ON THE MACROSCOPIC LIMIT OF NUCLEAR FRICTION COEFFICIENT*

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The surface vibrations in nuclear Fermi liquid are studied within a phase space approach which is based on the Landau-Vlasov kinetic equation. The linear response theory is used. We focus on the damping of low frequency surface vibrations and discuss their friction coefficient.

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1. Introduction

The nature of nuclear dissipation is still not completely clarified. The one-body friction (the wall formula) [1] is derived *disregarding* any traces of a collision term. All the derivations we know of involve the *macroscopic limit*: one first looks at a finite system and lets its size go to infinity afterward, as is done for instance in [2].

The question we are touching here is very closely related to how one introduces irreversibility into a dynamical description. We shall be able to perform the transformation to the macroscopic limit in two ways, firstly by simply letting the size of our system go to infinity, and secondly by performing energy or frequency smoothings keeping the size of the system finite.

The dynamics inside the bulk will be described with the Landau-Vlasov equation. It will have to be accomplished by an appropriate treatment of the surface. We look at the friction coefficient for surface motion of a slab of nuclear Fermi liquid [3] in Section 2. We extend our investigation by studying surface vibrations in the Fermi liquid drop in Section 3. Finally,

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in Section 4 the consequences are discussed for the understanding of the nature of dissipation in real nuclei.

2. Slab model

We discuss surface modes for an analytically solvable model of a slab in the hope that with this model we simulate some effects of the nucleus being a *finite system.* The latter is defined as a Fermi liquid bound by the two surfaces

$$z = \pm\frac{L}{2} + Z(\vec{r}_\perp, t) \tag{1}$$

parallel to the $xy-$ plane and, in equilibrium, a distance L apart. Here and below $\vec{r}_\perp = (x, y, 0)$. The macroscopic variable $Z(\vec{r}_\perp, t)$ describes the local displacement of the surfaces from its positions in equilibrium.

A change in Z induces motion of the fluid particles inside the slab. The latter is represented by the distribution function $\delta n(\vec{r}, \vec{p}, t)$ in the particle's phase space which is described by a linearized Vlasov equation

$$\frac{\partial \delta n(\vec{r}, \vec{p}, t)}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{r}}\left[\delta n(\vec{r}, \vec{p}, t) - \frac{dn_0}{d\varepsilon}\mathcal{F}_0 \int d\vec{p}\, \delta n(\vec{r}, \vec{p}, t)\right] = 0\,. \tag{2}$$

The $\mathcal{F}_0$ stands for $\mathcal{F}_0 = F_0 h^3/(4\pi m p_F)$, where F_0 is the Landau parameter.

This equation, valid inside the slab at $-L/2 < z < L/2$, is accomplished by "mirror reflection" boundary conditions at the moving surfaces (1) [4]

$$\begin{aligned}&[\delta n(\vec{r}, \vec{p}_\perp, p_z, t) - \delta n(\vec{r}, \vec{p}_\perp, -p_z, t)]\,\Big|_{z=\pm\frac{L}{2}+Z(\vec{r}_\perp,t)} \\ &= -2p_z \frac{dn_0}{d\varepsilon}\frac{\partial}{\partial t} Z(\vec{r}_\perp, t)\,.\end{aligned} \tag{3}$$

Like in the case of a finite nucleus, the motion of the surface must be determined "self-consistently". This can be achieved by asking for the following "subsidiary condition" [5, 6]:

$$\begin{aligned}&\frac{1}{2}\left(P_{zz}(\vec{r}, t)\Big|_{z=\frac{L}{2}+Z(\vec{r}_\perp,t)} - P_{zz}(\vec{r}, t)\Big|_{z=-\frac{L}{2}+Z(\vec{r}_\perp,t)}\right) \\ &= \sigma\,\frac{\partial^2}{\partial \vec{r}_\perp^{\,2}} Z(\vec{r}_\perp, t) \;-\; F(\vec{r}_\perp, t)\,.\end{aligned} \tag{4}$$

Here, σ is the surface tension and $P_{zz}(\vec{r}, t)$ is the normal component of the momentum flux tensor, see [7]. In (4) we have included an *external* force $F(\vec{r}_\perp, t)$. For obvious reasons, the $F(\vec{r}_\perp, t)$ is written in the following form

$$F(\vec{r}_\perp, t) = F(\vec{k}_\perp, \omega)\cos(\omega t - \vec{k}_\perp \vec{r}_\perp)\exp(\varepsilon t)\,. \tag{5}$$

Of interest are those solutions of Eqs. (2)–(3) which are tailored to the special form of the external force (5). Thus we may write:

$$Z(\vec{r}_\perp, t) = \mathrm{Re}[Z(k_\perp, \omega) \exp[-i(\omega t - \vec{k}_\perp \vec{r}_\perp)]], \tag{6}$$

$$\delta n(\vec{r}, \vec{p}, t) = \frac{dn_0}{d\varepsilon} \mathrm{Re}[f_{k_\perp,\omega}(z, \vec{p}) \exp[-i(\omega t - \vec{k}_\perp \vec{r}_\perp)]]. \tag{7}$$

We may now parametrize the solutions (6) in terms of response functions. The latter may be defined as [8]:

$$Z(k_\perp, \omega) = -R(\vec{k}_\perp, \omega) F(\vec{k}_\perp, \omega). \tag{8}$$

With the help of (4) the $R(\vec{k}_\perp, \omega)$ is found to be:

$$R(\vec{k}_\perp, \omega) = \left[-\chi_{\mathrm{int}}(\vec{k}_\perp, \omega) + \sigma k_\perp{}^2\right]^{-1}, \tag{9}$$

where

$$\chi_{\mathrm{int}}(\vec{k}_\perp, \omega) = \frac{2}{h^3} \frac{\omega}{L} \sum_{n=-\infty}^{\infty} \int d\vec{p} \, \frac{dn_0}{d\varepsilon} \, \frac{(v_z p_z)^2}{\omega - \vec{k}_\perp \vec{v}_\perp - k_n v_z + i\varepsilon}. \tag{10}$$

The poles of this function determine the possible surface modes of our system.

To define the friction coefficient we consider the rate of energy transfer from the external force (5) to the system in the terms of the response function (9) [9]

$$\frac{dE[Z(\vec{r}_\perp, t), \frac{\partial}{\partial t} Z(\vec{r}_\perp, t)]}{dt} = \frac{R'(\vec{k}_\perp, \omega)}{[R'(\vec{k}_\perp, \omega)]^2 + [R''(\vec{k}_\perp, \omega)]^2} Z(\vec{r}_\perp, t) \frac{\partial}{\partial t} Z(\vec{r}_\perp, t) + \frac{1}{\omega} \frac{R''(\vec{k}_\perp, \omega)}{[R'(\vec{k}_\perp, \omega)]^2 + [R''(\vec{k}_\perp, \omega)]^2} \left[\frac{\partial}{\partial t} Z(\vec{r}_\perp, t)\right]^2. \tag{11}$$

If we then average over one period the first term on the right of (11) vanishes. However the second one will survive. It is associated with the *irreversible* transfer of energy. We concentrate on the coefficient in front of the $(\dot{Z})^2$, which we want to call $\gamma(\vec{k}_\perp, \omega)$, and will calculate the latter in the so called "zero frequency limit". The friction will be given by evaluating $\gamma(\vec{k}_\perp, \omega)$ at $\omega = 0$:

$$\gamma_{\mathrm{fr}}(\vec{k}_\perp) = \gamma(\vec{k}_\perp, \omega)\Big|_{\omega=0} \tag{12}$$

One can find:

$$\gamma_{\rm fr}(k_\perp) = \gamma_{\rm w.f.}\frac{3}{2}(\frac{k_\perp L}{\pi})^4 \sum_{n=-\infty}^{\infty} [(2n+1)^2 + (\frac{k_\perp L}{\pi})^2]^{-5/2} \qquad (13)$$

This formula shows that for small sizes of the slab when $k_\perp L \ll 1$ the friction coefficient vanishes to leading order in $(k_\perp L)^4$. On the other hand for a *large system*, replacing the sum in (13) by an integral one finds

$$\gamma_{\rm fr}(k_\perp) = \gamma_{\rm w.f.}, \qquad k_\perp L \gg 1\,. \qquad (14)$$

We recover the wall formula as the macroscopic limit of nuclear dissipation.

The friction should not be expected to occur for systems whose modes have a finite spacing. It is necessary to employ some statistical averaging in the sense of an energy or frequency smoothing before it becomes physically meaningful to speak of a friction force.

In the spirit of [10, 11] we define for quantities like $\gamma(\vec{k}_\perp, \omega)$ the frequency average as

$$\{\gamma(\vec{k}_\perp, \omega)\}_{\rm av} = \int d\Omega\, {\rm f}_{\rm av}(\Omega, \omega, \gamma_{\rm av})\, \gamma(\vec{k}_\perp, \Omega)\,. \qquad (15)$$

The ${\rm f}_{\rm av}(\Omega, \omega, \gamma_{\rm av})$ is the weight function, which we will choose to be given by a Lorentzian with the averaging parameter $\gamma_{\rm av}$. This allows an analytical calculation.

One can find that the averaged friction coefficient is smaller or equal to the wall formula for any value of the averaging parameter $\gamma_{\rm av}$

$$\{\gamma_{\rm fr}(0)\}_{\rm av} \leq \gamma_{\rm w.f.}\,. \qquad (16)$$

3. Fermi liquid drop

Our approach can be extended to study surface vibrations in Fermi liquid drop [12]. We consider a Fermi liquid bound by the surface

$$r = R + \delta R(\vartheta, \varphi, t)\,, \qquad (17)$$

which is the sphere with radius R in equilibrium. The macroscopic variable $\delta R(\vartheta, \varphi, t)$ describes the local displacement of the surface $R(\vartheta, \varphi, t)$ from its equilibrium position.

To study small vibrations in this model we use linearized Vlasov equation (2) supplemented by boundary conditions like (3) and (4) on the moving surface and choose the external force in the form

$$F_L(\vartheta, \varphi, t) = \sum_M F_{LM} Y_{LM}(\vartheta, \varphi) \cos(\omega t) \exp(\varepsilon t). \tag{18}$$

To find solutions it is convenient to change variables $(\vec{r}, \vec{p})$ to a new set of variables $(r, \varepsilon, l, \alpha, \beta, \gamma)$ as proposed in [13]. The new variables are particle energy ε, particle angular momentum $l = |\vec{r} \times \vec{p}|$, radius r and Euler angles (α, β, γ).

In the same way as in Section 2 we can find the collective response function and then determine the averaged friction coefficient. It can be found that the response function reads as

$$\chi_L(\omega) = \left[-\chi^L_{\text{int}}(\omega) + \frac{\sigma(L-1)(L+2)}{P_0 R} \right]^{-1} \tag{19}$$

with the intrinsic response function given by

$$\chi^L_{\text{int}}(\omega) = -\frac{60\pi}{2L+1} \omega \sum_{N=-L}^{L} |Y_{LN}(\tfrac{\pi}{2}, \tfrac{\pi}{2})|^2 \times \sum_{n=-\infty}^{\infty} \int_0^1 d\lambda \, \lambda \frac{(1-\lambda^2)^{1/2}}{\omega - (v_F/R)(n\pi + N \arccos \lambda\pi)/(1-\lambda^2)^{1/2} + i\varepsilon}. \tag{20}$$

Here L is the multipolarity of vibrations, $P_0 = (2/5)\varepsilon_F \rho_0$ and $\lambda = l/(p_F R)$. Using (19) we can calculate the averaged friction $\{\gamma^L_{\text{fr}}(0)\}_{\text{av}}$ like (12) for the slab model. The numerical results show that for octupole vibrations ($L = 3$) in a system of A=208 nucleons the averaged friction coefficient approaches the wall formula when γ_{av} is of order 10 MeV. For quadrupole vibrations γ_{av} should be larger: the averaged friction coefficient amounts about 70% of the wall formula when $\gamma_{\text{av}} \approx 10$ MeV and approaches the latter for $\gamma_{\text{av}} \geq 40$ MeV.

4. Conclusions

We have been able to prove that the macroscopic limit can be obtained for the finite system if we apply an appropriate frequency averaging. This result has some important conclusions with respect to the real nuclei. It

appears evident that nuclear dissipation cannot be of pure one-body nature. The presence of the width γ_{av} in formulas represents some uncertainty in the single particle energies of our fluid. This has been introduced by way of our smoothing procedure. But this γ_{av} may also be understood to simulate effects of couplings from the 1p-1h excitations to more complicated ones. Indeed, had we taken into account a collision term in a simple relaxation time approximation the γ_{av} would be nothing else but the inverse relaxation time. Certainly, a realistic treatment of the effects of collisions will be much more involved.

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