

ON THE STAGGERING EFFECT OF DYNAMICAL MOMENTS OF INERTIA IN SUPERDEFORMED BANDS*

P. MAGIERSKI

Institute of Physics, Warsaw University of Technology
ul. Koszykowa 75, PL-00-662 Warsaw, Poland

(Received December 18, 1995)

The possibility of the appearance of the C_4 symmetry in the rotational bands is studied within the particle-rotor model. The role of the triaxiality of the rotor is analyzed.

PACS numbers: 21.60.Ev

1. Introduction

In the spectroscopy of superdeformed (SD) bands in ^{149}Gd [1], ^{153}Dy [2], ^{194}Hg [3], and $^{131,132}\text{Ce}$ [4] the $\Delta I = 4$ staggering of the dynamical moment of inertia has been observed. It manifests itself in systematic shifts of the energy levels which are alternately pushed down and up with respect to a purely rotational sequence. The amplitude of this staggering is of the order of 50 eV. It was suggested that their origin could be associated with the presence of the C_4 symmetry.

To date some models have been proposed to explain the experimental data. Hamamoto and Mottelson [5, 6] have studied the properties of a quartic rotational Hamiltonian. In their approach the C_4 perturbation coincides with the symmetry axis of a nucleus. Staggering appears then as a result of tunneling between the four equivalent minima of the total energy surface of the Hamiltonian due to the K -mixing. Nevertheless microscopic approaches do not confirm the presence of such perturbation in the mean field [7-10], or they found it to be too small to generate the effect.

On the other hand Pavlichenkov and Flibotte [11] proposed the model in which C_4 perturbation appears along the rotation axis. They employ the

* Presented at the "High Angular Momentum Phenomena" Workshop in honour of Zdzisław Szymański, Piaski, Poland, August 23-26, 1995.

rotational Hamiltonian which generate the C_{4v} bifurcation. In this scenario staggering effect does not originate from a static hexadecapole deformation. Rather, it arises from a dynamical effect that involves the alignment of an angular momentum vector.

One should also mention that the effect of staggering has been generated by the hexadecapole-hexadecapole interaction in the simple model consisting of a single j -shell filled by N identical nucleons [12, 13].

In the present paper I consider the triaxial version of the particle plus rotor model. Allowing for a slight triaxiality of the rotor it is shown that the lowest states of the system for a sufficiently high spin possess the symmetry which allows to distinguish states differing by two units of angular momentum.

1. The model

The model consists of a single particle occupying j -shell and coupled to the triaxial core in the strong coupling limit:

$$\hat{H} = \sum_{\alpha} \left[A_{\alpha} (\hat{I}_{\alpha} - \hat{j}_{\alpha})^2 + Q_{\alpha} \hat{j}_{\alpha}^2 \right] + \chi (\hat{j}_1^2 - \hat{j}_2^2)^2. \quad (1)$$

In the above formula \hat{I}_{α} denotes the total angular momentum component of the rotor in the body-fixed frame, whereas \hat{j}_{α} stands for the single-particle spin component. The coefficients A_{α} are inversely proportional to moments of inertia according to the relation $A_{\alpha} = 1/2\mathcal{J}_{\alpha}$. The Q_{α} are the mean-field parameters describing the quadrupole deformation of the shell whereas χ is the strength of the C_4 type of deformation along the third axis¹. In the further considerations I will assume that $A_2 > A_1 \geq A_3$. The diagonalization of the above Hamiltonian is performed in the basis of states:

$$|IMKjk\rangle = |IMK\rangle |jk\rangle, \quad (2)$$

where $|IMK\rangle$ denotes the state of the rotor and can be expressed as Wigner functions depending on Euler angles. By M I denoted the projection of the total angular momentum I on the third axis in the laboratory frame whereas K denotes the projection on the third axis in the body-fixed frame. Since the Hamiltonian is rotationally invariant I will omit the quantum number M in the formula (2). The $|jk\rangle$ describes the single particle state with spin j and projection on the third axis in the body-fixed frame equal to k .

¹ This model has been first studied (without the last term in (1)) by Pashkevich and Sardaryan [14].

If one assumes that the particle is coupled to a core rotational band comprising spins $0, 2, 4, \dots$ then the basis will be restricted to states for which the following relation holds:

$$|K - k| = 0, 2, 4, \dots \quad (3)$$

Hence the result eigenfunctions of (1) corresponding to the total spin I will have the form

$$|\Psi_I\rangle = \sum_{K,k} a_{K,k} |IjKk\rangle. \quad (4)$$

The classical motion of the vectors \mathbf{j} and \mathbf{I} can be determined from the following relations:

$$\begin{aligned} \dot{j}_i &= \{j_i, H\} \\ \dot{I}_i &= \{I_i, H\}, \end{aligned} \quad (5)$$

where $\{.,.\}$ denotes the Poisson bracket. Thus the time derivatives of the components of \mathbf{I} and \mathbf{j} follow from the transformation of the Hamiltonian under infinitesimal rotations. The qualitative nature of the motion corresponds to a periodic precession of \mathbf{I} and \mathbf{j} around the minimum for sufficiently low energies. If the energy becomes high enough the structure of orbits will change dramatically. This critical value of energy defines the separatrix on the total energy surface which divides the phase space into separated regions².

In order to investigate the small amplitude motion near the minimum point one can assume that the angular momentum of the core $\mathbf{R} = \mathbf{I} - \mathbf{j}$ is aligned along the third axis. This is the case when $A_3 < A_{1,2}$. Thus one can put $I_2 - j_2 \approx 0$ and $I_1 - j_1 \approx 0$. The above equations can be then simplified to the form:

$$\begin{aligned} \dot{j}_1 &= 2A_3 j_2 (I_3 - j_3) + 2j_2 j_3 (Q_2 - Q_3) - 4\chi j_2 j_3 (j_1^2 - j_2^2), \\ \dot{j}_2 &= -2A_3 j_1 (I_3 - j_3) - 2j_1 j_3 (Q_1 - Q_3) - 4\chi j_1 j_3 (j_1^2 - j_2^2), \\ \dot{j}_3 &= 2j_1 j_2 (Q_1 - Q_2) + 8\chi j_1 j_2 (j_1^2 - j_2^2), \\ \dot{I}_1 &= -2A_3 I_2 (I_3 - j_3), \\ \dot{I}_2 &= 2A_3 I_1 (I_3 - j_3), \\ \dot{I}_3 &= 0. \end{aligned} \quad (6)$$

One can see that in case of $Q_1 = Q_2$ the equations involving vector \mathbf{j} possess the symmetry associated with the rotation around the third axis

² It has been studied in case of an axially symmetric rotor by Bohr and Mottelson [15].

about the $\pi/2$ angle (*i.e.* C_4 symmetry). That means that the transformation

$$\begin{cases} j_1 \rightarrow j_2 \\ j_2 \rightarrow -j_1 \\ I_1 \rightarrow I_2 \\ I_2 \rightarrow -I_1 \end{cases} \quad (7)$$

leaves the equations (6) unchanged. Therefore the orbits around the minimum will possess the additional symmetry originating from the local properties of the total energy surface in the vicinity of the minimum point. In our case the transformations (7) form the local symmetry group which is suitable for the description of a part of the rotational multiplet levels. Obviously such symmetry will be satisfied only approximately and the amplitude of the symmetry breaking components will depend on the relative magnitude of symmetry breaking and restoring terms in (6) as well as the energy of the motion.

3. Results and conclusions

In the current section I will present the results of quantal calculations performed within the model described in the previous section. The diagonalization has been performed for a given spin I with one particle occupying the level $j = (11/2)$. In order to estimate whether the symmetry is broken completely or it survives one can introduce the measure of C_4 symmetry possessed by a given state. It is defined as the absolute value of a difference between the components of a wave function associated with different representations of this group. Thus for the wave function

$$|\Psi_I\rangle = a|\Psi_I^1\rangle + b|\Psi_I^2\rangle \quad (8)$$

the quantity $||a|^2 - |b|^2|$ will be the measure of the C_4 symmetry. As an example one can consider Fig. 1 where this quantity is plotted as a function of the total angular momentum. Calculations were performed for $A_3 = 10$ keV, $A_1 = 21$ keV, $A_2 = 240$ keV for three different sets of parameters. One can see that the shape of curves looks similarly. As the spin increases the motion of the system approaches the aligned regime discussed in the previous section. In spite of the fact that the Hamiltonian does not possess the C_4 symmetry the lowest levels can be characterized by a suitable quantum number associated with this symmetry. Moreover one should emphasize that the quantum number associated with the C_4 symmetry will change when the spin increases. This can be easily understood if one considers the lowest state of the Hamiltonian to be approximately

$$|\psi_I\rangle \approx |IjIj\rangle \quad (9)$$

which corresponds to the aligned configuration. One can see that $|\psi_I\rangle$ and $|\psi_{I+2}\rangle$ belong to different representations of the C_4 group (this fact has been pointed out in [11]). Moreover one need not necessarily employ the C_4 term ($\chi \neq 0$) to generate this symmetry.

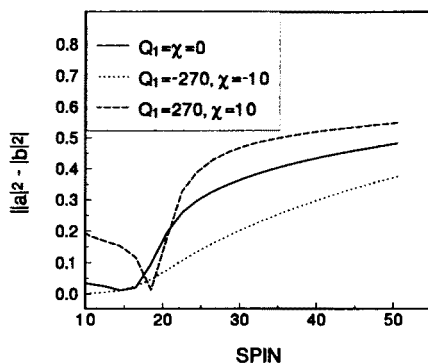


Fig. 1. The quantity $||a|^2 - |b|^2|$ plotted for the lowest states versus total angular momentum for three different sets of parameters. $Q_2 = 200$, $Q_3 = 0$, $A_3 = 10$, $A_1 = 21$, $A_2 = 240$ keV

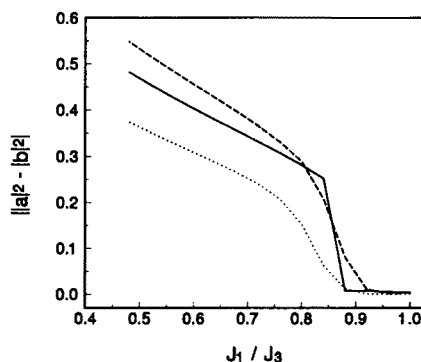


Fig. 2. The quantity $||a|^2 - |b|^2|$ plotted versus J_1/J_3 for the same sets of parameters as in the figure 1 and $I = 101/2$.

It is obvious that the symmetry can manifest itself only in the case when we allow for a small triaxiality of the rotor. This is shown in Fig. 2 where the quantity $||a|^2 - |b|^2|$ for the lowest states is plotted versus J_1/J_3 . Calculations have been performed for the same sets of parameters as before. One can see that for an axial rotor the symmetry is completely broken. This is the consequence of the fact that in this case the spin of the core R is delocalized and therefore breaks the rotational symmetry around the third axis.

This research was supported in part by the Polish State Committee for Scientific Research under Contracts No. 2 P302 056 06 and 2 P30 B 09809, and by the computational grant from the Interdisciplinary Centre for Mathematical and Computational Modelling (ICM) of Warsaw University.

REFERENCES

- [1] S. Flibotte *et al.*, *Phys. Rev. Lett.* **71**, 4299 (1993).
- [2] B. Cederwall *et al.*, submitted.
- [3] B. Cederwall *et al.*, *Phys. Rev. Lett.* **72**, 3150 (1994).
- [4] A.T. Semple *et al.*, preprint, 1995.
- [5] I. Hamamoto, B. Mottelson, *Phys. Lett.* **B333**, 294 (1994).
- [6] I. Hamamoto, B. Mottelson, *Physica Scripta* **56**, 27 (1995).
- [7] I. Ragnarsson, private communication, 1994.
- [8] S. Frauendorf, J. Meng, J. Reif, Proc. Int. Conf. on Physics from Large γ -ray Detector Arrays, Berkeley, 1994, LBL-35687, p. 52.
- [9] P. Magierski, P.-H. Heenen, W. Nazarewicz, *Phys. Rev.* **C51**, R2880 (1995).
- [10] W.D. Luo, A. Bouguettoucha, J. Dobaczewski, J. Dudek, X. Li, to be published.
- [11] I.M. Pavlichenkov, S. Flibotte, *Phys. Rev.* **C51**, R460 (1995).
- [12] K. Burzyński, P. Magierski, J. Dobaczewski, W. Nazarewicz, *Physica Scripta* **56**, 228 (1995).
- [13] P. Magierski, K. Burzyński, J. Dobaczewski, W. Nazarewicz, *Acta Phys. Pol.* **B26**, 261 (1995).
- [14] V.V. Pashkevich, R.A. Sardaryan, *Nucl. Phys.* **A65**, 401 (1965).
- [15] A. Bohr, B. Mottelson, *Physica Scripta* **22**, 461 (1980).