QUADRUPOLE PAIRING VERSUS δ-INTERACTION. COMPARATIVE STUDY*,**

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Pairing and deformation selfconsistent total routhian surface type calculations with different residual interactions in the particle-particle channel are presented. Calculations involving a separable pairing force of monopole plus quadrupole type are compared to those where a contact force (δ-force) is used. The calculations were performed for the superdeformed band ^{192}Hg and different rotational bands in ^{120}Cs, ^{133}Nd and ^{135}Sm to demonstrate the superiority of state dependent over seniority pairing.

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1. Introduction

The mean-field plus BCS model is one of the most exploited microscopic model in high-spin physics. This is not only due to its relative practical

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simplicity but most of all due to its predictive power. Within a rather simple concept a variety of different phenomena can be understood qualitatively. However, a more detailed comparison of model results to data shows that the agreement in many cases is not satisfactory, calling for extensions of the model. The main efforts for improvements are going towards (*i*) an inclusion of more realistic forces in the pairing channel as well as (*ii*) towards restoration of broken symmetries (mainly particle number symmetry), see *e.g.* [1–3] and references quoted therein.

The aim of this work is to present and briefly compare the results of pairing and deformation selfconsistent calculations involving a separable monopole plus quadrupole pairing force (s-force) and a contact force (δ-force).

2. The model

Our model hamiltonian involves a phenomenological mean-field potential (of Woods–Saxon type) and a residual two-body interaction. The standard solution to this problem is known as total routhian surface (TRS) calculations. This technique is based on the Strutinsky shell correction method where the total routhian of the nucleus is divided into a macroscopic (liquid-drop) and microscopic part accounting for quantal shell effects (shell-correction) and pairing correction. In almost all standard applications the treatment of the pairing channel is limited to a seniority force and the pairing energy is calculated via a non-selfconsistent BCS formalism. The selfconsistency enters the model only via the minimization of the total routhian with respect to deformation parameters.

Recently, we extended the TRS technique by enforcing rigorously the selfconsistent treatment of the pairing channel. To avoid the well known problems of early superfluid-to-normal phase transition generated by the mean-field solution and to make our calculations feasible over wide frequency range we approximately restore the particle number symmetry using the Lipkin–Nogami method [4].

The new method appears to have good convergency properties and allows relatively easy to go beyond the simple seniority pairing. Below we show the results of our calculations for two different types of interactions involving a separable monopole plus quadrupole pairing model (s-force) and a contact force (δ-force). The model based on the s-force was already introduced and discussed in detail previously [4, 2]. It is worth to stress again that (*i*) it involves the concept of a *shape independent* quadrupole force and (*ii*) it is essentially free of adjustable parameters. On the other hand the calculations involving the δ-force are presented for the first time and our results are to be regarded as preliminary. Due to the limited space we are not

able to go into details of the implementation. It is worth mentioning that: (*i*) the two-body matrix elements are calculated using the technique based on the transformation to the center of mass and relative motion coordinates [5, 6] (*ii*) the strength of the force is estimated by comparing the pairing energy to the one obtained in the calculations with monopole pairing (*iii*) the single particle potential, Γ, is disregarded (the force is volume-active and the Γ potential produces too strong modifications of the Woods-Saxon spectrum) (*iv*) 50 deformed states are taken in the calculations.

3. Numerical results and discussion

The results of our calculations for the yrast superdeformed band in ^{192}Hg are presented in Fig. 1. The left panel compares the δ-force calculations (dashed line) to seniority pairing (solid line). Surprisingly, both curves show similar, incorrect slope at low frequencies. A closer examination of the diagonal values of the pairing potential reveals relatively small variations of $\Delta_{\alpha\bar{\alpha}}$ for orbitals close to the Fermi level, resembling strongly the monopole Δ (seniority) approximation.

Satula & Wyss – Figure 1

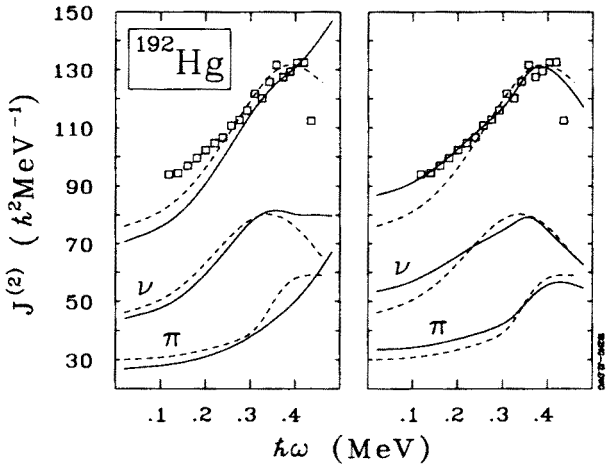

Fig. 1. The dynamical moment of inertia for the yrast superdeformed band in ^{192}Hg. The left panel shows the results of the δ-force (dashed line) and seniority pairing (solid line) calculations. Right panel shows the results of δ-force calculations with full (dashed line) and reduced (solid line) strengths.

In order to improve the agreement, we reduced the pairing strength by a factor 0.9 (right panel, solid curve). There, the agreement to the data

is quite superior but it is not clear why rescaling the strength so strongly influences the final result. One should also point out, that the pairing energy in this case is much smaller then the pairing energy for monopole pairing calculations.[1]

Satula & Wyss - Figure 2

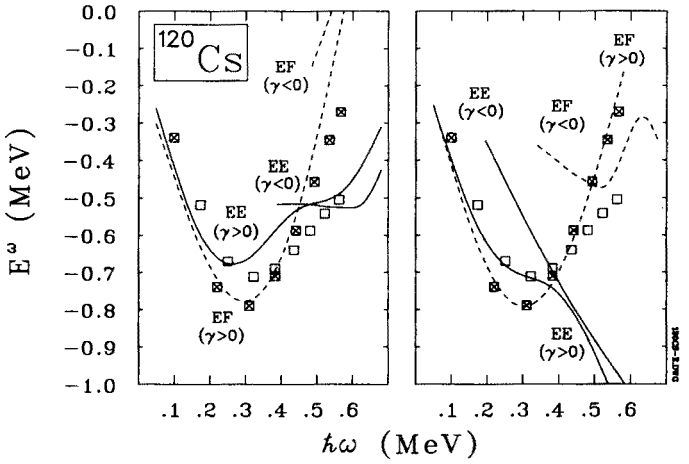

Fig. 2. Total routhians for $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in ^{120}Cs. The left (right) part shows the calculations with s-force (δ-force). The different signature configurations are marked as EF ($\alpha = 0$) and EE ($\alpha = 1$) and denoted by dashed (solid) lines respectively. The excited bands with negative triaxiality ($\gamma < 0$) are also shown.

The $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in ^{120}Cs form one of the most spectacular examples of the signature inversion phenomenon. The old TRS calculations were not able to reproduce the signature inversion and an interpretation involving the proton-neutron interaction was proposed in [8]. Fig. 2 shows the total routhian calculated with the present approach for s-force (left panel) and δ-force (right panel). Both forces reproduce the signature inversion. Note, that the signature inversion is not a result of large $\gamma-$ deformation, but due to the residual interaction. The agreement for the s-force calculations is very satisfactory for both EE and EF configurations. However, the first unblocked alignment of a pair of $\nu h_{11/2}$ for EE-configuration appears too early in the δ-force calculations in serious disagreement to the data.

The last example shows the dynamical moments of inertia for the well deformed bands in ^{133}Nd and ^{135}Sm (Fig. 3). Again the old TRS calculations were not able to reproduce the data encountering serious problems

[1] The latter is fitted to account for odd-even mass difference, see [7]. Nevertheless, the reduced strength is used in the following.

Satula & Wyss – Figure 3

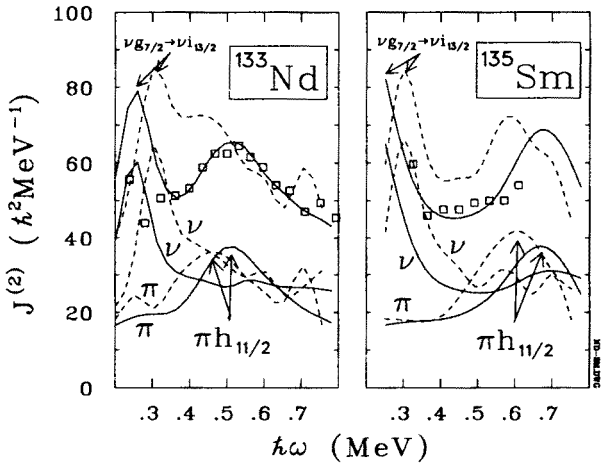

Fig. 3. Dynamical moments of inertia for the well deformed bands in ^{133}Nd (left panel) and ^{135}Sm (right panel). Solid (dashed) lines denote the results for s-force (δ-force) calculations respectively. The proton and neutron contributions are also depicted separately.

with $\pi h_{11/2}$ alignment [9, 10]. The s-force based model works surprisingly well. The crossing of the $\nu g_{7/2}$-and $\nu i_{13/2}$-band (being a consequence of adiabatic blocking) is pushed systematically towards higher frequencies for the δ-force as compared to the s-force calculations. Simultaneously, the $\pi h_{11/2}$ alignment appears too low in frequency, hence overestimating the dynamical moment of inertia.

4. Conclusions

The monopole plus quadrupole interaction is usually seen as a submodel to the δ-interaction. The argumentation is based on the formal mathematical property of the δ-function which can be expanded into an infinite serie of spherical harmonics. One should bear in mind, however, that this separability of the δ-function is restricted to its angular part only. In fact any δ-function based interaction with arbitrary radial form-factor does posses this property. Therefore, depending on the radial form factor of the quadrupole interaction (in our implementation it is generated by the double stretched quadrupole operators $Q''_{2\mu}$, see [2]) the separable force can be quite different from the pure δ-interaction.

Another source of differences relates to the separability. The separability induces the existence of common (state independent) mean-value gap

parameter. In the case of separable interactions, the state dependence of the Δ potential enters only via a single particle matrix element of the generator of the force and is independent on the local value of the pairing tensor. In contrary, for non-separable interactions the values of the pairing potential are sensitive to both two-body matrix element and the local value of the pairing tensor.

The calculations presented above clearly indicate that the monopole plus quadrupole pairing model (at least in the version we use) cannot be considered as a submodel to the δ-interaction. It agrees systematically better to the data than the δ-interaction. To draw further general conclusions more systematic calculations are needed, also including issues like the influence of the strength and/or basis cut-off on the results,

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