EXTENDED THOMAS–FERMI DESCRIPTION OF ROTATING NUCLEI*

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We present a semiclassical description of rotating nuclei in the framework of the Extended Thomas-Fermi density functional theory up to order $\hbar^2$. It leads to functional expressions of quantities such as the kinetic energy, current and spin vector densities in terms of the local (spin-scalar) density $\rho(\vec{r})$ alone. For effective nucleon-nucleon interactions of the Skyrme type a simple analytical expression is obtained for moments of inertia. It consists, at lowest order, of the Thomas-Fermi (TF) term which is shown to be identical to the rigid-body moment of inertia and semiclassical corrections of order $\hbar^2$ which are small. The importance of the Thouless-Valatin selfconsistency terms which are included in our approach, is pointed out. Within this approach we have performed selfconsistent semiclassical calculations in the restricted space of diffuse (Fermi type) densities and ellipsoidal triaxial shapes. Our analysis is qualitatively consistent with LDM results as in the paper by Cohen, Plasil and Swiatecki. However a significant dependence of the LDM parameters as function of the angular momentum has been pointed out. Generalizing our method to finite temperature we recover functional expressions formally similar to the $T = 0$ case with temperature dependent coefficients. The above formalism has also been extended to the semi-quantal description of other large amplitude collective modes and of their couplings. In particular it has been applied to the dynamics of a rigid rotation coupled with a simple (uniform) intrinsic vortical motion.

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A simplified (even classically) way of treating nuclear rotations is constituted by the so-called routhian approach consisting in *cranking* a nucleus around a given fixed axis. In Quantum Mechanics this approach (inspired by the Hamiltonian formalism of canonical transformations in Classical Mechanics) may be derived *e.g.* as the solution of the Schrödinger equation under a constraint on the total angular momentum $\vec{J} = \vec{L} + \vec{S}$. Many groups have worked within this framework upon using phenomenological one-body Hamiltonians. More microscopical approaches have also been developed (see [1–3] and references therein). In view of the numerical task which they involve, simpler hence more transparent approaches, of a semiclassical type, are worth attempting. *Semiclassical* should be understood here in the Wigner sense [4] of a $\hbar$ expansion beyond the lowest order term referred to here as the Thomas–Fermi (TF) approximation. Various authors have performed semiclassical calculations at the TF order [5, 6] or beyond [7, 8] but dealing with simple one-body potentials, without effective mass and, most importantly, without the Thouless–Valatin [9] selfconsistency correction generated as a response of the mean field to the time-odd part of the density generated by the cranking piece of the Hamiltonian. In the present contribution we will consider in the framework of the Extended Thomas–Fermi (ETF) approach [10] the semiclassical solutions of a one-

body routhian derived from a usual Skyrme force parametrization including effective mass, Thouless-Valatin terms and full spin contributions.

Solving the variational problem yields for a Slater determinant the following one-body routhian h_q (the index q refers to the two different charge states of the nucleon) whose action on a single-particle wavefunction is given by

$$h_q\varphi_q(\vec{r},\sigma)=\left\{-\frac{\hbar^2}{2m}f_q(\vec{r})\vec{\nabla}^2+\left[U_q(\vec{r})+\frac{\hbar^2}{8m}\Delta f_q(\vec{r})\right]-i\hbar\vec{\alpha}_q(\vec{r})\cdot\vec{\nabla}\right\}\varphi_q(\vec{r},\sigma)$$
$$+\sum_{\sigma'}\hbar(\vec{S}_q-i\hbar\vec{W}_q\times\vec{\nabla})\cdot<\sigma|\vec{\sigma}|\sigma'>\varphi_q(\vec{r},\sigma')\,, \qquad (1)$$

where appear the form factors of the effective mass f_q, the central potential U_q, the cranking field $\vec{\alpha}_q$, the spin field $\vec{S}_q$ and the spin-orbit field $\vec{W}_q$. All these form factors are functionals of the following densities: the local (spin-scalar) density $\rho_q(\vec{r})$, the kinetic energy density $\tau_q(\vec{r})$, the current density $\vec{j}_q(\vec{r})$, the spin-vector density $\vec{\rho}_q(\vec{r})$ and the spin-orbit density $\vec{J}_q(\vec{r})$. Quite some time ago Grammaticos and Voros [11] have given functional relations for these densities in terms of the above form factors. Our aim, however, is to give functional expressions of these densities in terms of the densities $\rho_q(\vec{r})$ alone. To disentangle this complicated system of equations we will take advantage of the following simple yet crucial remark: in a functional expression of order $\hbar^2$ it is sufficient to replace all quantities, for instance the cranking field $\vec{\alpha}_q$, by their TF approximations. One starts by showing that at the TF level the cranking field $\vec{\alpha}_q$ which includes a contribution from the orbital part of the constraint and another which represents the Thouless-Valatin selfconsistent field, is simply given by

$$\vec{\alpha}_q = f_q(\vec{r}\times\vec{\omega})\,. \qquad (2)$$

From there we calculate the explicit functional expressions for the spin vector densities $\vec{\rho}_q$ and then the current densities $\vec{j}_q$ and the full cranking form factor $\vec{\alpha}_q$. One finally obtains the functional expressions for τ_q and $\vec{J}_q$. One further demonstrates that the spin vector density is proportional to the angular velocity $\vec{\omega}$

$$\vec{\rho}_q = \hbar\chi_q\vec{\omega}\,, \qquad (3)$$

where, in analogy with magnetism, we have defined a spin susceptibility χ_q. It turns out that for all Skyrme force parametrizations in common use this quantity is of the *Pauli paramagnetic* type. The current density corresponds at the TF level to that of a rigid rotor, with semiclassical corrections due to the orbital motion and spin degrees of freedom. The semiclassical corrections for the orbital motion yield a surface-peaked counter rotation with

respect to the rigid body current which may be viewed, within the same analogy, as a Landau diamagnetism proper to finite fermionic systems. It has to be emphasized that the semiclassical functionals derived in this way are *universal* in nature and valid for every nucleus, at every deformation and for every effective interaction used.

Once these functional relations are established the total energy density in the laboratory system can be written as a functional of the local densities ρ_q alone. It has formally (as expected for a two-body force) a quadratic dependence on the rotation velocity $\vec{\omega}$ thus defining the dynamical moment of inertia [12]

$$\mathcal{J}^{(2)} = m \sum_q \int \left\{ r_\perp^2 \rho_q - (3\pi^2)^{-(2/3)} f_q \rho_q^{1/3} + \left[\frac{\hbar^2}{2m} + W_0(\rho + \rho_q) \right] \chi_q \right\} d^3r \,. \quad (4)$$

The TF term comes from the orbital motion and turns out to be the rigid-body moment of inertia. Semiclassical corrections arise from both orbital motion and spin degrees of freedom. The orbital and spin corrections are individually small (of the order of 5-10 % in heavy nuclei) and cancel each other to a large extent. This makes the TF (or rigid-body) value constitute a good approximation to the full semiclassical moment of inertia. It should be noticed that the Thouless–Valatin selfconsistency terms are fully included in our approach. When neglecting these terms the dynamical moment of inertia is found to be underestimated by as much as 25 % in heavy nuclei demonstrating in this way the importance of these terms [12].

We have also undertaken selfconsistent semiclassical calculations using the Skyrme interaction SkM* [13] known for its excellent surface and deformation properties, at least not too far from the β-stability valley. Limiting ourselves to generalized Fermi functions which are very close to the full variational solutions [14], we have performed these calculations in the restricted subspace of ellipsoidal axial and triaxial deformations [15]. Our analysis is qualitatively consistent with the LDM results of Ref. [16], showing *e.g.* for the nucleus ^{90}Zr a Jacobi bifurcation from oblate into triaxial deformations at $I \approx 55\hbar$. For $I \geq 70\hbar$ the nucleus becomes prolate tending to the instability against centrifugal fission. Our calculations establish the validity of the very concept of a rotating liquid drop which has been widely used for the determination of average shapes of rotating nuclei. However, a significant dependence of the LDM parameters as function of the angular momentum is observed [17].

The Extended Thomas–Fermi method of rotating nuclei has also been generalized to finite temperatures [18] along lines similar to those sketched in Ref. [14]. One again can derive functional relations of all quantities as a function of the local densities ρ_n and ρ_p alone, except that their coefficients are given in terms of Fermi integrals involving the nuclear temperature.

We observe *e.g.* that in the large T limit the moments of inertia tend to their rigid body value, indicating the decreasing importance of semiclassical corrections with increasing excitation.

The above formalism has also been extended to the semi-quantal description of other large amplitude collective modes and of their couplings [19, 20]. In particular it has been applied to the dynamics of a rigid rotation coupled with a uniform vortical motion. Such a description has been tentatively proposed as an explanation for the staggering phenomenon observed recently in the rotational spectra of superdeformed states of ^{159}Dy and ^{194}Hg [19], as well as for the identical bands found in both superdeformed and normally deformed nuclei [20].

REFERENCES

[1] B. Gall, P. Bonche, J. Dobaczewski, H. Flocard, P.-H. Heenen, *Z. Phys.* **A348**, 183 (1994).
[2] M. Girod, J.P. Delaroche, J.F. Berger, J. Libert, *Phys. Lett.* **B325**, 1 (1994).
[3] J. König, P. Ring, *Phys. Rev. Lett.* **71**, 3079 (1993).
[4] E.P. Wigner, *Phys. Rev.* **40**, 749 (1932).
[5] C. Bloch, *Phys. Rev.* **93**, 1094 (1954).
[6] J. da Providencia, *Nucl. Phys.* **A510**, 322 (1990).
[7] M. Brack, B.K. Jennings, *Nucl. Phys.* **A258**, 264 (1976).
[8] M. Durand, P. Schuck, J. Kunz, *Nucl. Phys.* **A439**, 263 (1985).
[9] D.J. Thouless, J.G. Valatin, *Nucl. Phys.* **A31**, 211 (1962).
[10] M. Brack, C. Guet, H.-B. Håkansson, *Phys. Rep.* **123**, 275 (1985).
[11] B. Grammaticos, A. Voros, *Ann. Phys.* **123**, 359 (1979); **129**, 153 (1980).
[12] K. Bencheikh, P. Quentin, J. Bartel, *Nucl. Phys.* **A571**, 518 (1994).
[13] J. Bartel, P. Quentin, M. Brack, C. Guet, H.-B. Håkansson, *Nucl. Phys.* **A386**, 79 (1982).
[14] J. Bartel, M. Brack, M. Durand, *Nucl. Phys.* **A445**, 263 (1985).
[15] E. Chabanat, J. Meyer, K. Bencheikh, P. Quentin, J. Bartel, *Phys. Lett.* **B325**, 13 (1994).
[16] S. Cohen, F. Plasil, W.J. Swiatecki, *Ann. Phys.* (NY) **82**, 557 (1974).
[17] P. Quentin, D. Samsoen, K. Bencheikh, E. Chabanat, J. Meyer, J. Bartel, *Nuclear Shapes and Nuclear Structure at Low Excitation Energies*, Frontières, Paris, 1994, pp. 283-294; D. Samsoen, J. Piperova, J. Bartel. J. Meyer, P. Quentin, in preparation.
[18] K. Bencheikh, J. Bartel, P. Quentin, in preparation.
[19] I.N. Mikhailov, P. Quentin, *Phys. Rev. Lett.* **74**, 3336 (1995).
[20] I.N. Mikhailov, P. Quentin, to appear in Proc. Int. Conf. on Low Energy Nuclear Dynamics, St. Petersburg, April 1995.