GIANT MONOPOLE RESONANCES AND THE COMPRESSIBILITY OF NUCLEAR MATTER*,**

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The procedures to deduce the incompressibility modulus of nuclear matter, K_0, from the measured energies of the Isoscalar Giant Monopole Resonances, E_{GMR}, are critically reviewed. A simple model is developed in which the isoscalar density oscillations in finite nuclei are described as being due to the coupling of two modes: the bulk (scaling) mode and the surface mode. Two energy solutions are obtained for the coupled mode: the low energy solution, which is presumably the one observed experimentally, and the high energy one, well above the range of the present experiments.

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1. Introduction

The observed giant monopole energies, $\hbar\omega$, contain information on the compressibility coefficient of nuclear matter, K_0. The relation between $\hbar\omega$ and K_0 is not, however, straightforward on two counts. First, the compressibilities K_{AZ} of the finite nuclei A, Z for which $\hbar\omega$ is measured can be considerably different from K_0. Second, the determination of the lowest mode of monopole oscillation of a finite nucleus is not easy, except under the assumption of a "scaling" mode, which can be quite inaccurate.

The first difficulty appears to have been largely resolved in [1], where it is argued that the ratio K_{AZ}/K_0 is well approximated by E/E_0, E being the binding energy per particle of the finite nucleus in question and E_0

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the binding energy per particle of nuclear matter. On the other hand, the often used scaling assumption for the monopole vibration represents a constrained solution (see *e.g.* [2]) and as such it is bound to overestimate the true frequency. For example, in the hydrodynamical model of a compressible fluid drop with a sharp surface, the ratio of the constrained scaling frequency to the true frequency is $\sqrt{15}/\pi^2 = 1.22$ ([1]). This overestimate is due to the disregard in the scaling solution of the deviations from uniformity of the bulk density during the monopole oscillation. Another degree of freedom on which the scaling assumption imposes an arbitrary constraint is that associated with variations of the surface diffuseness during the oscillation. As discussed in Refs [3] and [4] this latter effect can lead to a significant lowering of the calculated value of ω, especially for light nuclei.

Our objective is to study systematically the giant monopole frequency in a macroscopic model which includes the above three principal degrees of freedom: scaling, diffuseness changes and bulk non-uniformities. The present contribution is an interim report on a first step in this direction which still disregards the bulk non-uniformity degree of freedom.

2. The model

We parametrize the density of the oscillating diffuse nucleus by starting with a uniform generating density inside a sharp radius R_0 and then diffusing the density by means of a short-ranged folding function given by a spherical form factor $f(r)$, equal to 1 for $r < a_0$ and zero for $r > a_0$. By scaling all linear dimensions by a scaling factor λ and simultaneously changing the range of the folding function according to $\alpha = a/a_0$, we obtain a two-degree-of-freedom family of density distributions, specified by the time dependent variables $\lambda(t), \alpha(t)$. Explicitly, the following expression is found for the density $\rho(r, \lambda, \alpha)$ of a spherical nucleus:

$$\frac{\rho(r,\lambda,\alpha)}{\rho_0} = \begin{cases} \frac{1}{\lambda^3} & \text{for } r < \lambda(R_0 - \alpha a_0) \\ \frac{1}{16}\frac{(\zeta-1)^2[\zeta^2+(4y+2)\zeta+8y-3]}{\lambda^3(y+\zeta)} & \text{for } \lambda(R_0+\alpha a_0) > r > \lambda(R_0-\alpha a_0) \\ 0 & \text{for } r > \lambda(R_0+\alpha a_0) \end{cases}, \quad (1)$$

where $y = R_0/\alpha a_0$, $\zeta = (r - \lambda R_0)/\lambda\alpha a_0$ and ρ_0 is the equilibrium density in the bulk.

In order to find the normal modes of small oscillations of the density we need to write down the potential energy as function of the small deviations of λ and α from their original (equilibrium) values $\lambda = 1$, $\alpha = 1$, *viz.*,

$$E_{\text{pot}} = \tfrac{1}{2}K_{11}(\lambda-1)^2 + K_{12}(\lambda-1)(\alpha-1) + \tfrac{1}{2}K_{22}(\alpha-1)^2, \quad (2)$$

where K_{11}, K_{12}, K_{22} are appropriate stiffness coefficients. Similarly the kinetic energy is written in the form

$$T = \tfrac{1}{2} M_{11} \dot{\lambda}^2 + M_{12} \dot{\lambda}\dot{\alpha} + \tfrac{1}{2} M_{22} \dot{\alpha}^2. \tag{3}$$

where M_{11}, M_{12}, M_{22} are the inertia coefficients.

The inertia coefficients are found by deducing the (irrotational) current j from the continuity equation

$$\nabla \cdot (\rho \boldsymbol{v}) = -\frac{\partial \rho}{\partial t} \tag{4}$$

with the result: $\boldsymbol{j} = \rho \boldsymbol{v} = j \hat{e_r}$, where

$$j = \frac{\rho_0}{32} \frac{(\zeta^2 - 1)^2 (\zeta^2 + 6y\zeta + 6y^2 - 1)}{(y + \zeta)^2} a_0 \dot{\alpha} + r\rho\dot{\lambda}, \tag{5}$$

for $-1 < \zeta < 1$ and $j = r\rho\dot{\lambda}$ otherwise. Using the equation

$$T = \tfrac{1}{2} \int d^3 r \frac{j^2}{\rho} \tag{6}$$

one obtains the following expressions for M_{11} and M_{12}:

$$\begin{aligned} M_{11} &= \frac{3}{5} M R_0^2 (1 + (\frac{a_0}{R_0})^2) \\ M_{12} &= \frac{3}{5} M {a_0}^2 \end{aligned} \tag{7}$$

and a more complicated expression for M_{22} which, however, can be easily computed numerically. Here M is the mass of the nucleus. The coefficient M_{22} can be approximated by a calculation in which, instead of the spherical geometry, one considers an infinite slab of width $2R_0$ for which the diffuse density as a function of the normal distance z from the middle of the slab is given by:

$$\frac{\rho(z, \lambda, \alpha)}{\rho_0} = \begin{cases} \frac{1}{\lambda} & \text{for} \quad z < \lambda(R_0 - \alpha a_0) \\ \frac{(2-u)^2(1+u)}{4\lambda} & \text{for} \quad \lambda(R_0 + \alpha a_0) > z > \lambda(R_0 - \alpha a_0) \\ 0 & \text{for} \quad z > \lambda(R_0 + \alpha a_0) \end{cases} \tag{8}$$

In this case

$$M_{11}^{\text{slab}} = MR_0^2(1 + \frac{3}{5}(\frac{a_0}{R_0})^2),$$
$$M_{12}^{\text{slab}} = \frac{3}{5}Ma_0^2,$$
$$M_{22}^{\text{slab}} = \frac{27}{64}M\frac{a_0}{R_0}^3(9\ln(3) - \frac{28}{3}). \tag{9}$$

The leading term in M_{11} (Eq. (7)) is, of course, the correct one to be used for the spherical geometry. The corrections of order $(a_0/R_0)^2$ in Eqs (9) are, as expected, in agreement with Eqs (7), while Eq. (9) for M_{22} is a good approximation to M_{22} calculated for the sphere.

The stiffness coefficient K_{11} was taken from the Thomas–Fermi expression Eq. (29) in [1], which represents the nuclear matter compressibility $K_0 = 234$ MeV multiplied by the ratio of the binding energy per particle of the nucleus in question to the binding energy per particle of nuclear matter (-16.24 MeV). In practice this gives for K_{AZ} a value close to 150 MeV except for the lightest nuclei (Ref. [1], Table II).

The coefficients K_{12} and K_{22} can be numerically calculated by using the energy density underlying the Thomas–Fermi model of [7], applied to the density distribution given by Eq. (1). These coefficients, being associated with the surface region, are approximately proportional to $A^{2/3}$. Typical values were $K_{12} = 59A^{2/3}$ MeV, $K_{22} = 11A^{2/3}$ MeV for ^{90}Zr and $K_{12} = 67A^{2/3}$ MeV, $K_{22} = 11A^{2/3}$ MeV for ^{208}Pb. One should not take these estimates too seriously since there are reasons to expect that quantum corrections beyond Thomas–Fermi model will increase stiffness coefficient K_{22} and probably also K_{12}. At the moment we adopted the procedure of treating K_{12} and K_{22} as independent parameters whose values are established by a fit to experimental data.

The equation for the normal modes ω of the system defined by Eqs (2), (3) is

$$\det|M|(\omega_2^2 - \omega_1^2) = \frac{(K_{12} - M_{12}\omega_1^2)^2}{\omega_1^2 - \omega^2} - \frac{(K_{12} - M_{12}\omega_2^2)^2}{\omega_2^2 - \omega^2}, \tag{10}$$

where ω_1 and ω_2 are the frequencies of the scaling and diffuseness modes in the absence of coupling, *i.e.*

$$\omega_1 = \sqrt{\frac{(K_{11}}{M_{11})}}, \quad \omega_2 = \sqrt{\frac{(K_{22}}{M_{22})}}. \tag{11}$$

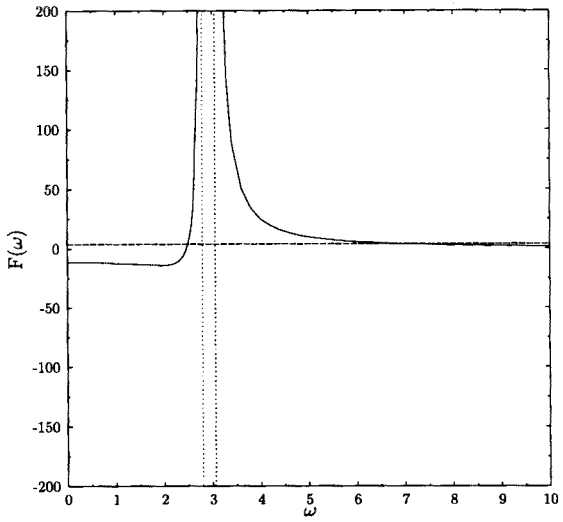

Fig. 1. The two solutions of eq.10 are illustrated graphically by the intersections of the full curves with the dashed horizontal line.

The solution of Eq. 10 is illustrated graphically in Fig. 1 in the case of ^{90}Zr. The full curves give the right hand side of Eq. (12) in its dependence on ω. The dashed horizontal line is the left hand side (a constant independent of ω). The intersections give the two normal mode frequencies or energies (frequencies multiplied by $\hbar$). One is a little lower than the energy $\hbar\omega_1$ shown by the left vertical dotted line, the other very much higher than the energy $\hbar\omega_2$ (the right vertical dotted line).

2. The data

The data included in the fits presented below were selected from the systematics of Refs [5, 6]. In selecting the entries we have followed the procedure suggested in [6], in which the errors assigned to each data point, σ_{GMR}, were related to the observed fraction of the energy weighted sum rule, f, according to the formula

$$\sigma_{GMR} = \left(\sigma_{\mathrm{exp}}^2 + (2(1-f))^2\right)^{1/2}, \tag{12}$$

where σ_{exp} is the error of the centroid, E_{exp}, of the observed Giant Monopole Resonance, GMR. The data points with errors exceeding 1.2 MeV were rejected. The adopted GMR energies were corrected for the measured widths, Γ, according to

$$\hbar\omega = \left(E_{\mathrm{exp}}^2 + 3\left(\frac{\Gamma}{2.35}\right)^2\right)^{1/2} \tag{13}$$

(a correction typically of about 200–300 keV).

The measured GMR energies for deformed nuclei were shifted downward by the amount resulting from introducing into the liquid drop formula for K_{AZ} a deformation dependent term $K_{\rm def}\beta_2$ with the value $K_{\rm def} = 35$ MeV adopted from [5].

4. Results

The triangles in Fig. 2 demonstrate the lowering of the resonance energies resulting from unfreezing the surface diffuseness mode. The dots are the measured energies. The values of the stiffness coefficients K_{12} and K_{22} obtained from a fit are equal to $50A^{2/3}$ and $25A^{2/3}$, respectively. For the diffuseness a_0 we have taken the value as to reproduce experimental root mean square radii along the periodic table. This gives for a_0 a value of 2.2 fm. A more nearly unambiguous comparison with measurements awaits the inclusion of the bulk non-uniformity degree of freedom. Fig. 3 illustrates the trend of the second, high energy monopole resonance energy corresponding to the second solution of Eq. 10.

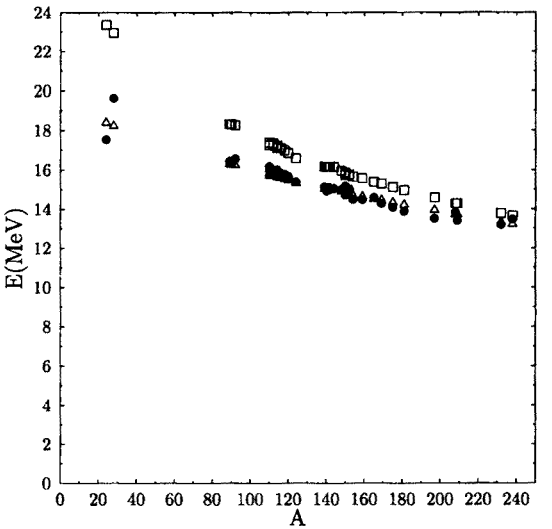

Fig. 2. Measured giant monopole resonance energies (dots) and calculated values using only the scaling degree of freedom (squares). Triangles show the lowering of the calculated resonance energies resulting from unfreezing the diffuseness degree of freedom.

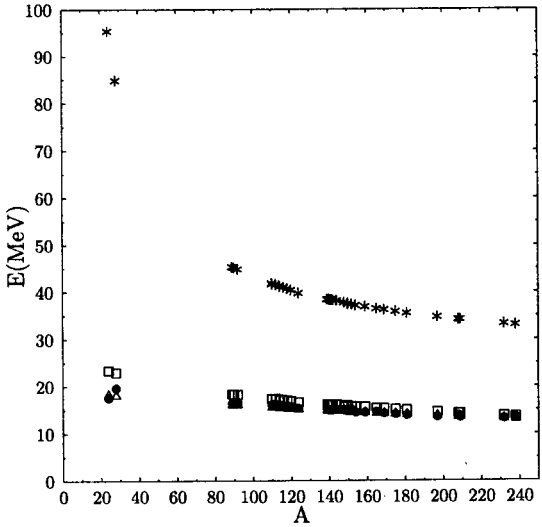

Fig. 3. This is like Fig. 2 drawn to a different scale in order to show the trend of the second, high energy giant monopole resonance energy (asterisks).

6. Summary

The use of a scaling assumption in the interpretation of giant monopole resonances is bound to overestimate the resonance energies. The inclusion of a diffuseness degree of freedom lowers the energies to the vicinity of the measured values. Since the present results point only to the possibility of reproducing GMR energies by fitting K_{12} and K_{22} it still remains to be seen if the inclusion of the bulk non-uniformity degree of freedom combined with some nonambiguous calculation of the stiffness coefficients will lead to a correct physical description.

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