ON THE QUALITY OF MICROSCOPIC DESCRIPTIONS OF NUCLEAR MASS*

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The quality of the description of nuclear masses by various microscopic approaches is studied. Hartree–Fock–Bogolubov, Extended Thomas–Fermi model with Strutinski Integral, Relativistic Mean Field and Macroscopic-Microscopic approaches are considered. Spherical even-even nuclei (116 nuclides) from light ($A = 16$) to heavy ($A = 220$) ones with known experimental mass are chosen for the study.

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1. Introduction

There is a continuing increase of number of nuclei far from stability, for which the mass has been measured [1]. A fast progress in the development of techniques of radioactive beams gives good perspectives for the continuation of this process. There is also an impressive increase of the accuracy of measurements of nuclear mass. For example, the use of the Penning trap leads to the accuracy of about 10 keV [2]. All these challenge the theory.

The objective of the present paper is to give an illustration how well nuclear mass can be reproduced by the present-day microscopic calculations. It is also aimed to show, how much various approaches to the mass differ from one another when one goes far from the β-stability line.

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The self-consistent Hartree–Fock–Bogolubov (HFB) approach is mostly used for the illustration. Three variants of the zero-range (Skyrme) effective interactions, which have been most frequently exploited in the literature, and the finite-range (Gogny) force, are taken.

The results obtained within the Extended Thomas–Fermi model with Strutinski Integral (ETFSI), Relativistic Mean Field (RMF) and Macroscopic-Microscopic (MM) approaches are also discussed. The illustration is limited to the case of spherical nuclei. Even-even nuclei (116 nuclides) with mass number $A \geq 16$ are considered.

The approaches used in the calculations are shortly described in Sect. 2. The results of the calculations and their discussion are presented in Sect. 3.

2. Description of the calculations

Three standard Skyrme forces are used in our calculations: SIII [3], SkM* [4] and SkP [5]. These forces are taken for the calculations in the particle-hole (p-h) channel, *i.e.* for the generation of the mean field. For each force, however, three different interactions are used for the calculations in the particle-particle (p-p) channel [6] , *i.e.* for the generation of the pairing correlations. The first interaction is just the same as that used in the p-h channel. The second is the contact δ force

$$V^{\delta}(\boldsymbol{r},\boldsymbol{r}') = V_0\delta(\boldsymbol{r}-\boldsymbol{r}') \tag{1}$$

and the third is the δ force which depends on nuclear density

$$V^{\delta\rho}(\boldsymbol{r},\boldsymbol{r}') = (V_0 + \tfrac{1}{6}V_3\rho^{\gamma})\delta(\boldsymbol{r}-\boldsymbol{r}')\,. \tag{2}$$

Thus, for each standard force, three effective interactions are finally used. They are denoted [6], *e.g.* in the case of the SIII force, by SIII, SIII$^{\delta}$ and SIII$^{\delta\rho}$, respectively.

A motivation for introducing the interactions (1) and (2) in the p-p channel is that only the SkP force is chosen in such a way as to give reasonable pairing correlations. The SIII and SkM* forces are repulsive in the p-p channel and lead to a vanishing or very weak pairing.

The parameters of the interactions (1) and (2) are taken the same as in [6], where they were adjusted to reproduce the experimental neutron pairing gap for the nucleus ^{119}Sn. The HFB equations are solved in the way described in [5].

The finite-range Gogny force (similarly as the zero-range SkP force) has been specially devised to describe the pairing properties simultaneously with the mean field. With this force, one avoids divergencies in the pairing calculations, in contrast with the zero-range forces, for which the energy cut-off is necessary and plays the role of an additional parameter. The interaction D1S [7, 8] has been used in the present study. The HFB equations are solved in the way described in the paper [7].

Nuclear masses calculated by the ETFSI model have been obtained in [9].

The RMF approach (*e.g.* [10, 11]) is used by us with three variants of parameters: NL1 [12], NL2 [13], and NLSH [14]. Three variants of the Macroscopic-Microscopic model are used for the present comparisons: the Finite-Range Droplet Model (FRDM) [15], the Finite-Range Liquid-Drop Model (FRLDM) [15] and the Thomas-Fermi model (TF) [16]. The respective values of nuclear mass are simply taken from the mass tables given in [15, 16].

3. Results and discussion

Table I presents the rms deviations of the calculated masses from experimental values. The HFB and RMF results are obtained in the present study, the ETFSI masses are taken from [9], two variants (FRDM and FRLDM) of the MM masses from [15], and the TF variant from [16]. The experimental values are taken from [17].

TABLE I

Mass rms deviations in MeV

SIII:	4.74	SkP:	2.37	SkM*:	6.32
SIII^{δ}:	3.07	SkP^{δ}:	2.53	$\mathrm{SkM}^{*\delta}$:	5.36
$\mathrm{SIII}^{\delta\rho}$:	2.26	$\mathrm{SkP}^{\delta\rho}$:	2.32	$\mathrm{SkM}^{*\delta\rho}$:	4.74
Gogny:	2.07	ETFSI:	0.80		
RMF(NL1):	3.94	RMF(NL2):	11.24	RMF(NLSH):	3.40
MM(FRDM):	0.65	MM(FRLDM):	0.76	MM(TF):	0.57

One can see that the smallest deviations from experimental masses are obtained in the MM approach. Within the HFB approximation the best results are obtained with the SkP force. One could expect this, as the parameters of this force have been chosen so as to describe possibly well the residual pairing interactions, which are important in the description of nuclear mass. Variants SkP^{δ} and $\mathrm{SkP}^{\delta\rho}$ are of about the same quality as the variant SkP. This differs from the case of the SIII (and also SkM*) force, for which the variant $\mathrm{SIII}^{\delta\rho}$ is much better than SIII^{δ}. The results obtained with the Gogny force are rather good, while those of RMF are poor, especially for the NL2 variant of the parameters.

Dependence of the mass deviations on the neutron number N, calculated for isotopes of lead, is illustrated in Fig. 1. Six variants of the calculated values are chosen for the illustration. One can see that the largest deviations are obtained for the RMF approach, although the variant RMF (NLSH), which gives the smallest rms in Table I, is taken here. The deviations obtained in the SIII$^\delta$ case are also large. They fast increase with the increase of the distance, in the neutron number N, from the magic value N=126. The small deviation of mass for ^{208}Pb comes from the fact that ^{208}Pb is one of the nuclei to the properties of which the parameters of the SIII force have been adjusted [3]. The mass deviation obtained for the Gogny force is also fast increasing with N, beyond $N = 128$.

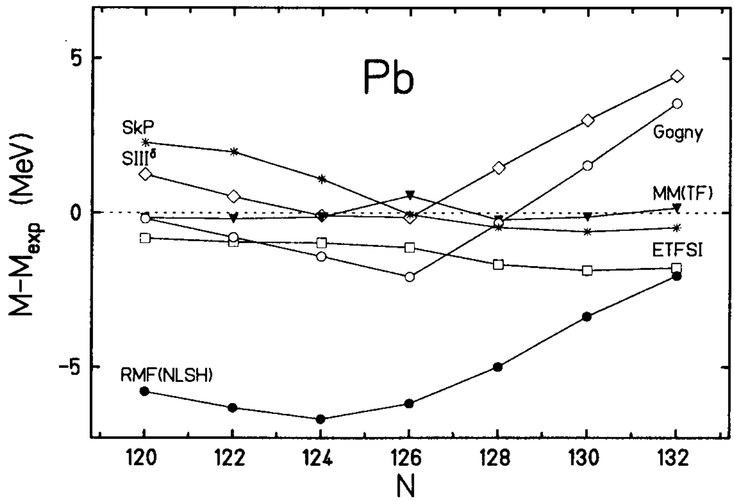

Fig. 1. The isotopic dependence of the deviation of theoretical mass from experimental values, calculated for lead. Six variants of the theoretical mass are considered.

Predictions of the considered mass models, when N becomes even more distant from the β-stability line, are shown in Fig. 2. As the region of N exceeds here the region for which the experimental values of mass are known, the calculated masses M_{SkP}, instead of the experimental ones M_{exp}, are taken as a reference. The reason for taking the mass SkP as a reference is that the SkP approach describes relatively well the experimental masses, and also that these values are calculated in the present analysis and are therefore available for all studied nuclei.

One can see that both MM variants considered and the ETFSI variant, which (together with MM (FRLDM)) have the smallest rms in Table 1, have similar behavior as functions of N. They also have the smallest deviations from the SkP values. "Asymptotically", for lowest and highest N, the ETFSI masses are closest to those of SkP. In distinction to that, the Gogny (and also RMF) masses have a very different "asymptotics" from

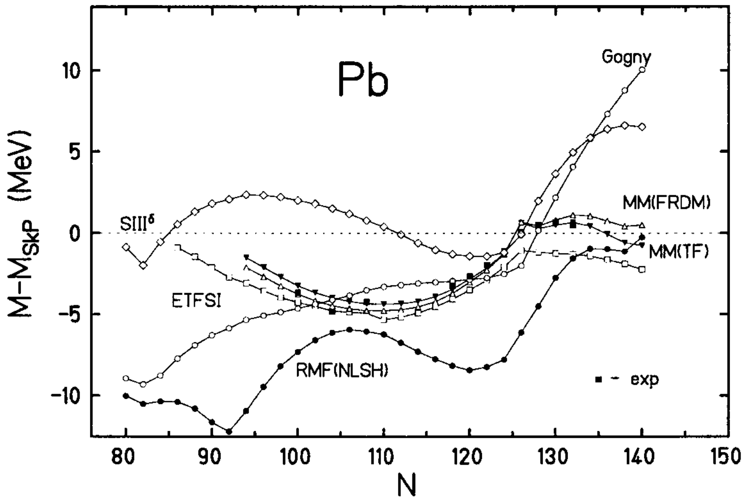

Fig. 2. Deviations of various theoretical masses from the SkP mass, calculated for a long chain of lead isotopes.

that of the SkP masses. For the lowest N considered in Fig. 2, the difference between the Gogny and SkP masses is about 9 MeV, and for the highest, about 10 MeV.

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