MICROSCOPIC BOHR HAMILTONIAN CALCULATIONS FOR Ba ISOTOPES*,**

L. PRÓCHNIAK AND K. ZAJĄC

Institute of Physics UMCS
pl. M. Curie-Skłodowskiej 1, 20-031 Lublin, Poland

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Low-lying energy levels and electromagnetic transitions in Ba isotopes were obtained using the new method of solving the general Bohr Hamiltonian. Inertial functions and collective potential were calculated within the cranking approximation with the projected BCS formalism and Seo parameters for the Nilsson single-particle potential.

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For many years the Bohr–Mottelson model remains still valuable and useful in investigations of the nuclear collective motion. Within this model nuclear vibrations and rotations are described in terms of intrinsic quadrupole variables β, γ and the Euler angles Ω. The collective Bohr Hamiltonian can be written (cf. [2, 6]) in the form:

$$H = T_{\rm vib} + T_{\rm rot} + V\,, \tag{1}$$

$$\begin{aligned} T_{\rm vib} =& -\frac{1}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\partial_\beta\left(\beta^4\sqrt{\frac{r}{w}}B_{\gamma\gamma}\partial_\beta\right) - \partial_\beta\left(\beta^3\sqrt{\frac{r}{w}}B_{\beta\gamma}\partial_\gamma\right)\right]\right. \\ &\left. + \frac{1}{\beta\sin 3\gamma}\left[-\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\partial_\beta\right) + \frac{1}{\beta}\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\right)\partial_\gamma\right]\right\}, \\ T_{\rm rot} =& \frac{1}{2}\sum_{k=1}^{3} I_k^2/J_k\,, \\ V =& V(\beta,\gamma)\,. \end{aligned} \tag{2}$$

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The mass parameters are denoted as $B_{\beta\beta}$, $B_{\beta\gamma}$ and $B_{\gamma\gamma}$ while the principal moments of inertia are expressed as $J_k = 4B_k(\beta,\gamma)\beta^2\sin^2(\gamma - 2\pi k/3)$, $k = 1,2,3$ and

$$w = B_{\beta\beta}B_{\gamma\gamma} - B_{\beta\gamma}^2\,,$$
$$r = B_1B_2B_3\,. \tag{3}$$

The eigenvalue problem of Bohr Hamiltonian has been solved in the two main ways: the first is based on direct numerical solution of a system of partial differential equations resulting from eigenequation for Hamiltonian (1) while the second one involves calculation of the matrix elements of (1) in properly chosen basis and then diagonalization of the resulted matrix. The construction of such a basis is not a straightforward matter. For example, the problem of building of the eigenstates of five dimensional harmonic oscillator as functions of the intrinsic variables β, γ, Ω was not solved (for arbitrary value of angular momentum) until the half of the seventies. It must be stressed that basis states do not have be the eigenvectors of a known in advance Hermitian operator. One example of an application of such states is an approach proposed and developed by Kumar [3]. Another example, having greater significance for our considerations is a paper of Libert and Quentin [4]. The main idea of the method used here can be summarized as follows: one starts from a set of rather simple functions of variables β, γ, Ω being a basis of an appropriate Hilbert space and then imposes restrictions coming from various properties of the intrinsic variables and Hamiltonian (1). As an initial set of functions we take

$$e^{-Q/2}\beta^n\begin{Bmatrix}\cos m\gamma\\ \sin m\gamma\end{Bmatrix}D^I_{ML}(\Omega),\ n = 0,1,\ldots;\ m = n, n-2,\ldots,0 \text{ or } 1\,, \tag{4}$$

where

$$Q = \mu\beta^2\,. \tag{5}$$

The more general form $Q = \mu\beta^2 + \mu_3\beta^3\cos 3\gamma + \mu_4\beta^4$ with free parameters μ, μ_3, μ_4 can also be considered.

Functions (4) form a basis (nonorthogonal) in a space of square integrable functions of variables β, γ, Ω with respect to the large class of measures.

Imposing necessary symmetry conditions (as for example invariance under R_1, R_2 and R_3 transformations) and requirement of the proper behaviour on $\gamma = n\pi/3$ axes we finally obtain subset of linear combinations of (4) which can be used in calculations of matrix elements of Hamiltonian (1). Let us mention also about two advantages of the method described above. Firstly, we obtained analytic, compact formulas for the basis (nonorthogonal) functions with no use of numerical approximations. Secondly, these

formulas hold with no limits on n (the meaning of n is close to the principal number of harmonic oscillator) and with no limits on values of angular momentum I.

We plan to apply the method of a diagonalization of the Bohr Hamiltonian in systematic investigations of a wide range of nuclei. The collective potential, mass parameters $B_{qq'}$ ($q, q' = \beta$ or γ) and moments of inertia J_k appearing in (2) are obtained microscopically using the cranking approximation with the nuclear Hamiltonian containing a single particle potential and pairing forces. We want to study in particular the influence of a choice of a single particle potential and corrections to the standard BCS techniqe on the calculated nuclear properties.

The general features of the method applied here are known very well [5, 6]. Cranking formulas describing inertial functions after inclusion of the pairing interaction within the BCS formalism can be written [5] in the following form:

$$J_k = 2\hbar^2 \sum_{\nu,\nu'} \frac{|<\nu|j_k|\nu'>|^2}{E_\nu + E_{\nu'}} (u_\nu v_{\nu'} - u_{\nu'} v_\nu)^2 , \tag{6}$$

$$B_{qq'} = \frac{\hbar^2}{f(q)f(q')} \{ 2 \sum_{\nu,\nu'} \frac{<\nu|\frac{\partial H}{\partial q}|\nu'><\nu'|\frac{\partial H}{\partial q'}|\nu>}{(E_\nu + E_{\nu'})^3} (u_\nu v_{\nu'} + u_{\nu'} v_\nu)^2 + \frac{1}{4} \sum_\nu \frac{1}{E_\nu^5} [\Lambda_\nu^q \Lambda_\nu^{q'} - \Delta(\Lambda_\nu^q <\nu|\frac{\partial H}{\partial q'}|\nu> + \Lambda_\nu^{q'} <\nu|\frac{\partial H}{\partial q}|\nu>)] \} , \tag{7}$$

where $f(\beta) = 1$, $f(\gamma) = \beta$ and

$$\Lambda_\nu^q = \Delta \frac{\partial \lambda}{\partial q} . \tag{8}$$

In the above equations H is a single particle Hamiltonian with the eigenvalues e_ν and eigenstates $|\nu>$ while u_ν and v_ν are the BCS variational parameters, $E_\nu = \sqrt{(e_\nu - \lambda)^2 + \Delta^2}$ is the quasiparticle energy while λ and Δ are the Fermi energy and the pairing energy-gap parameter, respectively.

As the first area of our calculations we have considered barium isotopes, which had already been studied in the frame of the Bohr Hamiltonian [5, 6]. The general approach presented here is similar to that from [5, 6], however we would like to stress two differences. (I) The single-particle Hamiltonian part: we applied the Nilsson potential containing the shell-dependent correctional terms V_{ls} and V_{l^2} with Seo [7] parameters (regarded as the best fit for the Nilsson potential for all nuclear regions). This parametrization of

the Nilsson potential has given a remarkable improvement of the description of nuclear ground states. (II) The pairing interaction part: we adopted the approximate projection of BCS wave function on the particle number [8, 9]. This procedure is based on the Generator Coordinate Method (GCM) and the gaussian overlap approximation (GOA) but its results are easy to describe in terms of standard BCS formalism. The BCS energy and, as it follows, the total nuclear energy has to be only corrected by subtracting the zero-point term when the GCM method is used [8]. It was shown [9] that when the projection on the given particle number is taken into account the pairing strength parameter should be also corrected in order to reproduce experimental energy gaps. Our estimation:

$$G = \begin{cases} g_0/Z^{2/3} & \text{for protons} \\ g_0/(A-Z)^{2/3} & \text{for neutrons} \end{cases}, \tag{9}$$

where $g_0 = 0.285\,\hbar\omega_0$ seems to be quite appropriate.

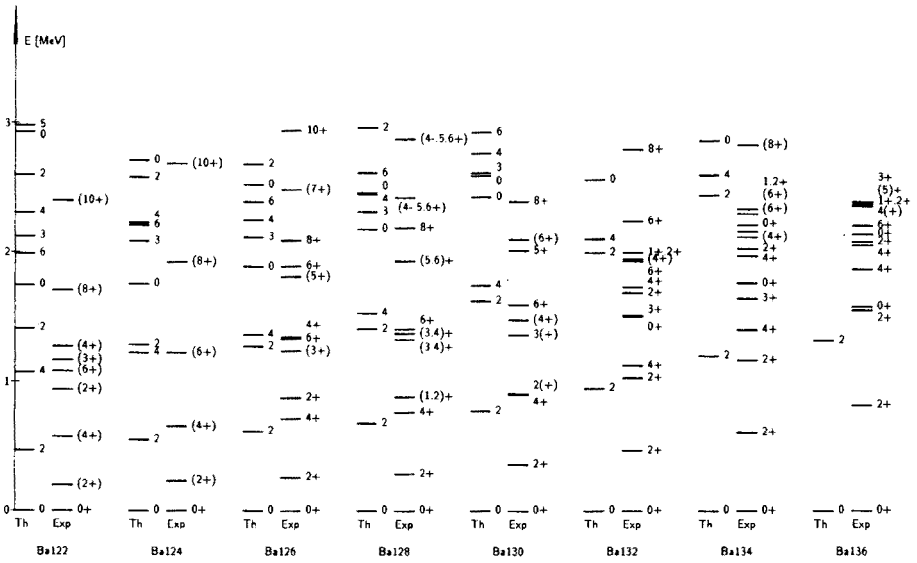

Fig. 1. See text.

Results of the diagonalization of Bohr Hamiltonian (1) with microscopic inertial functions are presented in Fig. 1. Experimental levels are taken from the NNDC database at Brookhaven National Laboratory (see Refs. given therein). The comparison of calculated (Th) and experimental (Exp) low-lying energy levels in Ba isotopes shows that even with the best fit of the

pairing strength we obtain still too large values of the mass parameters. The calculated energy levels are in general close to those obtained in [5, 6] with standard pairing strength and we must come to a conclusion that the new parametrization of the Nilsson potential and approximate particle number projection are insufficient to improve essentially an agreement with experimantal data.

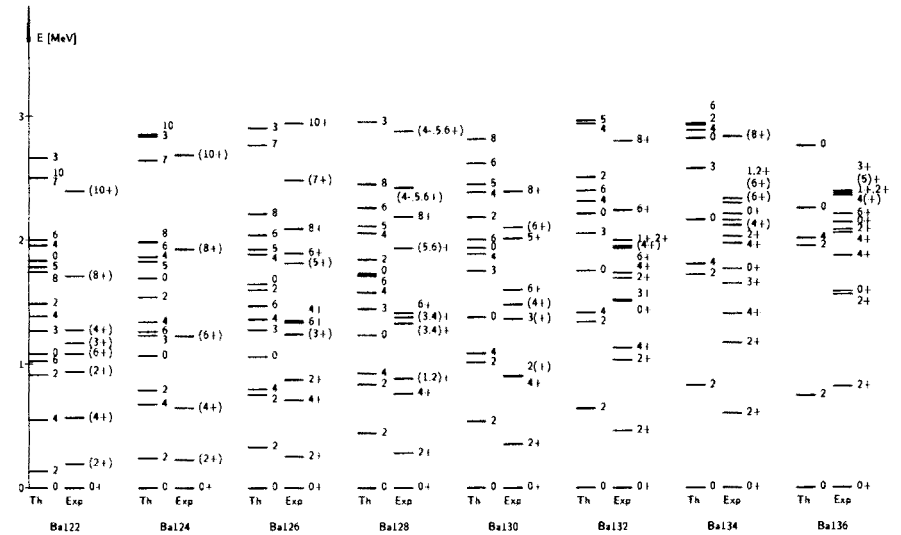

Fig. 2. See text.

It was found in [10] that the systematic shifting of the maximum of the ground state wave function towards smaller pairing gaps from the minimum of the potential is connected with the rapid increase of the mass parameters for decreasing Δ values. In consequence, this effect is due to the coupling of pairing vibrations with the nuclear shape oscillations. It seems that this coupling is too strong to be neglected. So while we are not able to include pairing gaps Δ_p, Δ_n and gauge angles ϕ_p, ϕ_n into the set of collective variables, we treat the effect of the coupling with the pairing vibrations in the static way. Such an effect can be described by systematic lowering the values of gap parameter. We have repeated our calculations with average dynamical Δ (taking 60% of the statical value for each nucleus) with the effect which is shown in Fig. 2. The density of collective levels is reproduced now correctly but many discrepancies still occur in their positions. More extended calculations including also electromagnetic transitions are in progress.

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