DYNAMICS OF CATASTROPHIC PROCESSES IN NUCLEAR PHYSICS*

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A brief overview is given of recent advances towards extending the nuclear Boltzmann model to processes exhibiting instabilities and associated catastrophic bifurcations, by incorporating the fluctuating part of the collision term in the equation of motion for the one-body phase-space density.

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1. Introduction

Many natural processes exhibit catastrophic transformations, in which a given system undergoes an irreversible change and acquires a qualitatively different appearance, often leading to a higher degree of complexity. In physics, this type of phenomenon may be associated with phase transitions, but its occurrence is much more general. The dynamics of such processes is further complicated by the presence of instabilities and bifurcations and the development of appropriate formal descriptions poses significant challenges. The general topic of catastrophic trajectory branching is especially important in the context of nuclear collisions, since detailed microscopic simulations are required for the extraction of the key physics from experimental data. Figure 1 displays schematic illustrations of catastrophic processes in nuclear dynamics.

This presentation summarizes recent advances towards extending the effective one-body description, the nuclear Boltzmann equation, to accommodate catastrophic processes, such as occur when the expanding participant matter condenses into fragments. By incorporating the fluctuating

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part of the collision term in the equation of motion for the one-body phase-space density, one obtains the Boltzmann–Langevin model without introducing any new physical parameters. This extended model produces the correct relaxation dynamics for both averages and fluctuations, and it also describes the spontaneous agitation and amplification of unstable collective modes. Thus, the model appears to provide a physically sound basis for addressing actual nuclear collision dynamics and may be useful for interpreting multifragmentation data.

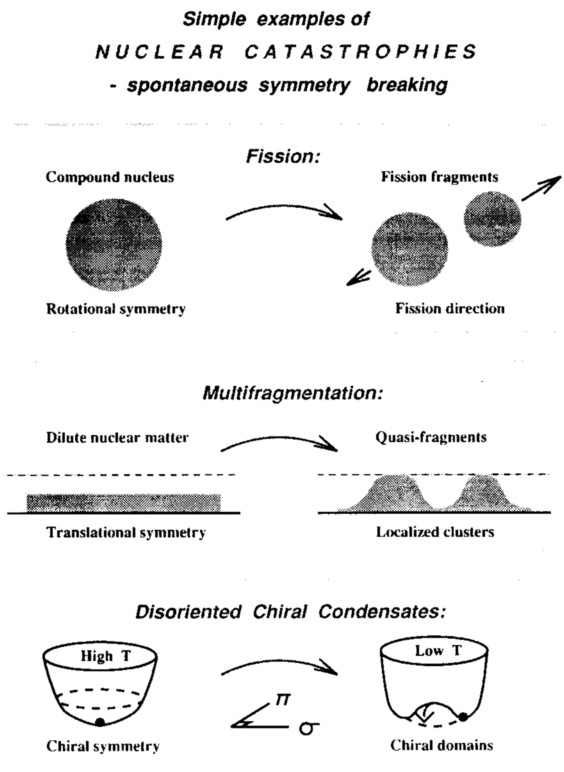

Fig. 1. Three simple examples of catastrophic transformations in nuclear dynamics. The top panel shows how the low-energy fission process transforms a hot compound nucleus into two receding fragments, whose direction of relative motion breaks the initial rotational symmetry. The center panel is relevant for multifragmentation at intermediate energies, showing the spontaneous clusterization of dilute nuclear matter inside the zone of spinodal instability in the density-temperature phase diagram and the associated breaking of the initial translational symmetry. The bottom panel illustrates the formation of disoriented chiral domains following a high-energy collision, in which the effective potential becomes unstable as the system cools, causing the chiral field to select a particular orientation in isospace and thereby breaking the chiral symmetry prevailing at high temperature.

2. Boltzmann–Langevin dynamics

Over the past several years, the nuclear Boltzmann–Langevin model has emerged as a promising microscopic tool for nuclear dynamics at intermediate energies. This model describes the one-body phase-space density $f(\boldsymbol{r},\boldsymbol{p},t)$ for the nucleons (and any other hadrons present), as it evolves in the self-consistent effective one-body field $h[f]$ while subjected to the effect of occasional Pauli-suppressed two-body collisions,

$$\dot{f}(\boldsymbol{r},\boldsymbol{p}) \equiv \frac{\partial}{\partial t}f - \{h[f],f\} = \begin{cases} 0 & \text{(Vlasov)} \\ \bar{I}[f] & \text{(Boltzmann)} \\ I[f] & \text{(Boltzmann–Langevin)} \end{cases} . \quad (1)$$

In the simplest description the residual interactions are neglected and the Vlasov equation emerges, the semi-classical analog to TDHF [1]. The effect of the two-body collisions is described by the collision integral $I[f] = \bar{I}[f] + \delta I[f]$ which can be decomposed into an average part $\prec I \succ = \bar{I}$ and its fluctuating remainder δI.

The Pauli-blocked average collision integral was first employed by Nordheim [2]. Later on, the approach was adapted to hadronic gases [3] and subsequently augmented with the mean field [4], leading to the nuclear Boltzmann model. The average effect of the two-body collisions on the phase-space occupancy is

$$\bar{I}[f]: \quad \dot{f}_1 = \int \frac{dp_2}{h^3}\, v_{12} \int d\Omega'_{12}\, (\underbrace{\bar{f}_1\bar{f}_2 f_{1'} f_{2'}}_{\text{gain}} - \underbrace{f_1 f_2 \bar{f}_{1'}\bar{f}_{2'}}_{\text{loss}})\, \frac{d\sigma_{12\leftrightarrow 1'2'}}{d\Omega'_{12}} . \quad (2)$$

The last factor is the differential cross section for (elastic) nucleon-nucleon scattering in the medium. Thus the in-medium properties must be known to allow quantitative results to be calculated. Conversely, in so far as the calculated observables depend on this physical ingredient, confrontations with data may help to extract this microscopic information from experiment.

The correlation function of the collision term was first considered by Ayik *et al.* [5]. The inclusion of $\delta I[f]$, the fluctuating part of the collision integral, produces a continual rearrangement of the phase-space occupancy and so is akin to the Langevin force acting on a Brownian particle. Therefore, an entire *distribution* of possible phase-space densities emerges, $\phi[f]$. This obviously presents a formidable task, both formally and numerically. The analysis is greatly aided by recasting the transport problem in terms of a generalized Fokker–Planck equation [6],

$$\dot{\phi}[f] = -\int ds \frac{\partial}{\partial f(s)} V[f](s)\phi[f] + \int ds \int ds' \frac{\partial^2}{\partial f(s)\partial f(s')} D[f](s,s')\phi[f] , \quad (3)$$

where s denotes the phase-space point $(\boldsymbol{r}, \boldsymbol{p})$, with $ds = d\boldsymbol{r}d\boldsymbol{p}/h^3$ and V and D denote the generalized transport coefficients. The drift coefficient $V[f](s) = \bar{I}[f](s)$ represents the average rate of change, while the $2D[f](s,s') = \prec \delta I \delta I' \succ$ gives the diffusive growth rate of the correlated fluctuations. This formulation of the problem has made it possible to develop and test a numerical simulation method on a phase-space lattice [7], so far the only demonstrably valid approach [8].

It is instructive to study the equilibrium features of the model. Under suitable conditions of confinement and stability, the BL equation of motion (1c), as well as the corresponding Fokker–Planck transport equation (3), yield dynamical solutions that approach stationarity and display properties characteristic of the associated quantum-statistical equilibrium. Any physically acceptable simulation code must also do so.

It is especially simple to consider a uniform Fermi–Dirac gas of nucleons that interact only via their stochastic two-body collisions, because the correlated equilibrium fluctuations are given by a simple analytical expression [9], a novel realization presenting a general contribution to statistical physics. The correlations between the fluctuations in phase space have the form $\prec \delta f_1 \delta f_2 \succ = f_1^0 \bar{f}_1^0 \delta_{12} - f_1^0 \bar{f}_1^0 S_{12} f_2^0 \bar{f}_2^0$, where the coefficient S_{12} can be expressed as a bilinear form in those observables that are left invariant by the two-body collision process (density, current, and pressure),

$$S_{12} = \frac{1}{\rho \phi_0} \left[1 + \frac{\varepsilon_1 - \varepsilon_0}{\sigma_T} \frac{\varepsilon_2 - \varepsilon_0}{\sigma_T} + 3 \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{2m\varepsilon_0} \right], \tag{4}$$

where $\sigma_T^2 \approx \frac{\pi^2}{3} T^2$ and $\phi_0 \approx \frac{3}{2} \frac{T}{\varepsilon_F}$. It has been shown explicitly that the Fokker–Planck equation for the correlation function leads to the appropriate equilibrium form [9]. This provides an important test of both the transport model per se and its specific numerical lattice implementation. However, the method is too computer demanding to be practical for realistic collision scenarios. Its primary value therefore is to establish a well-based reference against which to test simpler approximate methods.

3. Triggering of catastrophes in unstable matter

Fluctuations are of crucial importance when instabilities occur. Nuclear matter in the unstable spinodal zone of the phase diagram therefore provides a useful testing ground for candidate dynamical models. Considerable progress has been made over the past few years in understanding the spontaneous agitation of unstable modes [10].

The problem is most conveniently discussed in terms of the Fourier components of the phase-space fluctuations δf. Uniform matter is mechanically unstable against density undulations if $F_0 < -1$, where $F_0(k,T) \equiv$

$\phi_0(\rho_0/T)(\partial h_k/\partial \rho)$ is the generalized Landau parameter [11] associated with the finite wave number k and the finite temperature T. The corresponding spinodal boundaries are displayed in Fig. 2.

Inside the spinodal region there exists, for each wave number $\boldsymbol{k}$, a pair of collective modes, $f_k^{\pm}(\boldsymbol{p})$. They have an exponential time development, with an e-folding time t_k determined by the dispersion relation,

$$1 = g \int \frac{d\boldsymbol{p}}{h^3} \frac{\partial h_k}{\partial \rho} \frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\boldsymbol{k}\cdot\boldsymbol{v} \pm i t_k^{-1}} \frac{\partial f^0}{\partial \varepsilon} \approx -F_0(k,T)\left(1 - \gamma_k \arctan\frac{1}{\gamma_k}\right), \quad (5)$$

where $\gamma_k = 1/kV_F t_k$ is a dimensionless measure of the growth rate. Figure 3 shows the growth times and rates for a range of temperatures and wave lengths, as calculated with a density and momentum dependent effective two-body interaction that leads to a good reproduction of macroscopic nuclear properties, such as densities, bindings, barriers, and optical potential [11].

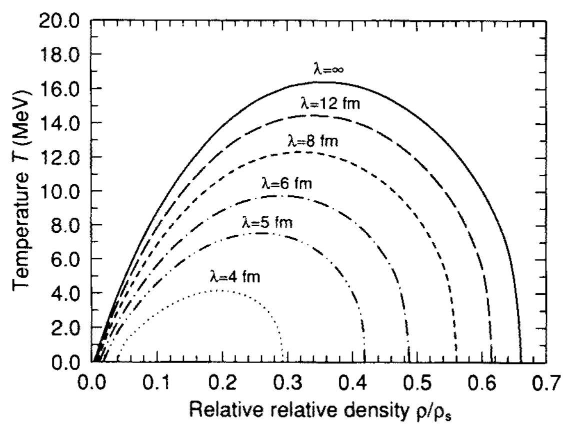

Fig. 2. Spinodal boundary. The region of spinodal instability is delineated in the density-temperature phase-plane for harmonic density undulations for a specified wave length λ.

The collective part of the density fluctuations can be expressed as

$$f_k^{\rm coll}(\boldsymbol{p},t) = A_k^+(t) f_k^+(\boldsymbol{p}) + A_k^-(t) f_k^-(\boldsymbol{p}) \,. \quad (6)$$

The time development of the amplitudes is governed by $dA_k^\nu/dt = \nu A_k^\nu/t_k + F_k^\nu$, where the stochastic force F_k^ν arises from the Langevin part of the collision term. It agitates the mode and thereby exposes it to amplification by the unstable selfconsistent effective field. The Langevin term vanishes on the average, $\prec F_k^\nu \succ = 0$, and so the amplitude A_k^ν will remain zero on the average if we start from uniform matter. However, its correlation coefficients give rise to source terms, $\mathcal{D}_k^{\nu\nu'}$, which govern the rate at which the magnitude of the density fluctuations grow,

$$\frac{d}{dt}\,\sigma_k^{\nu\nu'}(t) \;=\; 2\mathcal{D}_k^{\nu\nu'} \;+\; \frac{\nu+\nu'}{t_k}\sigma_k^{\nu\nu'}(t)\,, \tag{7}$$

where $\sigma_k^{\nu\nu'} \equiv \prec A_k^\nu A_k^{\nu'} \succ$ are the correlation coefficients. The density fluctuations are described by the density-density correlation function, $\sigma(\boldsymbol{r}_{12}) \equiv \prec \delta\rho(\boldsymbol{r}_1)\delta\rho(\boldsymbol{r}_2) \succ$. It is instructive to consider its Fourier transform,

$$\sigma_k \;\equiv\; \int d\boldsymbol{r}\; e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\sigma(\boldsymbol{r}) \;=\; \sum_{\nu\nu'}\sigma_k^{\nu\nu'}\,, \tag{8}$$

which is displayed in Fig. 4 at various times.

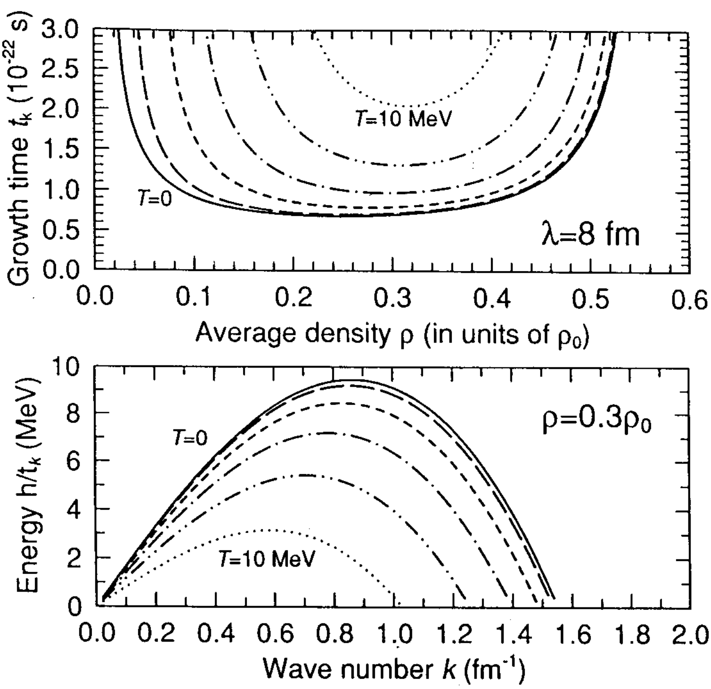

Fig. 3. Collective amplification rates. The top panel shows the e-folding time t_k characterizing the amplification of harmonic density undulations having the particular wave length $\lambda = 8$ fm, as a function of the relative density ρ/ρ_0 and for various temperatures T. The bottom panel shows the corresponding characteristic energy $E_k = \hbar/t_k$ for the particular density $\rho = 0.3\rho_0$, as a function of the wave number $k = 2\pi/\lambda$ and for the same temperatures T.

For early times, a broad range of wave numbers are present in the fluctuation spectrum, reflecting the fact that the collision integral is local in space and so produces white noise. As more time elapses, the most rapidly amplified modes grow progressively more dominant (recall that the plot is logarithmic), so that only a relatively narrow band of wave lengths remains significant by the time the amplitudes have grown beyond the regime of linear response. This feature may suggest that the spectrum of final fragment sizes is correspondingly narrow. In actual nuclear collisions, this would mean that the massive fragments tend to have the same size, an effect that can be subjected to experimentally verification by means of modern multidetector arrays.

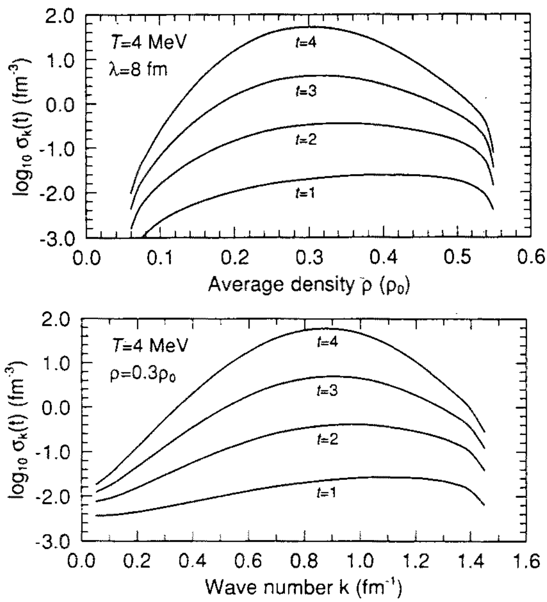

Fig. 4. Growth of density fluctuations. The evolution of the average magnitude of the density fluctuations is illustrated by plotting the (logarithm of the) Fourier transform of the density-density correlation function σ_k defined in (8) for various times t, either as a function of density ρ for a fixed wave length $\lambda = 8$ fm (top) or as a function of wave number k for a fixed density $\rho = 0.3\rho_0$ (bottom).

4. Memory time

Current models of nuclear dynamics are all time local. In particular, the collision term in the one-body approaches is assumed to be local in both space and time, in accordance with Boltzmann's original treatment for dilute gases. However, the fastest-growing collective modes have fairly high characteristic energies $E_k = \hbar/t_k$, as is evident from Fig. 3. Consequently, one must expect non-local effects to be important. Recently, the collision term in the Boltzmann–Langevin model was augmented by a finite memory time by Ayik [12].

The inclusion of a finite memory time leads to a rather simple modification of the feed-back equation (7) governing the collective correlation coefficients [13],

$$\frac{d}{dt}\,\sigma_k^{\nu\nu'}(t) \;=\; 2\,\mathring{\mathcal{D}}_k^{\nu\nu'}\,\chi_k^{\nu\nu'}(t) \;+\; \frac{\nu+\nu'}{t_k}\sigma_k^{\nu\nu'}(t)\,, \tag{9}$$

where the time-dependent coefficients $\chi_k^{\nu\nu'}(t)$ express the effect of the memory time.

This modulation of the effective source terms causes the evolution of the covariance coefficients $\sigma_k^{\nu\nu'}(t)$ to deviate from what would be obtained without a memory time, $\sigma_k^{\nu\nu'}(t) = \mathring{\sigma}_k^{\nu\nu'}(t)\bar{\chi}_k^{\nu\nu'}(t)$. These deviations can be significant, particularly in the domain of fastest growth, as is illustrated in Fig. 5. It, therefore, appears important to incorporate such memory effects in dynamical simulations. Fortunately, this can be done without increasing the computational effort significantly.

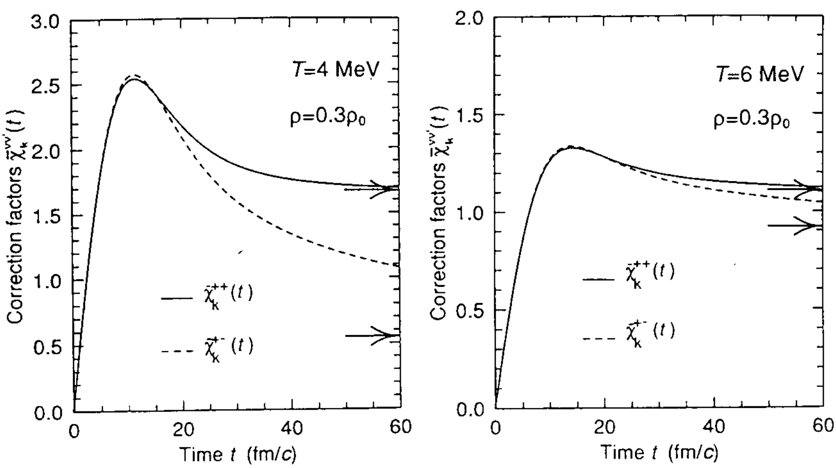

Fig. 5. Memory correction factors. The correction factors $\bar{\chi}_k^{\nu\nu'}(t)$ determining the time dependence of the covariance coefficients describing the agitation of collective modes in unstable nuclear matter, for the density $\rho = 0.3\rho_0$ and two temperatures, $T = 4,\ 6$ MeV. The arrows indicate the values approached at large times, $t \gg t_k$.

5. Concluding remarks

Significant progress has been made over the past several years towards extending the nuclear Boltzmann model to scenarios in which the dynamics displays catastrophic bifurcations. The ensuing Boltzmann–Langevin model appears to be on solid formal ground, with physically reasonable properties, and the efforts are now being focussed on the practical implementation the model for the purpose of applying it to nuclear collisions at intermediate energies where multifragmentation phenomena provide a promising area of confrontation between theory and experiment.

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REFERENCES

[1] P. Bonche, S.E. Koonin, J.W. Negele, *Phys. Rev.* **C13**, 1226 (1976).
[2] L.W. Nordheim, *Proc. Roy. Soc.* **A119** (1928) 689; E.A. Ühling, G.E. Uhlenbeck, *Phys. Rev.* **43**, 552 (1933).
[3] J. Randrup, *Nucl. Phys.* **A314**, 429 (1979).
[4] G.F. Bertsch, H. Kruse, S. Das Gupta, *Phys. Rev.* **C29**, 673 (1984).
[5] S. Ayik, C. Gregoire, *Phys. Lett.* **B212**, 269 (1988); *Nucl. Phys.* **A513**, 187 (1990).
[6] J. Randrup, B. Remaud, *Nucl. Phys.* **A514**, 339 (1990).
[7] G.F. Burgio, Ph. Chomaz, J. Randrup, *Nucl. Phys.* **A529**, 157 (1991).
[8] F. Chapelle, G.F. Burgio, Ph. Chomaz, J. Randrup, *Nucl. Phys.* **A540**, 227 (1992).
[9] B. Benhassine, M. Farine, E.S. Hernandez, D. Idier, B. Remaud, F. Sebille, *Nucl. Phys.* **A545**, 81c (1992); J. Randrup, Interdisciplinary Workshop on Statistical Description of Transport in Plasma, Astro, and Nuclear Physics, Les Houches, France 1992, Nova Science; G.F. Burgio, B. Benhassine, B. Remaud, F. Sebille, *Nucl. Phys.* **A567**, 626 (1994).
[10] M. Colonna, Ph. Chomaz, J. Randrup, *Nucl. Phys.* **A567**, 637 (1994).
[11] J. Randrup, preprint LBL-35848 (1995), in preparation.
[12] S. Ayik, *Z. Phys.* **A350**, 45 (1994).
[13] J. Randrup, S. Ayik, *Phys. Rev.* **C50**, 2947 (1994).