GLOBAL DESCRIPTION OF NUCLEAR PROPERTIES AT HIGH ANGULAR MOMENTUM — PROPERTIES OF UPSLOPING ORBITALS*

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For nuclei with only a few particles outside closed shells, rotational bands might be formed in configurations with holes in the core. Common properties of bands of this kind with hole(s) in the high-j shells, $f_{7/2}$, $g_{9/2}$ and $h_{11/2}$ are discussed and specific examples are compared with experiment. At superdeformation it is suggested that an approximate decoupling factor for the $\pi[301\ 1/2]$ and $\nu[411\ 1/2]$ orbitals can be extracted from existing data

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1. Introduction

It is well-known that nuclei with proton and neutron numbers in between closed shells are deformed in their ground states. Indeed, the deformed regions show up in a very regular way where the degree of deformation is essentially governed by the number of particles in open shells. This can be seen for example from a contour plot in the (N, Z)-plane [1] of the E_{2+} energy or the E_{4+}/E_{2+} ratio or from the fact that these such quantities can be parametrized by the number of valence protons and valence neutrons [2]. The regular features should hold as long as no particles are put into orbitals from the next higher shell or particles are excited from the core. For example, in the heavy rare earth region, rotational bands are known with protons in the downsloping $[541\ 1/2]$ or $[660\ 1/2]$ orbitals, which emerge from the $h_{9/2}/f_{7/2}$ and $i_{13/2}$ subshells above the $Z = 82$ gap. The occupation of these orbitals will in general lead to a shape polarization towards

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larger deformations [3]. In a similar way, a nucleus in the $Z = 50 - 82$ region will become more deformed if an upsloping orbital emerging from subshells below the $Z = 50$ gap is emptied. In would be an interesting project to study if the resulting deformations can be parametrized in a similar way as in Ref. [2]. Here, however, we will confine ourselves to give examples of "non-collective" nuclei with one or two protons outside the $Z = 28$ and $Z = 50$ shells where collective bands are built from configurations with holes in the high-j shells below the gap. Similarly, we will indicate how holes in $h_{11/2}$ orbitals below the $N = 82$ gap increases the collectivity in nuclei with a few neutrons outside $N = 82$. Indeed, it is in the nuclei with N =80–86 that the best examples of high spin superdeformed bands are known [4]. In these bands, several upsloping orbitals are emptied (and down-sloping orbitals are filled) relative to the states of small or no deformation. Of special interest are the bands built on upsloping $j_z = \Omega = 1/2$ orbitals which are only weakly disturbed by the non-diagonal Coriolis interaction. Thus, with a small decoupling factor, a, they should have a small but finite signature splitting which could make it possible to observe both signature branches at high spin in the same nucleus. If so, it should be possible to extract a numerical value for a by extrapolating back to $\omega = 0$. This is then one of the few quantities which might give some hint about wave-functions at superdeformation. We will briefly discuss cases where it seems that both signatures of the [301 1/2] proton orbital and the [411 1/2] neutron orbital have been observed.

2. Configuration dependent Nilsson–Strutinsky calculations

The calculational results presented here are obtained using the Nilsson–Strutinsky cranking method where fixed configurations are traced as functions of spin [5]. The fixing of configurations is facilitated by the fact that we use the wave functions of the rotating harmonic oscillator as basis states when determining the single-particle eigenstates and eigenvectors of the modified oscillator potential. Thus, the quadrupole deformation and the rotational degrees of freedom are fully included already in the basis and only the hexadecapole deformation and the $\ell \cdot s$ and ℓ^2 terms must be taken care of through diagonalisation. Because of this special feature, we can to a good approximation treat the principal quantum number of the rotating oscillator, $N_{\rm rot}$, as pure. This makes it much easier to keep track of different configurations because we can label the single-particle orbitals not only by signature and parity but also with their $N_{\rm rot}$ quantum number. In addition, after the diagonalization, we identify [6, 7] the high-j orbitals within the different $N_{\rm rot}$ shells (*e.g.* the orbitals in the $N = 4$ shell with major components in $g_{9/2}$). This makes it possible to specify the number of high-j

orbitals occupied in each N-shell, something which is very important for the present applications.

3. Collective bands with holes in high-j subshells — terminating bands

In the single-particle diagram of Fig. 1, we have specifically indicated the upsloping [404 9/2] orbital which emerges from the $g_{9/2}$ high-j shell. The role of holes in this orbital can be illustrated from the so called superdeformed bands in the $Z = 58$ Ce nuclei. According to our calculations [8], these bands are formed with 2 proton holes in the $g_{9/2}$ orbitals and consequently 10 valence protons in the $d_{5/2}$, $g_{7/2}$ and $h_{11/2}$ orbitals, see Fig. 1. The large deformation results from these proton holes and from the neutrons in the downsloping [541 1/2] and [660 1/2] orbitals. In a similar way, it seems that the collective bands seen in $Z = 46$ Pd nuclei with $N \approx 58$ are formed because of holes in the neutron [404 9/2] orbital [9].

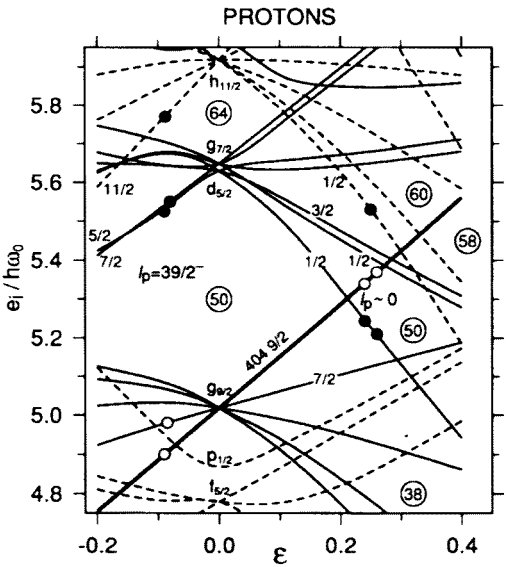

Fig. 1. Single-proton orbitals drawn as function of quadrupole deformation ε with the upsloping $g_{9/2}$ orbital [404 9/2] drawn by a thick line. Note that for $\varepsilon \approx 0.35$ there is a gap for $Z = 58$ with two holes in this orbital and another gap for $Z = 60$ where these holes are filled. At smaller deformation, the filling of the most favoured $Z = 51$ $(g_{9/2})^{-2}$ configuration is shown relative to the $Z = 50$ core at low proton spin I_p (prolate shape) and at the maximum spin $I_p = 39/2^-$ (oblate shape). This configuration is also illustrated in Fig. 2.

Our main interest here is however nuclei around closed shells which are non-collective in their ground state configurations but which become collective at higher spins due to particle-hole excitations across the closed shell. This is exemplified in Fig. 2 for ^{109}Sb, and ^{64}Zn in the form of the filling of spherical subshells. These two configurations are very similar. The proton numbers are close to magic, but hole(s) in the proton high-j shells, $g_{9/2}$ and $f_{7/2}$, increase the active number of valence particles + valence holes so that collective rotational bands are formed.

In the nucleus ^{109}Sb, three rotational bands have been observed [10, 6] up to very high spins. A further observation is that their highest spins are about as large as can be formed in configurations with two $g_{9/2}$ proton holes (Fig. 2). Thus, if there are not more than 2 holes in the core, these rotational bands must come close to termination. Indeed, as discussed in detail in Refs. [6, 7], the $\pi(g_{9/2})^{-2}$ configurations in this region of nuclei are rather special cases where it appears possible to observe how collective rotational bands gradually lose their collectivity until they terminate in non-collective states corresponding to the highest possible spin values for that specific distribution of neutrons and protons over the j-shells.

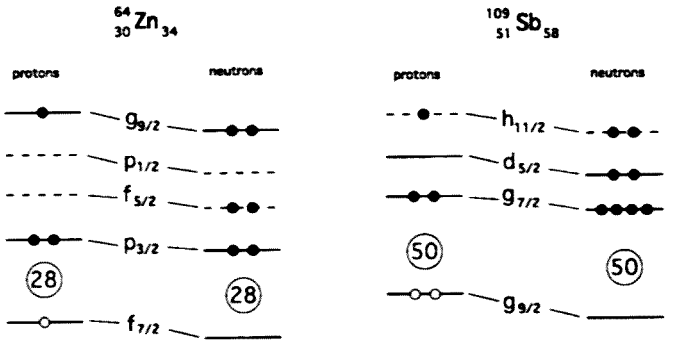

Fig. 2. Filling of spherical subshells of collective bands in ^{109}Sb and ^{64}Zn. It seems that the holes in $\pi g_{9/2}$ ($Z \approx 50$) and $\pi f_{7/2}$ ($Z \approx 28$) have very similar roles increasing the collectivity and creating configurations which are essentially uncoupled from the configurations without these holes.

When comparing experiment and calculations in ^{109}Sb, one difficulty is that the high-spin bands are not connected to the low-spin configurations so neither their spin nor their excitation energies are known. It is possible to find quite a good agreement but this is with special assignments of spins and excitation energies. This comparison suggests that all three bands have been observed to their terminating spins which is in the range $83/2 - 89/2\hbar$. A real test of this interpretation would be to measure spins and excitation energies of these bands. Before this becomes possible, another partial test

could be to compare with rotational bands in neighbouring nuclei. It is then very fortunate that two high spin bands in the nucleus ^{108}Sn [11] have been connected to the low-spin structures. These bands are compared with calculations in Fig. 3. There is an excellent agreement between them and the two bands calculated to be lowest in energy in the $I = 30 - 35$ range. In addition one more band has been observed in ^{108}Sn. This band can be interpreted as the other signature of band 2 which is consistent with calculations. These data for ^{108}Sn, as well as data for other nuclei in this mass region (*e.g.* [12], do thus support our interpretation of the rotational bands in ^{109}Sb.

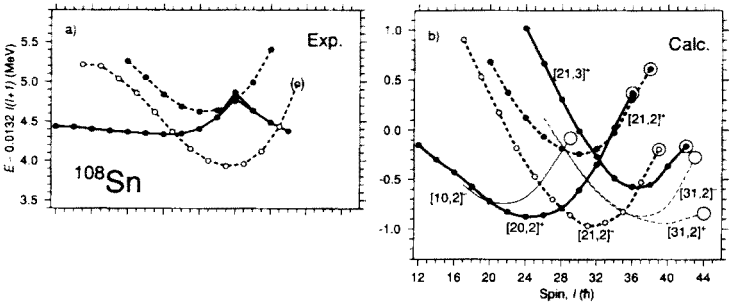

Fig. 3. Experimental and calculated excitation energies normalized to rigid rotation drawn vs. spin I for ^{108}Sn [11]. Positive parity is drawn by full lines, negative parity by dashed lines while closed and open symbols are used for signature $\alpha = 1/2$ and $\alpha = -1/2$, respectively. Large circles indicate terminating states. The zero points of the energy scales are different corresponding to the observed ground state and the liquid-drop energy at spin zero, respectively. The experimental band shown for $I = 20 - 36$ has not been connected to the lower spin states so its spin values and excitation energy are tentative. The nomenclature used in the theoretical calculations is $[p_1\ p_2, n]^\alpha$, where p_1 is the number of proton $g_{9/2}$ holes, p_2 is the number of $h_{11/2}$ protons, n the number of $h_{11/2}$ neutrons and $\alpha = +$ and $-$ corresponds to signature $\alpha = 0$ and 1, respectively. The calculations suggest strongly that the experimental bands correspond to the $(g_{9/2})^{-2}$ configurations drawn by thick lines. Examples of calculated low-lying $(g_{9/2})^{-1}$ and $(g_{9/2})^{-3}$ configurations are shown by thinner lines.

The next major shell below $N = Z = 50$ is $N = Z = 28$. Recently, a high-spin rotational band has been found [13] in the ^{64}Zn nucleus. It goes to $I \approx 20$ with two signatures which are essentially degenerate. This means that there must be a signature degenerate orbital which is occupied by one "odd" particle. There seems to be only one reasonable candidate for such an orbital, namely the proton [303 7/2] orbital from the $f_{7/2}$ shell below the $Z = 28$ gap. Thus, according to our interpretation, the valence particles are distributed around the $Z = 28$ gap in very much the same way

as they are distributed around the $Z = 50$ gap in the ^{109}Sb nucleus, see Fig. 2. The similarity between ^{109}Sb and ^{64}Zn is supported by the fact that the experimental $\mathcal{J}^{(2)}$ moments of inertia behave in a similar way as shown in Fig. 4. The fact that both in experiment and in calculations, the $\mathcal{J}^{(2)}$ value drops to unusually small values, less than half the rigid body value, is a nice support for our interpretation. The low value of $\mathcal{J}^{(2)}$ indicates the high energy cost to align the last spin units in these configurations. A qualitative understanding of the high energy at termination is obtained from Fig. 1, where it is seen that at maximal spin for oblate shape, the holes are in orbitals far below the Fermi surface.

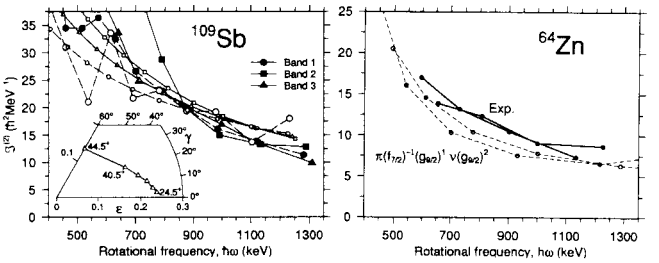

Fig. 4. Experimental (filled symbols) and calculated (open symbols) $\mathcal{J}^{(2)}$ moments of inertia for collective bands in ^{109}Sb and ^{64}Zn. The configurations are of the type illustrated in Fig. 2 with two $h_{9/2}$ holes and one $f_{7/2}$ hole respectively. The calculated values are smoothed by fitting and $I(I+1)$ expansion to the excitation energies except for one band in ^{109}Sb where also unsmoothed values are shown (larger symbols). The calculated change in deformation as function of spin is illustrated in the (ε, γ)-plane for one band in ^{109}Sb up to the terminating value, $I_{max} = 44.5$. Analogous shape changes are calculated in ^{64}Zn ($I_{max} \approx 25$) but the ε-values are somewhat larger.

In the ^{155}Dy nucleus, bands which go towards termination are known since long [14, 15], while it has been speculated [16] if other bands stay more collective, namely bands with neutron holes in the [505 11/2] orbitals emerging from the $h_{11/2}$ shell below the $N = 82$ gap. It seems that this speculation is now supported by recent results [17] from Gamma-sphere. In Fig. 5, the experimental energies with a rigid rotation term subtracted are plotted vs. I. Selected calculated rotational bands are shown in a similar way in Fig. 6. The correspondence between calculations and experiment appears quite good. The bands which are down-sloping for spins $I > 40$ do then approach termination and the $101/2^-$ state is the highest spin which can be build from 3 neutrons in $i_{13/2}$ and 4 in ($h_{9/2}$,$f_{7/2}$) combined with four proton in $h_{11/2}$ and two holes in $g_{7/2}/d_{5/2}$ (relative to the $Z = 64$ subgap). Then there are also configurations which go away from yrast at the highest spins but which can still be followed in experiment. This suggests that they

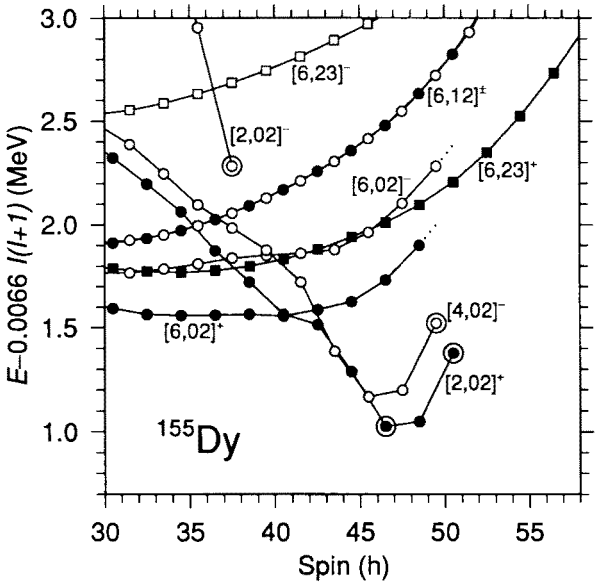

Fig. 5. Observed high-spin bands in ^{155}Dy normalized to rigid rotation drawn vs. spin ([17]).

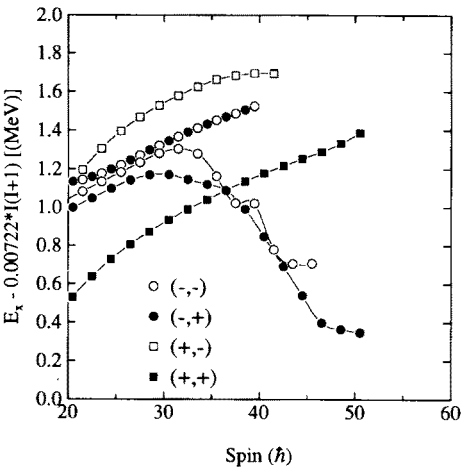

Fig. 6. Calculated high-spin configurations of ^{155}Dy drawn in a similar way as the observed bands in Fig. 5 but with a slightly different $I(I+1)$ term subtracted. The symbols are chosen to indicate the similarities with the observed spectrum. The nomenclature $[p, n_1\ n_2]^\alpha$ is similar as used for ^{108}Sn in Fig. 3; p is the number of $h_{11/2}$ protons, n_1 is the number of $h_{11/2}$ neutron holes and n_2 the number of $i_{13/2}$ neutrons.

have a considerably different configuration, for example hole(s) in the $h_{11/2}$ shell compared with the terminating bands. Indeed, the calculations suggest that the observed bands are of this type. Thus, with one hole in $h_{11/2}$, a signature degenerate band is formed while with two holes, the odd neutron will be in $i_{13/2}$ with a large signature splitting but even so, it appears that both signatures of this configuration is seen in experiment.

4. Rotational bands based on upsloping $\Omega = 1/2$ orbitals at superdeformation — the decoupling parameter

Also at superdeformation (SD), one can see different roles of upsloping and downsloping orbitals. Here, we will only discuss the $A \approx 150$ Dy/Gd region. There, the downsloping high-N orbitals appear to give the special features of the different bands and it is now standard to characterize these bands by their high-N content [18]. The upsloping orbitals on the other hand are very interesting because they appear to give rise to so called identical bands. Typically, upsloping orbitals with $K \geq 3/2$ give signature degenerate bands whereas those with $K = 1/2$ have the additional interesting property that they give a signature splitting. Two orbitals are then of special interest, namely the [301 1/2] proton orbital and the [411 1/2] neutron orbital. These orbitals are situated just below the $Z = 62 - 66$ and $N = 80 - 86$ 2:1 SD gaps.

Bands interpreted as the favoured signature of the [301 1/2] proton orbital at SD are known [19] since several years. These bands have transition energies E_γ which are essentially identical to those of the yrast SD bands in the '$A + 1$' nucleus and it has been noted [20] that they can be explained in the pure pseudospin limit in which case the [301 1/2] orbital has a decoupling factor, $a = 1$. The identical transition energies correspond to an effective alignment, $i_{\text{eff}} = -0.5$ for the [301 1/2] orbital where i_{eff} is defined [21] as the difference in spin at a constant rotational frequency, $\omega = E_\gamma/2$, between the band in the 'A' nucleus with this orbital empty and the band in the '$A+1$' with this orbital filled (for the general classification of identical bands at SD, see also [22]). Recently, another band has been experimentally observed [23] in ^{151}Tb which has been interpreted as built from a hole in the unfavoured signature of the [301 1/2] orbital relative to the ^{152}Dy yrast SD band. The effective alignment given in Ref. [23] agrees quite well (except at the lowest spin values) with predictions [21] using the modified oscillator potential while it is rather far away from the pure pseudospin limit, $i_{\text{eff}} = 0.5$.

When the identical bands based on the [301 1/2] proton orbital were discovered it was soon pointed out [20] that one could also expect to see a bands of similar type based on the [411 1/2] neutron orbital. This orbital

has a decoupling factor $a \equiv -1$ in the pure pseudospin limit. Therefore, the transition energies formed with a hole in the favoured signature of this orbital should come exactly half-way between the transition energies in the '$A+1$' nucleus [20, 22]. It is now quite interesting that a band has recently been found [24] in ^{151}Dy (band 4) which roughly agrees with this prediction, *i.e.* with specific assumptions about the relative spin values, it has $i_{\rm eff} = -0.5$ relative to the ^{152}Dy yrast SD band.

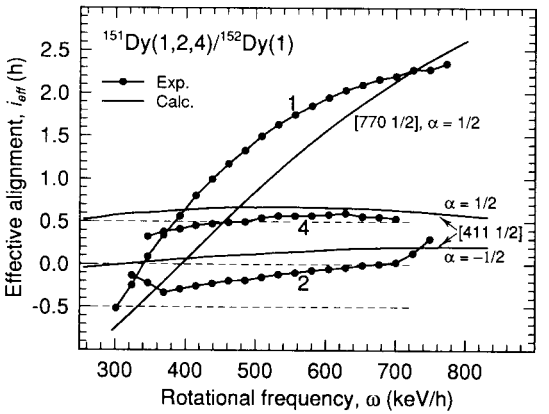

Fig. 7. Observed (filled points) effective alignments $i_{\rm eff}$ for 3 SD bands in ^{151}Dy ([24]) relative to ^{152}Dy compared with the calculated (full lines) configurations we assign to these bands.

Calculations using the modified oscillator potential do however suggest a different explanation. Effective alignments extracted in an analogous way from calculated total energies and observed energies are compared in Fig. 7. The calculations do suggest that the favoured and unfavoured signature bands of the [411 1/2] orbital have $i_{\rm eff} \approx 0$ and $i_{\rm eff} \approx 0.5$, respectively. As seen in the figure, this is roughly consistent with experiment if band 2 is assigned as the favoured signature and band 4 as the unfavoured signature. It also requires that the spin values of band 4 are shifted by $1\hbar$ relative to the pure pseudospin interpretation [24] (in [24], band 2 is suggested to correspond to a hole in an $N = 6$ orbital). One should note that for band 4, there is no specific preference for one or the other spin value from the experimental point of view.

In our opinion, the agreement between experiment and theory in Fig. 7 is a strong argument in favour of our interpretation but it is certainly not conclusive. Another argument is that excited bands have been found in ^{147}Gd [25] and ^{148}Gd [26] with $i_{\rm eff} \approx 0$, *i.e.* similar to band 2 in ^{151}Dy, and which have been assigned to the favoured signature of the [411 1/2] orbital. Furthermore, independent of calculations and of spin assignments,

we note that the difference in effective alignments between bands 2 and 4 is almost constant over the range in rotational frequency ω where the bands are observed. This does strongly suggest that these two bands are formed from signature partner orbitals. We also note that the constant difference in $i_{\mathrm{eff}} \sim 0.6\hbar$ does suggest a single-particle alignment of $\pm \approx 0.3\hbar$ at no rotation and thus a decoupling factor, $a \approx -0.6$, for the [411 1/2] orbital. In a similar way, the decoupling factor of the [301 1/2] orbital can be estimated as $a \approx 0.7$ if the assignments discussed for ^{151}Tb above are correct.

5. Summary

We have discussed the role of holes in the core for nuclei with only a few particles outside closed shells. These holes lead to a polarization towards larger deformation and specific bands are formed which are almost uncoupled from other states. In specific cases, collective rotational bands are formed which terminate in a smooth way while if the effective number of valence particles (*i.e.* valence particles + valence holes) becomes too big, the terminating spin values might become too high and therefore, at present inaccessible to experimental study. These features were exemplified on ^{64}Zn, ^{108}Sn, ^{109}Sb and ^{155}Dy. The superdeformed bands are specific examples of configurations with many holes in the core. In general, starting from a SD band in an 'A' nucleus, one additional hole in an upsloping orbital will lead to a very similar band in the '$A-1$' nucleus (identical bands). We discussed possible observations of holes in upsloping $\Omega = 1/2$ orbitals and how one might determine the decoupling factor at superdeformation from the observation of the two bands created from a hole in one or the other signature.

As seen in the list of references, the comparison between experiment and calculations has been carried out in cooperation with different experimental groups. I am also grateful to A.V. Afanasjev with whom I have collaborated on the theoretical calculations and to D.M. Headly who has carried out the calculations on ^{64}Zn. This research has been supported by the Swedish Natural Science Research Council.

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