PHYSICS OF STRONG ELECTROMAGNETIC FIELDS*

G. SOFF, T. BEIER, AND C. HOFMANN

Institut für Theoretische Physik, Technische Universität,
D-01062 Dresden, Germany

(Received December 8, 1995)

Supercritical electromagnetic fields are predicted to lead to spontaneous emission of positrons in nuclear systems with $Z > 173$. A possible route to identify spontaneous positron creation is discussed. The radiative quantum electrodynamical corrections are calculated. Their contribution amounts to about one per cent of the electron binding energy in nearly critical systems. The formation of supercritical high-Z quasiatoms in heavy-ion collisions is investigated, and the use of δ-electron spectra as measurement tool for nuclear delay times and electron binding energies in superheavy quasiatoms is pointed out. Positron creation by dynamical processes and internal pair conversion is evaluated.

PACS numbers: 23.20. Nx, 29.30. Dn, 31.30. Jv

1. Introduction

Heavy-ion collisions provide the strongest electromagnetic fields to which electrons can be exposed nowadays. Two uranium nuclei which, *e.g.*, approach to a distance of about 20 fm, create an electric field of $|\boldsymbol{E}| \sim 10^{19}$ V/cm and a magnetic field of $|\boldsymbol{B}| \sim 10^{11}$ T. For comparison, the strong magnetic fields generated in the superconducting bending magnets of recent accelerators reach only about $|\boldsymbol{B}| \sim 10$ T.

It is a stimulus to both experimentalists and theoreticians to detect and to understand the behaviour of an electron under these extreme external conditions. During the heavy-ion collision for a short period of time a quasiatom is formed with $Z_u = Z_1 + Z_2 = 184$ in the specific case of U + U. In these heavy systems the binding energy of the innermost electron states does not only exceed the electron's rest mass, but for very close approaches

* Presented at the XXIV Mazurian Lakes School of Physics, Piaski, Poland, August 23–September 2, 1995.

the states are thought to "dive" into the Dirac sea of the negative energy continuum, thus leading to the spontaneous creation of an electron-positron pair. The identification of this process represents one of the major challenges to experimentalists nowadays, and in our contribution we will emphasize a possible route to observe this phenomenon.

Furthermore, as $Z\alpha > 1$ in these systems, they also allow to test the theory of non-perturbative quantum electrodynamics (QED). Whereas QED modifications of binding energies are treated by perturbation theory in "conventional" low-Z atoms, a perturbation series in orders of $Z\alpha$ is not valid for systems under consideration here. Therefore, calculations have to include corrections to all orders in $Z\alpha$.

In the first part of our contribution we will discuss the QED of strong fields and the radiative corrections to the binding energies in superheavy atoms. It will be demonstrated that their influence on high-Z systems amounts to only a few per cent of the binding energy.

Then we will point out possible ways of the spectroscopy of quasiatoms during the heavy-ion impact which might be helpful in identifying candidates for spontaneous positron emission. In particular, δ-electron spectra are an extremely sensitive tool to determine nuclear delay times in collision processes and electron binding energies in the formed quasiatoms. Furthermore, we examine dynamical and nuclear processes, which also lead to the emission of positrons in heavy-ion collisions.

2. Quantum electrodynamics of strong fields

An electron in the Coulomb field of a nucleus is described by the Dirac equation

$$[\gamma^{\mu}(p_{\mu} - eA_{\mu}) - m]\,\psi(\boldsymbol{r},t) = 0\,. \tag{1}$$

Here, we use natural units with $\hbar = c = 1$. $A^{\mu} = \{A_0(\boldsymbol{r}), \boldsymbol{A}\}$. For a point-like nuclear charge we neglect $\boldsymbol{A}$. $A_0(\boldsymbol{r}) = Ze/r$ is the Coulomb potential. In this particular case the spectrum of (1) is given by the Sommerfeld fine-structure formula

$$E_{nj} = m \Big/ \sqrt{1 + \left(\frac{Z\alpha}{n - |\kappa| + \sqrt{\kappa^2 - (Z\alpha)^2}}\right)^2}\,, \tag{2}$$

where $\kappa = \pm 1, \pm 2 \ldots$; $|\kappa| = j + 1/2$ and $n = 1, 2, \ldots$. If no external field is present at all, *i.e.*, $A_{\mu} = 0$ in (1), one obtains the spectra of the positive and negative energy continuum of free particles,

$$E = \pm\sqrt{\boldsymbol{k}^2 + m^2}\,, \tag{3}$$

which are separated by an energy gap from $-m$ to m, in which the eigenvalues of the bound solutions (2) emerge. The negative energy continuum is thought to be filled completely by electrons and sometimes called Dirac sea. A hole state in this sea is identified with a positron. From (2) it is obvious that $E_{n=1,j=1/2} \to 0$ as $Z\alpha \to 1$. For large Z the binding energy therefore approaches the electron rest mass. For Z even larger than $1/\alpha \approx 137$, real values of (2) cease to exist for all $j = 1/2$-states. A detailed analysis [1] reveals that this problem vanishes, if the finite extent of the nuclear charge distribution is taken into account. This circumvents the singularity of the pure Coulomb potential at the origin. The binding energies of electron states then do not only exceed the electron rest mass, but even reach $E_{nj} = -m$ for a so-called "critical" value $Z_{nj,cr}$. In the case of $1s$-states, $Z_{\rm cr} = 173$ [2]. For $Z > Z_{\rm cr}$, the state is paraphrased to "dive into the Dirac sea".

This diving of a bound state into the continuum is predicted to have exciting consequences: The formerly localized bound state with fixed energy becomes a resonance embedded in the continuum with a width Γ. If this resonance state is empty, it may be spontaneously filled by an electron from the negative energy continuum, thus leaving a hole there, which is emitted as a positron. The process is therefore called spontaneous positron emission. Its time scale depends on the width Γ of the resonant state which amounts to about $\tau \sim 1/\Gamma \sim 10^{-19}$ s. The energy distribution of the emitted positrons forms a Lorentzian at $E = E_{\rm bind} - 2m$. In the field of a supercritical nucleus, this particle creation does not require any energy transfer out of the nuclear motion. Taking into account the spin degeneracy, the stable ground state of the vacuum consists of two bound electrons and two positrons in the continuum - contrary to the neutral vacuum ground state $|0\rangle$ of "normal" subcritical fields. Thus in Ref. [3] the supercritical vacuum ground state $|\tilde{0}\rangle$ is designated "charged vacuum".

In addition to the strong Coulomb field radiative quantum electrodynamical processes, namely self energy and vacuum polarization, can influence the binding energy of an electron by a considerable amount. Classically, the self energy is the interaction of a charge distribution with itself. In terms of QED, it implies the emission and reabsorption of a virtual photon by a charged particle. Vacuum polarization describes the coupling of a charged particle to virtual electron-positron pairs via photon exchange.

In the past, both processes were thought to eventually prevent the spontaneous emission of positrons. Could the repulsive self energy shift hinder the diving of energy levels into the Dirac sea? Could vacuum polarization prevent the spontaneous production of positrons? To answer these questions, calculations were performed for both the vacuum polarization [2, 4, 5] and the self energy [6–8] for systems up to $Z = 170$. As in these systems $Z\alpha > 1$, it is not sufficient to employ a perturbation expansion in orders of $Z\alpha$, and a non-perturbative approach had to incorporate all orders in $Z\alpha$. For the self energy, the results are depicted in Fig. 1. Various existing calculations for the energy shifts ΔE of $1s$-electrons are indicated, which agree fairly well for most of the values. The discrepancy between Mohr's data [6] and the calculation of Soff [8] for $Z = 130$ is simply caused by the neglection of nuclear size effects in [6].

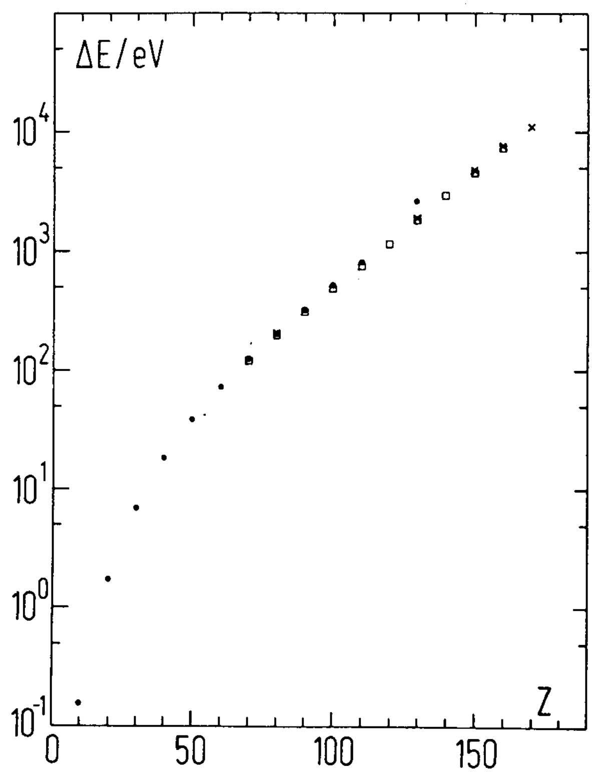

Fig. 1. The self-energy shift of K-shell electrons plotted versus the nuclear charge number Z. • represents numerical results of Mohr [6] for 1s-electrons in the field of point-like nuclei; □ shows computed values of Cheng and Johnson [7]; × represents results of Soff *et al.* [8].

The most important piece of information drawn from the picture is the self energy shift for the almost critical system $Z = 170$. The radius of this nucleus has been adjusted such that the energy of the $1s$-state differs only by 1 meV from the border line to the negative energy continuum. The calculation is therefore at the limit of subcriticalness. The obtained result for the self energy shift is $\Delta E_{\text{s.e.}} = 10.989$ keV. So, even for this nearly critical system the self energy correction amounts to only 1 % of the total binding energy. Consequently, the self energy correction does not prevent the state from diving into the negative continuum.

Similar calculations for the vacuum polarization [2, 5] yield a result of almost the same amount but of different sign: $\Delta E_{\mathrm{v.p.}} = -10.3$ keV. The sum of both radiative corrections therefore even almost cancels at the boundary to the negative energy continuum.

In conclusion we may state, that these corrections to do not hinder the spontaneous emission of positrons in the case of supercritical electromagnetic fields.

3. Spectroscopy of superheavy quasiatoms

Up to now, the charged vacuum ground state has not been observed. Nature does not provide us with stable atomic nuclei of $Z \geq Z_{\mathrm{cr}}$. Greiner *et al.* [3], therefore, proposed to utilize heavy-ion collisions for generating superheavy "quasi"atoms. Two nuclei with sufficiently large charge numbers Z_1 and Z_2 form a combined system characterized by the sum of both charges, $Z_u = Z_1 + Z_2$. Due to the strong localization of the $1s$-wave function in critical and nearly critical systems, a distance of closest approach comparable to the nuclear diameter has to be achieved during the impact.

In a quasielastic collision, the two nuclei can be treated to move on Rutherford trajectories. For these impacts, even at impact parameters b of a few fermi the time for supercriticalness is only about $2 \cdot 10^{-21}$ s, which is much shorter than 10^{-19} s corresponding to the width of the resonant state. To fill the resonant state by a Dirac sea electron it is absolutely necessary to keep both nuclei at a fixed distance for some time. The prolonged sticking time can be generated only by nuclear forces, which then necessarily cause deviations from a Rutherford trajectory. The time the nuclei stick together is called nuclear delay time. Reliable and also necessary delay times are in the order of a few times 10^{-21} s, which is still much less than 10^{-19} s. The expected positron peak is thus smeared out and shifted towards higher energies due to interference effects with other positron formation modes which also occur in heavy-ion collisions. An unambiguous identification of spontaneous positron emission is therefore not possible by the positron spectrum alone.

Further tools are required to provide sufficient evidence of the new atomic processes in heavy-ion collisions and the required sticking of both nuclei. The mutual velocity of the nuclei at a bombarding energy close to the nuclear Coulomb barrier is about $c/10$. In contrast, the electron velocity in the innermost shells under consideration is $v_{e^-} \approx c$. Therefore, quasimolecular states according to the instantaneous distance $\boldsymbol{R}(t)$ of the nuclei can be formed during the collision. As the internuclear distance changes with time, the electron states are described by the time-dependent Dirac equation

$$i\frac{\partial\psi_j(\boldsymbol{R}(t))}{\partial t} = H(\boldsymbol{R}(t))\,\psi_j(\boldsymbol{R}(t)) \tag{4}$$

for an electron state ψ_j. $H(\boldsymbol{R}(t))$ is the relativistic two-centre Hamiltonian,

$$H(\boldsymbol{R}(t)) = \boldsymbol{\alpha p} + \beta m + V(Z_1, Z_2, \boldsymbol{R}(t))\,. \tag{5}$$

As the electron velocity is much larger than $\boldsymbol{R}(t)$, it is evident to expand the total wave function $\psi_j(\boldsymbol{R}(t))$ in terms of the stationary eigenstates $\varphi_k(\boldsymbol{R})$ of the two-centre Dirac equation:

$$\psi_j(\boldsymbol{R}(t)) = \sum\!\!\!\!\!\!\int_k a_{jk}(t)\,\varphi_k(\boldsymbol{R})\,\exp(-i\int^t E_k\,dt')\,, \tag{6}$$

where the $\varphi_k(\boldsymbol{R})$ obey the stationary Dirac equation for fixed internuclear distances $\boldsymbol{R}$,

$$H(\boldsymbol{R})\,\varphi_k(\boldsymbol{R}) = E\,\varphi_k(\boldsymbol{R})\,. \tag{7}$$

Inserting Eq. (6) into (4) leads to a set of coupled first-order differential equations for the amplitudes $a_{jk}(t)$:

$$\dot{a}_{jk}(t) = \sum\!\!\!\!\!\!\int_l a_{jl}(t)\,\langle\varphi_k(\boldsymbol{R})\,|\partial/\partial t|\,\varphi_l(\boldsymbol{R})\rangle\exp\{-i\int^t (E_l - E_k)\,dt'\}\,. \tag{8}$$

Note, that $\partial/\partial t$ acts on the φ_l only via the t-dependence of $\boldsymbol{R}$ during the collision process.

The set of equations (8) is known as coupled-channel equations. For a more elaborated recent review we refer to [9]. The time derivatives of the occupation amplitudes a_{jk} describe the transitions of an electron from one state to another during the collision. As continuum states are also included in the sums in (6) and (8), Eq. (8) describes the vacancy formation in occupied shells during the collision, the spectrum of electrons emitted to the continuum and also the creation of holes in the negative energy continuum. We emphasize, that these positrons created by collision dynamics have not to be mixed up with those created by spontaneous pair production discussed above. The formation of electron-positron pairs in heavy-ion collisions takes place also for $Z < Z_{\mathrm{cr}}$, and it is essential to separate this dynamical contribution from the spontaneous positron creation to yield a proof for the predicted level diving.

Up to now we have considered one single electron. However, it is not yet possible to collide completely stripped heavy ions, and therefore the treatment in equations (4) to (8) has to be supplemented to be suitable for the actual many-electron problem. This implies the consideration of the Pauli principle, which forbids the transition to an occupied state, and the handling of electron-electron interactions, which can be accounted for within the Hartree–Fock–Slater (HFS) formalism. For a detailed survey of these extensions the reader is referred to [9] and to references therein.

Employing the formal language of second quantization, the solution of the system (8) leads to predictions for the following observables:

— number of inner shell vacancies,
— intensity and spectrum of δ-electrons,
— intensity and spectrum of created positrons.

Any of these observables depends on the chosen trajectory $\boldsymbol{R}(t)$. Therefore it is possible to deduce information of the collision process characterized by $\boldsymbol{R}(t)$ from one variable and predict the behaviour of the others. In particular, the K-shell vacancy production rate and the δ-electron spectrum may yield information about the nuclear delay time, thus serving as an atomic clock for nuclear processes, as was proposed in Ref. [10]. During the past decade, this principle was verified by numerous experiments, and with the help of experimental data it is possible to distinguish between different models for the nuclear reaction, as, *e.g.*, was verified in a recent experiment carried out by Rhein *et al.* [11].

Besides the nuclear delay time, also a variety of additional conclusions can be drawn form the observables mentioned above. Fig. 2 depicts the number of created holes in the K-shell of the heavier collision partner Cm depending on the impact parameter b. The theoretical results were evaluated by de Reus *et al.* [12], experimental data were measured by Liesen *et al.* [13] for $E_{\mathrm{lab}} = 5.9$ MeV and Ito *et al.* [14] for $E_{\mathrm{lab}} = 5.4$ MeV. The measured data are described fairly well by the HFS theory. For central encounters, about 10 % of all K-electrons are excited to high-lying states above the Fermi surface. This number exceeds the predictions of non-relativistic extrapolations by several orders of magnitude. The value measured has also an important consequence to the possible observation of spontaneous positron emission. To fill a dived resonant state by an electron from the Dirac sea, this state has to be empty, and the experimental data of Fig. 2 demonstrate, that it is possible to create a sufficient number of K-shell vacancies by the heavy-ion collision itself even if the ions are not completely stripped initially.

The binding energy of the quasiatomic shells is reflected in the ionization probability plotted in Fig. 2. The steeper the decline of $P(b)$ for a given b, the higher is the electron binding energy at the distance of closest approach

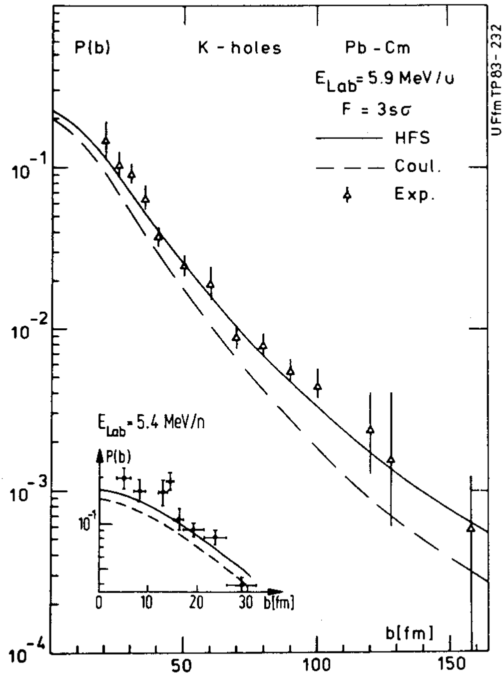

Fig. 2. Number of created $1s\sigma$-vacancies as a function of impact parameter b for the system Pb + Cm at a projectile energy of $E_{\rm lab}$ = 5.9 MeV/u [12] compared with experimental data from Liesen *et al.* [13]. Full lines: HFS calculations, dashed lines: calculations neglecting the electron-electron interaction. In the inset theoretical results [12] are compared with experimental data of Ito *et al.* [14] for $1s\sigma$-vacancy formation in 5.4 MeV/u Pb+Cm collisions. The theoretical values were obtained assuming the Fermi surface at the molecular $3s\sigma$-state.

R_0. Therefore it is possible to deduce the actual binding energy of the quasiatomic K-state from plots like Fig. 2. Details of this calculation are presented by Greiner *et al.* [3] and by Soff *et al.* [15].

Fig. 3 illustrates theoretical and experimental binding energies for different systems and distances of closest approach. Not only an excellent agreement between experiment and theory can be observed, but also an enormous $1s$-binding energy, which for the system Pb + Cm exceeds the electron rest mass.

In Fig. 4 [15], for head-on collisions the calculated number of $1s\sigma$-vacancies is plotted versus the combined nuclear charge number $Z_1 + Z_2$. This specific calculation is based on first-order time-dependent perturbation theory. The various curves display this quantity for different values of R_0 and thus for different impact energies. The most important feature is the decrease in vacancy formation for high $(Z_1 + Z_2)$ after reaching a maximum at

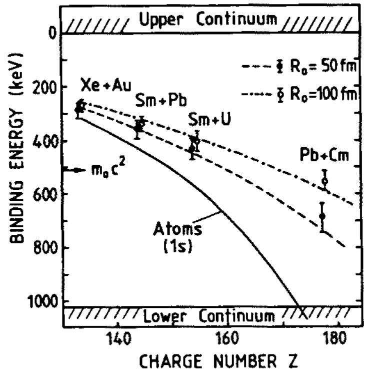

Fig. 3. Experimentally determined binding energies in different quasiatomic systems created in heavy-ion collisions for two distances of closest approach R_0. The lines indicate the corresponding theoretical values.

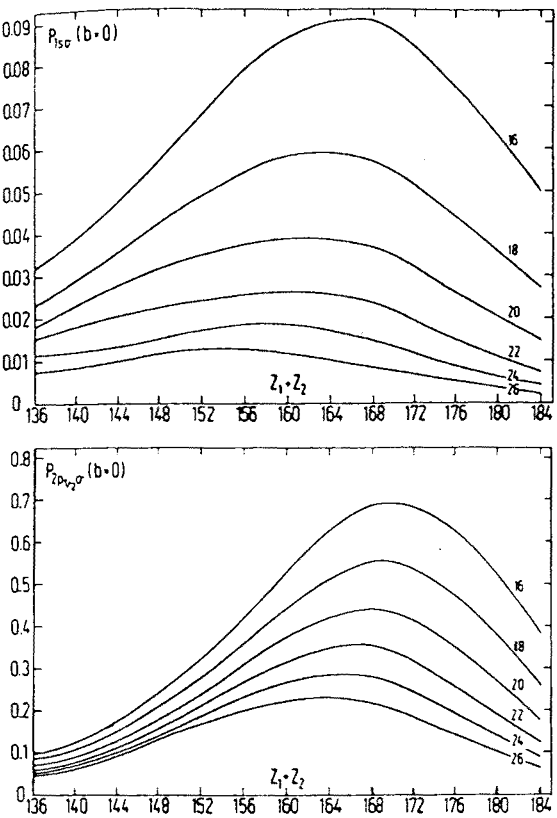

Fig. 4. Number of $1s\sigma$-vacancies (top) and $2p_{1/2}\sigma$-vacancies (bottom) per collision for head-on collisions as function of the total nuclear charge number $Z_1 + Z_2$ for different distances R_0 of closest approach given in units of fermi.

$Z_1 + Z_2 \approx Z_{cr}$. This behaviour is explained by the increasing localization of the electron wave function and also by the increase in binding energy. The wave function becomes more and more localized for $Z < 160$. This corresponds to higher Fourier frequencies in the wave function. Therefore, higher momenta can be transferred during the collision and the creation of vacancies is enlarged. For even larger Z, this effect is compensated by the rather strong increase of the binding energy. Fig. 4 was calculated using time-dependent perturbation theory. Therefore, only qualitative conclusions should be drawn from the plot. A more elaborated calculation based on coupled-channel calculations is presented in Fig. 5, where the δ-electron

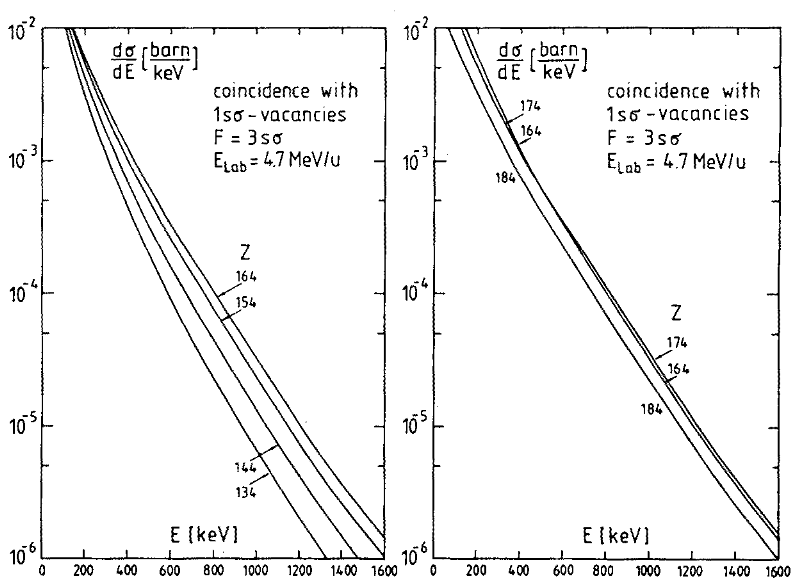

Fig. 5. Expected δ-electron distribution for coincidence measurements with $1s\sigma$-vacancies and a Fermi surface at $3s\sigma$. The curves correspond to different values of $Z_1 + Z_2$.

yield is shown for different values of Z. Again, a maximum is observed at $Z \approx Z_{cr}$, whereas the rate decreases again for $Z = 184$. This calculation also includes accidental coincidences and should therefore provide the correct experimental values. It represents a challenge to experimentalists to measure spectra like those in Fig. 5 and thus confirm the large increase in the binding energy for $Z > Z_{cr}$, which would be a strong evidence for the formation of a resonant state in the negative energy continuum.

4. Creation of positrons in heavy-ion collisions

The fundamental process we are looking for is the spontaneous creation of a positron by filling a vacancy in the K-shell of a supercritical atom by an electron from the Dirac sea. But positrons are also created by other processes, as was lined out above. When a heavy-ion collision takes place, according to Eq. (8) also electrons from states of the negative energy continuum can be excited to bound states or to free continuum states. Both mechanisms lead to the formation of a positron, called induced or dynamical positron creation. The total cross section for this type of formation is known to scale as

$$\sigma_{\text{dyn}} \sim (Z_1 + Z_2)^{20} , \tag{9}$$

which predicts a rather strong increase in the dynamical positron production for nearly critical and supercritical systems. In the past, experiments did not only verify this overall dependence, but also confirmed theoretical predictions of kinematical dependences of the positron yield. Therefore, the dynamical production of positrons is thought to be well understood.

An additional serious problem is the creation of positrons by nuclear processes. Any intermediate photon with an energy larger than 1022 keV can undergo pair conversion in the field of a nucleus. Electron-positron pairs originating from Coulomb excitation of nuclei in heavy-ion collisions contribute significantly to the total positron yield.

Here the electromagnetic de-excitation of the nuclei results in the emission of virtual photons which convert subsequently in electron-positron pairs. This process is known as internal pair conversion (IPC).

Heavy-ion collisions at energies in the vicinity of the nuclear Coulomb barrier lead to an alignment of colliding nuclei, *i.e.* to a non-uniform occupation of nuclear substates [16, 17]. The occupation probabilities of magnetic substates, *e.g.*, can be computed within the help of the COULEX code of Alder and Winther [18]. However, one should take into account the change of population by electromagnetic transitions from higher lying nuclear states. Similar to the emission of γ-rays [19], this results for internal pair conversion in an anisotropic spatial distribution of electron-positron pairs [20–22]. We calculated the pair emission probability in the case of internal pair conversion for nuclear transitions of electric (E) or magnetic (M) type and angular momenta $L > 0$ taking into account the nuclear charge number Z and the finite extension of the nucleus [23].

As is well-known from the investigation of the positron spectra calculated in Refs. [24–26], Born approximation cannot be applied to the internal pair conversion of very heavy nuclei. This can be illustrated, *e.g.*, by the opening angle distribution of the electron-positron pairs. For an E1 transition of an uranium-like nucleus this distribution exhibits the typical pattern expected for internal pair conversion (Fig. 6): The angular distribution decreases from its maximum at $\Theta = 0°$ to $180°$. The deviation from the Born approximation (B.A.) demonstrates the influence of the strong Coulomb field of the nuclear charge on the pair emission rate and the angular correlation.

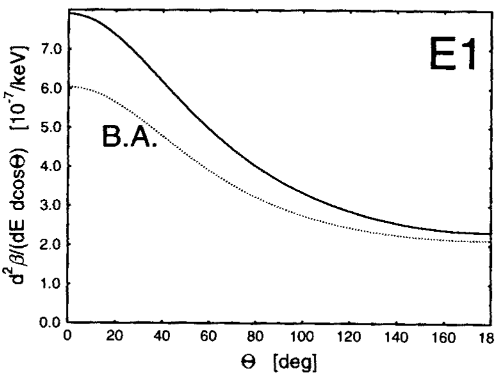

Fig. 6. Opening-angle distribution of electron-positron pairs emitted by an E1 transition of a set of randomly oriented uranium-like nuclei. The nuclear transition energy amounts to 2 MeV, the kinetic positron energy to $E = 800$ keV.

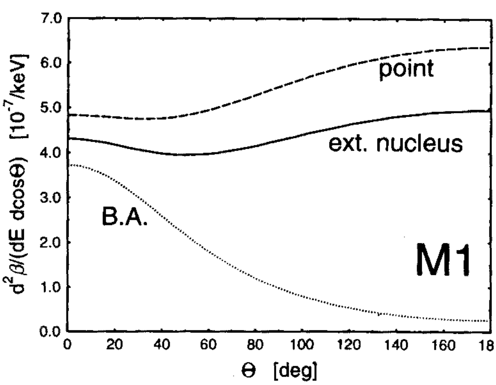

Fig. 7. The same as in Fig. 6 assuming a M1 transition of a set of randomly oriented uranium-like nuclei. In addition to the Born approximation (dotted line) and to the point-nucleus approximation (dashed line) we display the result for a finite nuclear extension (full line). One can recognize that the maximum of the angular distribution is shifted from $0°$ towards $180°$ for high nuclear charge numbers.

For magnetic transitions in heavy nuclei, the situation looks different. In Fig. 7 we plotted the opening-angle distribution for a M1 transition assuming a point-like nucleus as well as an extended nucleus. Here the angular distribution displays a maximum of the pair emission yield for a back-to-back emission of the electron and positron in contrast to the Born approximation (B.A.). Fig. 7 verifies also that the magnetic transitions are very sensitive to the finite extent of the nucleus, which was already stated in [25] for the positron spectra. For the E1 transition in Fig. 6, on the other hand, the effect of the finite nuclear size is negligible.

Internal pair conversion reveals thus a plethora of signatures depending on the charge number Z of the nuclei under consideration. For high nuclear charges the spectra as well as the angular correlations deviate drastically from what is expected from the Born approximation. This will allow to study properties of excited nuclei. For magnetic transitions of high-Z nuclei one has also to account for the finite-size effects. Especially M1 transitions are very sensitive on the nuclear charge distribution. The spectroscopy of Coulomb excited heavy nuclei is thus an interesting topic by itself.

As we have demonstrated, all positron formation processes competing with the spontaneous creation can be handled theoretically. For the unique identification of the spontaneous positron emission, we point to a calculation of Graf *et al.* [27]. They calculated the emission probability $N_{\boldsymbol{p}_{e^+}\boldsymbol{p}_{e^-}}$ of coincident electron-positron pairs with Energies E_{e^+} and E_{e^-} and directions $\boldsymbol{p}_{e^+}/|\boldsymbol{p}_{e^+}|$ and $\boldsymbol{p}_{e^-}/|\boldsymbol{p}_{e^-}|$. Considering only the monopole approximation of the nuclear charge distribution and restricting the calculation to the dominant angular momentum channels $\kappa = \pm 1$, they found

$$N_{\boldsymbol{p}_{e^+}\boldsymbol{p}_{e^-}} = a^0_{E_{e^+},E_{e^-}} \left(1 + a^1_{E_{e^+},E_{e^-}} \cos \Theta_{e^+ e^-}\right) . \tag{10}$$

Here $\Theta_{e^+ e^-}$ denotes the angle between the directions of electron and positron and $a^1_{E_{e^+},E_{e^-}}$ is the anisotropy coefficient. Fig. 8 depicts the anisotropy coefficient as a function of the nuclear delay time for the fully stripped systems Pb + Pb and U + U. For the subcritical system Pb + Pb, a^1 always remains positive, due to similar contributions from the $\kappa = +1$ channel and the $\kappa = -1$ channel. In supercritical systems, the spontaneous creation of positrons allows for a negative coefficient a^1 for increasing nuclear delay times. With a corresponding measurement it should be possible to proof the spontaneous positron creation without any doubt.

Unfortunately, there are still some major experimental difficulties to this approach. First of all, in collisions of nuclei surrounded by electrons, the large amount of statistical coincidences will dominate angular correlations. One therefore has to study collisions of bare nuclei. Secondly, the amount of leptons created by IPC may also cause a problem. Experimentally, IPC is found to be of minor importance compared to quasimolecular pair production in supercritical quasielastical collisions [9]. However, both processes contribute with the same amount in deep inelastic collisions. Graf

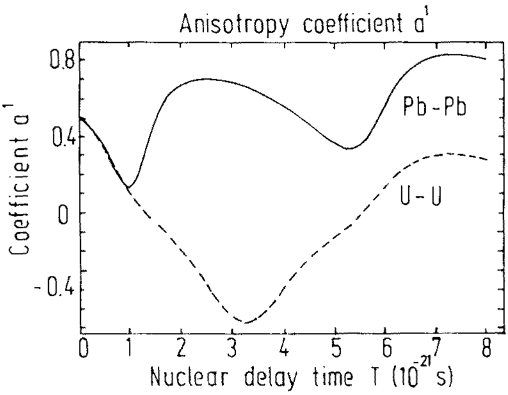

Fig. 8. Anisotropy coefficient a^1 defined by Eq. (10) as a function of nuclear delay time T for head-on collisions of Pb + Pb and U + U at a projectile energy of $E_{\text{lab}} = 6.2$ MeV/u [27]. While in the subcritical system Pb + Pb a^1 always remains positive, a^1 changes its sign in U + U for $2 \times 10^{21}\,\text{s} \lesssim T \lesssim 5 \times 10^{21}\,\text{s}$. The emitted leptons were integrated within an energy range of 102 keV $\leq E_{e^+}, E_{e^-} \leq$ 588 keV.

et al. therefore proposed collisions of bare ions close to the nuclear Coulomb barrier with a nuclear delay time of about $T \sim 2 - 3 \times 10^{-21}$ s to obtain a reliable signal.

5. A possible route to identify spontaneous positron creation

In the preceding sections it was indicated, that all processes competing with spontaneous positron emission can be well treated theoretically. Even more, one can line out a path to look for suitable nuclear collision systems, which may lead to the identification of spontaneous positron creation.

1. A nuclear delay time of $T \geq 2 \cdot 10^{-21}$ s is necessarily required in a heavy-ion collision. To determine nuclear systems with sufficient delay time, nuclear reactions have to be studied with optimal resolution in all variables, namely mass number A, charge number Z and Q-value. δ-electron spectra and inner-shell vacancies can point to nuclear delay times and thus identify adequate systems for further investigations.

2. δ-electron spectra and K-shell vacancy production yields should be measured as a function of Z, b and Q. The emission rates of the δ-electrons must be found to decrease for $Z \gtrsim Z_{\text{cr}}$, thus verifying the predicted increase in the binding energy.

3. Provided the strong binding energies are confirmed by experiment, an appropriate system with sufficient nuclear delay times can be utilized in a crossed beam collision of bare heavy ions to obtain an angular correlation signal of electron-positron pairs, which can be explained by spontaneous positron emission only.

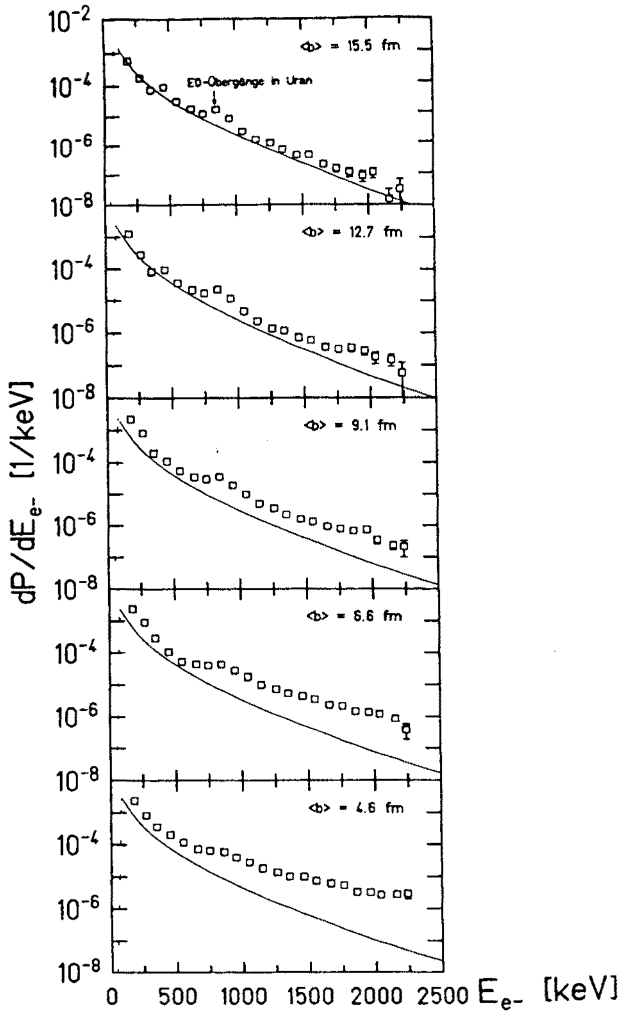

Fig. 9. δ-electron spectra for the collision system U + Pd at $E_{\text{lab}} = 6.1$ MeV/u. Five impact parameter intervals with mean values $\langle b \rangle$ were investigated by Mojumder [28] and Stroth [30]. The lines signify results of coupled-channel calculations [29] which were obtained under the assumption of pure Rutherford scattering for all impact parameters.

Up to here, the route of further investigations seems to be clear. It should be mentioned, however, that striking discrepancies exist for a few collision systems between theoretical predictions and experimental results. As an example, in Fig. 9 we present the δ-electron spectrum for the system U + Pd at $E_{\mathrm{lab}} = 6.1$ MeV [28]. Mojumder and coworkers measured δ-electron spectra for five impact parameter intervals. The comparison with coupled channel results of St. Graf *et al.* [29] is depicted. For large impact parameters the coupled channel results reproduce the measured δ-electron spectra quite well. The structure at 870 keV can be explained by Coulomb excited β-states, which decay to the ground state band in Uranium. As the transition is of multipolarity E0, it is not visible in the γ-spectrum and, therefore, no correction by anticoincidence is possible.

Still, a total emission probability of about 40 % is missing in the coupled channel results. The discrepancy increases tremendously, if smaller impact parameters are investigated. For δ-electron energies higher than 2 MeV and mean impact parameters $\langle b \rangle = 4.6$ fm, theory underestimates the experimental data by two orders of magnitude. Possible additional nuclear sources such as internal conversion or further E0 transitions were considered to account for the difference [30], but all attempts failed to reproduce the experimental values.

One can speculate, that deviations from pure Rutherford scattering occurs even for larger impact parameters. For the similar system Pb + Pd, longer nuclear delay times were found. From the discussion above, this can alter the δ-electron spectrum considerably. But neither a theoretical nor an experimental explanation could be given for the U+Pd collision system, and the indicated discrepancy up to now remains an unsolved puzzle.

6. Conclusions

Characteristic features of almost critical and supercritical electromagnetic fields were discussed, and the quantum electrodynamical corrections for nearly critical fields were evaluated. It was shown, that these corrections amount only to a few per cent of the electron binding energies and therefore do not hinder the atomic states from diving into the Dirac sea for $Z > Z_{\mathrm{cr}}$ and forming resonant states within the negative energy continuum. As major evidence for these resonances, spontaneous emission of positrons should occur.

Supercritical nuclear systems can be generated in heavy-ion collisions. A consistent theoretical framework exists, which allows to predict almost all observable features of atomic physics in heavy-ion collisions. One impressing result of the theory is the understanding of the dynamical positron production which is outstanding by a cross section depending on $(Z_1 + Z_2)^{20}$.

Another very useful tool is the use of δ-electron spectra, which allow to determine not only electron binding energy features of a quasiatomic system but can also serve as an atomic clock in the search and control for systems with nuclear delay times. It has to be mentioned, however, that for a few systems striking discrepancies exist between theoretical prediction and experimental result for the δ-electron emission, which require further investigations.

Nevertheless, in general it is possible to provide a detailed description of the atomic physics in heavy-ion collisions at the nuclear Coulomb barrier. Thus with the help of δ-electron spectra, it is furthermore possible to search systematically for suitable nuclear collision candidates, which might exhibit spontaneous positron emission and therefore a phase transition to a new vacuum ground state.

This work has been supported by the BMBF, by the Deutsche Forschungsgemeinschaft (DFG), and by GSI (Darmstadt).

REFERENCES

[1] W. Pieper, W. Greiner, *Z. Physik* **218**, 126 (1969).
[2] G. Soff, B. Müller, J. Rafelski, *Z. Naturforsch.* **29a**, 1267 (1974).
[3] W. Greiner, B. Müller, J. Rafelski, *Quantum Electrodynamics of Strong Fields*, Springer, Berlin 1985.
[4] M. Gyulassy, *Nucl. Phys.* **A244**, 497 (1975).
[5] G. Soff, P. J. Mohr, *Phys. Rev.* **A38**, 5066 (1988).
[6] P. J. Mohr, *Ann. Phys.* (N.Y.) **88**, 26 and 52 (1974).
[7] K. T. Cheng, W. R. Johnson, *Phys. Rev.* **A14**, 1943 (1976).
[8] G. Soff, P. Schlüter, B. Müller, W. Greiner, *Phys. Rev. Lett.* **48**, 1465 (1982).
[9] U. Müller-Nehler, G. Soff, *Phys. Rep.* **246**, 101 (1994).
[10] G. Soff, J. Reinhardt, B. Müller, W. Greiner, *Phys. Rev. Lett.* **43**, 1981 (1979).
[11] M. Rhein, R. Barth, E. Ditzel, H. Feldmeier, E. Kankeleit, V. Lips, C. Müntz, W. Nörenberg, H. Oeschler, A. Piechaczek, W. Polai, I. Schall, *Phys. Rev. Lett.* **69**, 1340 (1992).
[12] T. H. J. de Reus, J. Reinhardt, B. Müller, W. Greiner, G. Soff, U. Müller, *J. Phys.* **B17**, 615 (1984).
[13] D. Liesen, P. Armbruster, H. H. Behncke, F. Bosch, S. Hagmann, P. H. Mokler, H. Schmidt-Böcking, R. Schuch, in *Electronic and Atomic Collisions*, edited by N. Oda and K. Takayanagi, North-Holland, Amsterdam 1980, p. 337.
[14] S. Ito, P. Armbruster, H. Bokemeyer, F. Bosch, H. Emling, H. Folger, E. Grosse, R. Kulessa, D. Liesen, D. Maor, D. Schwalm, J. S. Greenberg, R. Schulé, N. Trautmann, GSI Scientific Report 1981, GSI-82-1, 141 (1982).
[15] G. Soff, B. Müller, W. Greiner, *Phys. Rev. Lett.* **40**, 540 (1978).

[16] K. Alder, A. Winther, *Electromagnetic Excitation*, North-Holland, Amsterdam 1975.

[17] R. A. Broglia, A. Winther, *Heavy Ion Reactions*, Addison Wesley, Redwood City, CA, 1991.

[18] *Coulomb Excitation*, edited by K. Alder and A. Winther, Academic Press, New York 1966.

[19] H.-J. Wollersheim, GSI-Report, GSI-93-22, (1993).

[20] G. Goldring, *Proc. Phys. Soc.* **A66**, 341 (1953).

[21] M. E. Rose, *Phys. Rev.* **131**, 1260 (1963).

[22] E. K. Warburton, *Phys. Rev.* **133**, B1368 (1964).

[23] C. Hofmann, *Elektron-Positron-Winkelkorrelation*, Harri Deutsch, Frankfurt am Main 1995.

[24] P. Schlüter, G. Soff, W. Greiner, *Z. Phys.* **A286**, 149 (1978).

[25] P. Schlüter, G. Soff, W. Greiner, *Phys. Rep.* **75**, 327 (1981).

[26] G. Soff, P. Schlüter, W. Greiner, Z. Phys. A **303**, 189 (1981).

[27] O. Graf, J. Reinhardt, B. Müller, W. Greiner, G. Soff, *Phys. Rev. Lett.* **61**, 2831 (1988).

[28] S. Mojumder, GSI Report, GSI-90-12 (1990).

[29] St. Graf, J. Reinhardt, U. Müller, T. H. J. de Reus, B. Müller, W. Greiner, G. Soff, Z. Physik **A329**, 365 (1988).

[30] J. Stroth, GSI Report, GSI-90-28 (1990).

[31] G. Soff, W. Greiner, W. Betz, B. Müller, *Phys. Rev.* **A20**, 169 (1979).