# STUDY OF THE SPONTANEOUS FISSION HALF-LIVES IN THE MULTIDIMENSIONAL COLLECTIVE SPACE\*,\*\*

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Spontaneous fission half-lives of the Fm isotopes are analyzed in a multidimensional collective space. The parameters describing the shape of nuclei  $\{\beta_{\lambda}\}$ ,  $\lambda=2,3,4,5,6,8$  and the pairing degrees of freedom  $\Delta_p$  and  $\Delta_n$  are taken into account. The half-lives  $T_{sf}$  are calculated by the WKB method. We propose the 'optimal' collective space  $(\beta_2,\beta_4,\beta_6,\Delta_p,\Delta_n)$  for dynamical calculations of the spontaneous fission half-lives of nuclei in this region. The  $T_{sf}$  for the even-even Fm isotopes are calculated.

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### 1. Introduction

In recent years a number of theoretical papers have been devoted to estimate the spontaneous fission half-lives of the heaviest nuclei with the Woods–Saxon potential [1–3]. The estimations were done mainly for the nuclei with the atomic number  $Z \geq 104$ , where the fission barriers are relatively simple and thin. The dynamical calculations require the very large computer time. Therefore this method has been used practically only for the two-dimensional deformations space ( $\beta_2$  and  $\beta_4$ ) with the simultaneous minimization of the potential energy V in the remaining degrees of freedom ( $\beta_3$  and  $\beta_5$  or  $\beta_6$  and  $\beta_8$ ) [3]. But a detailed study of the  $T_{sf}$  in deformation spaces of various dimensions has shown that this approximation is not good for lighter nuclei ( $Z \sim 100$ ), considered in the present paper. Therefore,

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in this paper we will consequently use only the full dynamical method. To minimize the action integral we have applied the dynamic-programing method [4].

In the present study we use a very reach collective space: except of the deformation parameters  $(\beta_{\lambda}, \lambda = 2, 3, 4, 5, 6, 8)$ , describing the shape of nuclei, we add the pairing proton and neutron gaps  $(\Delta_p \text{ and } \Delta_n)$  as a new collective coordinates [5]. The aim of the present paper is to examine the role of the all degrees of freedom used in the calculations. To this effect we extended the dynamic-programming method up to four collective degrees of freedom; the calculations in more-dimensional collective space are practically impossible. On this base, in full dynamic method we investigate successively:

- the role of the odd-multipolarity deformations ( $\beta_3$  and  $\beta_5$ ), using the four-dimensional deformation space ( $\beta_2, \beta_3, \beta_4, \beta_5$ ),
- the role of the higher even-multipolarity deformations ( $\beta_6$  and  $\beta_8$ ) using the ( $\beta_2,\beta_4,\beta_6,\beta_8$ ) space and
- the role of the pairing degrees of freedom  $(\Delta_p \text{ and } \Delta_n)$  using the  $(\beta_2, \beta_4, \Delta_p, \Delta_n)$  collective space.

The method of evaluation of the  $T_{sf}$  is described in Sect. 2. In Sect. 3 the main results are presented.

### 2. Theoretical model

We based on the single-particle Woods–Saxon potential with the 'universal' set of its parameters [6]. According to the Strutinsky model, the collective potential energy was split into a shell  $\delta E_{\rm shell}$  and the pairing correction parts  $\delta E_{\rm pair}$  and the smooth average background energy defined as the folded Yukawa plus exponential model with the standard values of its parameters [7]. The residual pairing interaction is treated in the BCS approximation with the pairing strength constants as in [8]. The collective mass B is calculated in the adiabatic cranking model. The fission process of a nucleus we describe as a tunnelling through the collective potential energy barrier using the classical WKB approximation.

### 3. Results

The spontaneous fission half-lives were calculated in the static and the dynamic approximation. The static value of the action integral  $S(L_{\rm stat})$  was obtained along the fission trajectory minimizing the potential energy only. In the dynamic approach the minimal value of  $S(L_{\rm dyn})$  was calculated by

minimization of the action integral with respect to all possible trajectories in our 4-dimensional collective space.

### 3.1. Role of the odd-multipolarity $\beta_3$ and $\beta_5$ deformations

The calculations have been performed in the four-dimensional deformation space  $(\beta_2,\beta_3,\beta_4,\beta_5)$ . Fig. 1 shows the spontaneous fission half-lives for Fermium isotopes calculated successively in the 4-, 3-, and 2-dimensional collective spaces. In this figure the experimental data are denoted by full circles,  $T_{sf}$  obtained in 2-dimensional deformation space  $(\beta_2,\beta_4)$  — by down-triangles and values obtained in full 4-dimensional deformation space  $(\beta_2,\beta_4,\beta_3,\beta_5)$  — by up-triangles . For clarity we do not show the  $T_{sf}$  in 3-dimensional collective space, because the dynamical results are the same as in 2- and 4- dimensional space. The full symbols denote the dynamical results, the open statical ones. From the figure we can see that in the dynamical calculations the results in 2- and 4-dimensional space are practically the same. It is seen that spontaneous fission process prefers the shapes of nuclei with  $\beta_3 = \beta_5 = 0$ . From this investigations one can draw a conclusion that odd-multipolarity deformations  $\beta_3$  and  $\beta_5$  do not play a role in the dynamical method of calculations of the  $T_{sf}$ .

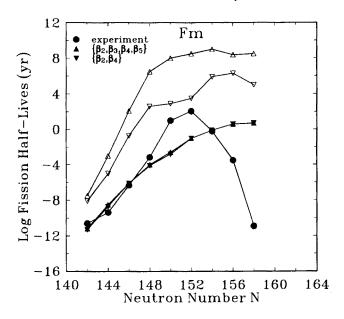


Fig. 1. Spontaneous fission half-lives calculated in  $(\beta_2,\beta_3,\beta_4,\beta_5)$  space.

## 3.2. Role of the higher even-multipolarity $\beta_6$ and $\beta_8$ deformations

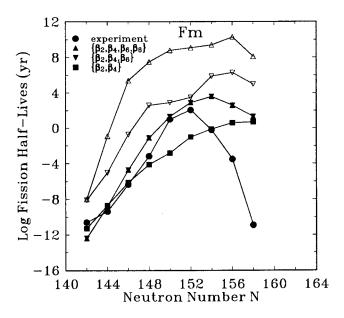


Fig. 2. Spontaneous fission half-lives calculated in  $(\beta_2, \beta_4, \beta_6, \beta_8)$  space.

The calculations have been performed in the four-dimensional deformation space  $(\beta_2,\beta_4,\beta_6,\beta_8)$ . Fig. 2 shows the theoretical  $T_{sf}$  in the deformation spaces of various dimensions. The experimental data are denoted by full circles, the results in 4-dimensional space — by up-triangles, the  $T_{sf}$  in 3-dimensional — by down-triangles and in the standard 2-dimensional  $(\beta_2,\beta_4)$  collective space — by squares. As above, the open symbols denote statical results, the full dynamical ones. One can see easily that only the deformation  $\beta_6$  affects the dynamical results of the  $T_{sf}$  (the calculations with and without  $\beta_8$  give almost the same results). The spontaneous fission half-lives with  $\beta_6$  increase  $T_{sf}$  about 1–4 orders of magnitude. This effect improves a little the agreement with experiment, but for nuclei with  $N \geq 154$  the difference between theory and experiment reaches 4–6 orders of magnitude. We can conclude that only  $\beta_6$  deformation is important in the dynamical calculations of the  $T_{sf}$ .

## 3.3. Influence of the pairing degrees of freedom on the $T_{sf}$ .

In order to examine the role of pairing degrees of freedom we have used the 4-dimensional collective space  $(\beta_2, \beta_4, \Delta_p, \Delta_n)$ . Fig. 3 presents the spontaneous fission half-lives in two cases: in 2-dimensional, standard

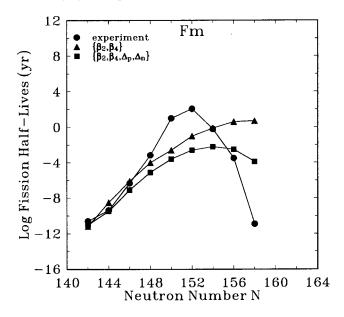


Fig. 3. Spontaneous fission half-lives calculated in  $(\beta_2, \beta_4, \Delta_p, \Delta_n)$  space.

space, when only  $\beta_2$  and  $\beta_4$  parameters are included (up-triangle) and in 4-dimensional collective space, with pairing degrees of freedom  $\Delta_p$  and  $\Delta_n$  (square). The differences between results of  $T_{sf}$  with  $\Delta_p$  and  $\Delta_n$  and without them represent the effect of the pairing degrees of freedom. As it is seen from the figure this effect is strongly isotopic dependent. For lighter isotopes the differences are about 0.5–1.0 order of magnitude and increase to 4–5 orders of magnitude for heavier Fm isotopes. The  $T_{sf}$  calculated without of the pairing degrees of freedom increase nearly monotonically with neutron number N. The investigations show that the pairing degrees of freedom are very important in calculations of the spontaneous fission lifetime.

## 3.4. The 'optimal' collective space

On the base of the above investigations we have proposed the 'optimal' collective space for calculations of the dynamical spontaneous fission half lives of the nuclei around fermium  $(Z \sim 100)$ . The our 'optimal' collective space contains the three parameters describing the shape of nuclei  $(\beta_2, \beta_4, \beta_6)$  and two pairing degrees of freedom  $(\Delta_p)$  and  $(\Delta_p)$ . It is full set of the collective degrees of freedom which influence on the spontaneous fission half lives of nuclei in this region. Fig. 4 shows (squares) the spontaneous fission half-lives estimated in the 'optimal' space (the results from  $(\beta_2, \beta_4, \beta_6)$  space corrected by the effect of the pairing degrees of freedom  $(\Delta_p)$  and  $(\Delta_n)$ . For comparison we have given the results in 3-dimensional deformation space

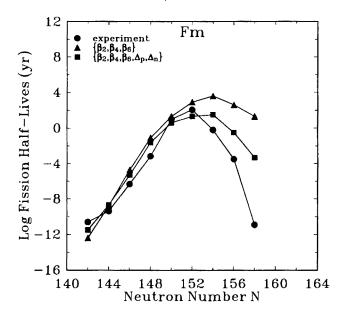


Fig. 4. Spontaneous fission half-lives estimated in  $(\beta_2, \beta_4, \beta_6, \Delta_p, \Delta_n)$  space.

 $\beta_2, \beta_4, \beta_6$  (down-triangle). From the figure we can see that in 'optimal' space the theoretical and experimental values of  $T_{sf}$  are in relatively good agreement. In particular we observe the well known effect of the decrease of the fission lifetimes for heavier Fm isotopes. However, for better description of the experimental spontaneous fission half-lives we have to correct the parameters of the Woods-Saxon potential.

#### REFERENCES

- [1] K. Böning, Z. Patyk, A. Sobiczewski, S. Ćwiok, Z. Phys. A325, 479 (1986).
- [2] Z. Lojewski, A. Baran, Z. Phys. A329, 161 (1988).
- [3] R. Smolańczuk, J. Skalski, A. Sobiczewski, Report GSI-94-77, November 1994.
- [4] A. Baran, Phys. Lett. 76B, 8 (1978).
- [5] A. Staszczak, S. Piłat, K. Pomorski, Nucl. Phys. A50, 589, (1989).
- [6] S. Ćwiok, J. Dudek, W. Nazarewicz, J. Skalski, T. Werner, Comput. Phys. Commun. 46, 379 (1987).
- [7] H.J. Krappe, J.R. Nix, A.J. Sierk, Phys. Rev. C20, 992 (1979).
- [8] J. Dudek, A. Majhofer, J. Skalski, J. Phys. G 6, 447 (1980).