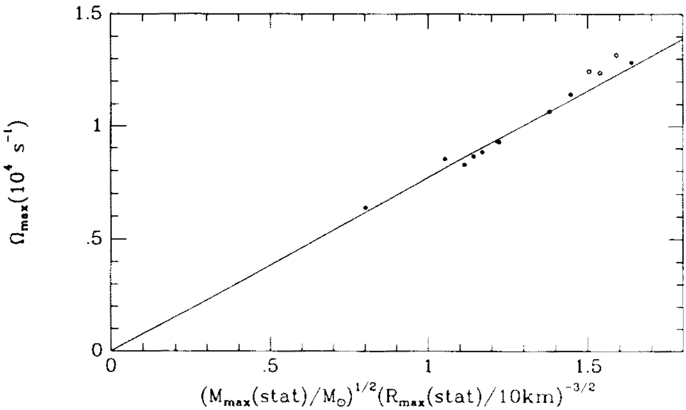

Fig. 4. Empirical formula fit to the exact results obtained for EOS of Table I, with $C = C_{causal} = 0.77 \times 10^4\ s^{-1}$. Small filled circles correspond to EOS which are causal within neutron star models. Numerical results for EOS which are non-causal within central cores of massive neutron stars are represented by small open circles.

rejected as non-physical. In view of this the value of the empirical constant C, recommended in [22], is $C = C_{causal} = 0.77 \times 10^4\ s^{-1}$.

It is interesting to note, that this recent value of C, obtained in [22], coincides with the original value, proposed by Haensel and Zdunik in 1989 [21], which was based on the set of older results for rapidly rotating neutron stars, obtained by Friedman *et al.* [23]. The values of C based on the exact results of other groups turn out to be rather close to those obtained in [21, 22]. Namely, exact results of Lattimer *et al.* [18] lead to $C = 0.76 \times 10^4\ s^{-1}$, while the causal subset of results of Cook *et al.* [19] corresponds to $C = 0.78 \times 10^4\ s^{-1}$ (see [22] for a detailed comparison of C's obtained by different authors).

The empirical formula, Eq. (4.2), establishes thus a rather precise correspondence between two extremal configurations: one from the set $\{\mathcal{M}^{stat}\}$, and the other from $\{\mathcal{M}^{rot}\}$. While the form of empirical formula, Eq. (4.2), is identical to Eq. (4.1), its physical content is thus very different. It is interesting to note, that the empirical value of C is 2/3 of the Newtonian coefficient, appearing in Eq. (4.1).

The practical importance of empirical formula consists in allowing a rapid and precise determination of Ω_{max} (with typical error of only a few percent), using easily available (calculated) parameters of the non-rotating maximum mass model. This was actually the motivation behind the introduction of the empirical formula by Haensel and Zdunik [21], but it should be stressed that the precision and universality of the formula by far exceeded the original expectations of these authors. In practice, the empirical formula for Ω_{max} was widely used for the calculation of the maximum neutron star rotation frequency for EOS, for which only results of static calculations were available (see, *e.g.*, [18, 24–27].

It turns out to be possible to derive other empirical formulae, relating the masses and equatorial radii of the extremal configurations from the $\{\mathcal{M}^{stat}\}$ and $\{\mathcal{M}^{rot}\}$ sets.

Recent analysis shows, that for realistic EOS of dense matter one has $M_{max}(\mathrm{rot}) = 1.18\ M_{max}(\mathrm{stat})$ and $R_{max}^{equator}(\mathrm{rot}) = 1.34\ R_{max}(\mathrm{stat})$ [10]. These empirical relations hold within the relative error of only 3% and 4%, respectively !

5. Conclusions

Limiting mass shedding frequency for uniformly rotating, stable neutron star models, Ω_{max}, calculated for realistic EOS of dense matter using precise numerical methods, is represented, within better than 5 %, by an empirical formula, relating it to mass and radius of static configuration with maximum allowable mass.

The value of Ω_{max}(EOS) is an important parameter characterizing the EOS of dense matter. An EOS, which yields Ω_{max} such that $\Omega_{max} < 2\pi/P_{obs,min}$ (where $P_{obs,min}$ is the shortest observed pulsar period) should be rejected as inconsistent with observations. Future detection of sub-millisecond pulsars may thus be expected to put interesting constraints on the EOS of superdense matter.

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