EXPERIMENTAL TEST OF BELL'S INEQUALITY USING ANNIHILATION PHOTONS*,**

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The correlation of polarization of annihilation quanta has been measured in order to test the Bell's inequality. Significant violation of the Bell's inequality and a good agreement with predictions of quantum mechanics have been ascertained.

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1. Introduction

Bell's inequality apply to any correlated measurement on an ensemble of two correlated systems (for instance pair of particles), for which the measurement taken separately appear random. In 1935 Einstein, Podolsky, and Rosen introduced their thought experiment (EPR) [1]. In an effort to rescue local reality they proposed that quantum mechanics (QM) could be incomplete. In order to account for the observed correlations, many authors introduced supplementary parameters in addition to the state vector, which is the same for all emitted pairs. This natural supplement, called hidden variables, leads to the conclusion that quantum mechanics is not a complete description of physical reality.

Thanks to Bell's work [2] we have got a theoretical tool to discriminate in the experiment between predictions of QM and local hidden variable theory (LHV). In order to test the Bell's inequality we consider the combination S of four possible correlation coefficients, evaluated at one setting of

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four polarymeters. The polarymeters at adjusted orientations $\vec{a}, \vec{a'}, \vec{b}, \vec{b'}$ will measure the correlation coefficients $E(\vec{a},\vec{b}), E(\vec{a},\vec{b'}), E(\vec{a'},\vec{b})$ and $E(\vec{a'},\vec{b'})$. Each correlation coefficient can be determined as:

$$E(\vec{a},\vec{b}) = \frac{R(\vec{a}_{\|},\vec{b}_{\|}) + R(\vec{a}_{\perp},\vec{b}_{\perp}) - R(\vec{a}_{\|},\vec{b}_{\perp}) - R(\vec{a}_{\perp},\vec{b}_{\|})}{R(\vec{a}_{\|},\vec{b}_{\|}) + R(\vec{a}_{\perp},\vec{b}_{\perp}) + R(\vec{a}_{\|},\vec{b}_{\perp}) + R(\vec{a}_{\perp},\vec{b}_{\|})}, \quad (1)$$

where $R(\vec{a}_{\|},\vec{b}_{\|}), R(\vec{a}_{\perp},\vec{b}_{\perp}), R(\vec{a}_{\|},\vec{b}_{\perp})$ and $R(\vec{a}_{\perp},\vec{b}_{\|})$ denotes respective coincidence counting rate between two polarymeters set at opposite sides of the source with orientations (a,b). Assuming existence of hidden variables Bell showed that the combination of four correlation coefficients defined as:

$$S = E(\vec{a},\vec{b}) - E(\vec{a},\vec{b'}) + E(\vec{a'},\vec{b}) + E(\vec{a'},\vec{b'}) \quad (2)$$

is limited by the inequality:

$$-2 \leq S \leq +2\,. \quad (3)$$

Bell proved that any LHV theory based on the concepts of locality and reality would be inconsistent with some predictions of quantum mechanics [2, 3], for which the quantity S predicted by quantum mechanics violates Bell's inequality reaching the value of $2\sqrt{2}$. The essential assumption made in LHV theory is locality which states that the result of a measurement by a polarymeter cannot be directly influenced by the result obtained on the other, distant polarymeter. This assumption can be considered to be a consequence of Einstein's causality. In order to check Einstein's causality one should consider experiments in which the settings of the polarizers are changed in time which is short compared with time of flight of photon at a distance between polarizers. The Compton scattering, offers unique opportunity to set a scheme in which several two-channel polarymeters can be adjusted to measure independently polarizations of two-photon state (EPR state).

Several experiments testing the Bell's inequality were already performed. Some of them were based on the measurements of polarization of annihilation quanta [4, 5]. In the experiments of Clauser *et al.* [6, 7] and Aspect *et al.*[8, 9], the pairs of EPR photons produced in radiative cascade in ^{40}Ca were employed. However an experiment which fulfills all requirements of locality has been needed.

2. The experiment and results

The experimental arrangement used to measure the relative linear polarization of γ-rays in order to investigate the Bell's inequality is shown

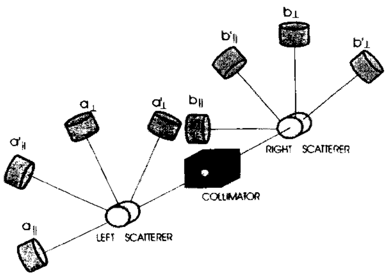

Fig. 1. Schematic diagram of scattering geometry for the experiment of the test of Bell's inequality violation.

schematically in Fig. 1. Positrons were emitted by ^{22}Na radioactive source of activity $A = 170$ MBq. The source was placed in a cylindrical container made of copper which slid into rectangular hole, to fix it in the lead collimator.

The annihilation photons are emitted in opposite directions along the symmetry axis which is selected by the lead collimator. Both annihilation photons are scattered by small cylindrical plastic scintillators of dimensions Φ=3cm $\times$ 5cm, placed symmetrically at the distance of 18.5 cm from the source. The dimensions of the scatterer were limited in order to minimize the chance of multiple Compton scattering. The scattered photons are registered by eight BaF_2 scintillators set on both sides at polar angle 82°, at the distance from the scattering axis of 36 cm, forming set of four dichotomous polarization analysers at selected orientations $\vec{a}, \vec{a'}$, and $\vec{b}, \vec{b'}$. Each pair of γ-detectors registering Compton scattered γ-rays at polar angle 82° and relative azimuthal scattering angles of 90°, establish one two-channel (dichotomous) polarymeter. For example, polarymeter $\vec{a}$ has only two possible outcomes: coincidence count either in scatterer-analysing detector at $\vec{a}_{||}$ or scatterer-analysing detector at $\vec{a}_{\perp}$. The polarization measurements then become similar to Stern–Gerlach measurements for spin 1/2 particles. In our set-up the polarization analysers are kept at two fixed orientations $\vec{a}, \vec{a'}$ on one side, and $\vec{b}, \vec{b'}$ on the other side, what allows to determine two different orientations of polarization. Four identical polarymeters adjusted at orientations $\vec{a}, \vec{a'}$ and $\vec{b}, \vec{b'}$ measure polarization of both annihilation photons emitted in opposite directions. In our set-up action-at-a-distance in the relativistic sense is precluded, since both photon striking the scatterers have choice to demonstrate various polarizations until they impinge on scatterer. It is easy to imagine that instrumental mechanism in producing highly correlated coincidence events is excluded. To test locality condition, we do not require to change the orientations $\vec{a}$ and $\vec{a'}$ of the polarymeters while the correlated photons are in flight.

The function of the electronics was to select the events corresponding to the coincidence between two scatterers and two analysing detectors, responding on opposite sides of the symmetry axis, selected by the collimator. We have registered time of response and energy deposit in both scatterers, four analysing detectors on the left side and four of them on the right side.

In off-line analysis the time gates were set on all spectra of analysing detectors. In order to accept the event we required also relevant energy deposited in each detector, what is equivalent to requirement of the total energy loss equal to the energy of the annihilation photon. These energy conditions can eliminate multiple Compton scattering which leads to higher energy deposited in the scatterer. Selecting fourfold coincidence, we have reduced data from the single run to the 16 coincidence counting rates R, which determines four correlation coefficients yielding directly S values.

For our convenience we set the polarymeters in the sequence $\vec{a}, \vec{b}, \vec{a}', \vec{b}'$. The successive polarymeters were set at azimuthal angles nϕ where n=0,1,2,3 for polarymeters $\vec{a}, \vec{b}, \vec{a}', \vec{b}'$, respectively. The coincidence rates were evaluated as a function of the parameter ϕ. For ideal polarymeter the prediction of quantum mechanics gives the S function as:

$$S = 3\cos(2\phi) - \cos(6\phi)\,. \tag{4}$$

Because our experiment was carried out with a non ideal polarymeter, the theoretical values of correlation coefficient E(a,b) and function S were corrected for taking into account experimental value of the analyzing power of our polarymeter. Corrections introduced for non ideal polarimeters will modify Bell's limit for LHV theory and QM predictions. These modified values can be compared with the experimental values.

In order to determine the corrections for our polarymeters, the correlation function $E(90°)$ was measured. It can be shown that the Bell's inequality corrected for the properties of the polarymeter reduces to:

$$\mid S_r \mid \leq 2 \cdot E(90°)\,. \tag{5}$$

As the measured value of $E(90°) = 0.401$, thus for our polarymeters the Bell's inequality is given by:

$$\mid S_r \mid \leq 0.802\,. \tag{6}$$

The experimental values of "S" function determined from the measurements of the coincidence rates at different orientations of the polarymeters parametrized by the azimuthal angle ϕ are displayed on Fig. 2. As it is seen a good agreement between the prediction of quantum mechanics and experimental data is observed ($\chi^2 = 1.34$). Our data show also that the

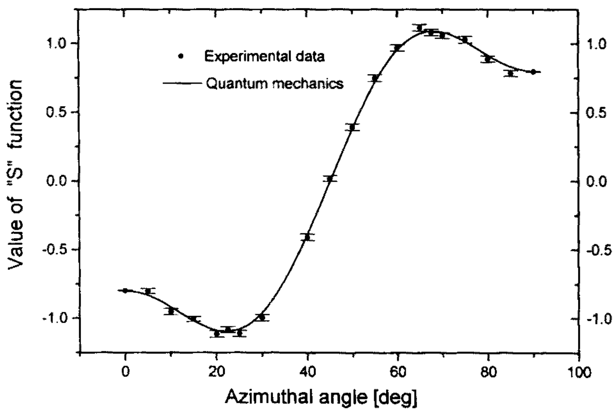

Fig. 2. Plot of experimental values "S" *vs.* relative azimuthal angle ϕ. The solid line represents the function "S" calculated according to quantum mechanics with corrections for the analysing power of the polarymeter.

Bell inequality, is violated by twelve standard deviations, what can lead to the rejection of the local hidden variable hypothesis.

In our experimental set-up we have registered $3 \cdot 10^4$ fourfold events for each setting of the polarymeters. We have measured and analysed the experimental results for 20 settings of our polarymeters. For every one of settings we have determined the experimental value of the function S (dots in Fig. 2.), in order to compare them with the calculations based on QM and the limit given by the Bell's inequality. The calculations of correlation coefficients and the function S based on QM, as well as the Bell's limit, were corrected due to analysing power of the polarymeters (solid curve). We have obtained significant violation of the Bell's inequality $| S_r | \leq 0.802$ (corrected for the properties of the polarymeter), and a good agreement with quantum mechanics.

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