MICROSCOPIC STERN–GERLACH EFFECT AND SPIN–ORBIT PENDULUM*

P. Rozmej

Institute of Physics, University Maria Curie–Skłodowska
20–031 Lublin, Poland
<rozmej@tytan.umcs.lublin.pl>

AND

R. Arvieu

Institut des Sciences Nucléaires
53 Avenue des Martyrs, 38026 Grenoble–Cédex, France
<arvieu@frcpn11.in2p3.fr>

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The motion of a particle with a spin in spherical harmonic oscillator potential with spin–orbit interaction is discussed. The attention is focused on the spatial motion of wave packets. The particular case of wave packets moving along the circular orbits for which the most transparent and pedagogical description is possible is considered. The splitting of the wave packets into two components moving differently along classical orbits reflects a strong analogy with the Stern-Gerlach experiment. The periodic transfer of average angular momentum between spin and orbital subspaces accompanying this time evolution is called the *spin–orbit pendulum.*

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1. Introduction

The recent development of short and intense laser pulses renewed the interest in wave packet dynamics. In atomic systems electrons can be excited to a coherent mixture of many Rydberg states and move almost classically for many Kepler periods [1]. Long range time evolution of such wave packets

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exhibits spreading and revivals according to the universal scenario described in [2]. These quantum effects arise from nonequidistant spectrum of energy levels in the Coulomb field.

In our paper we want to point out another interesting feature of wave packet dynamics which shows up when strong enough spin–orbit interaction is present in a physical system. The detailed theory of the phenomenon is given in [3–5], here we only give the brief description and explanation of main results. The physical system, which properties we want to discuss, is one fermion in a spherical oscillator potential with a spin-orbit interaction,

$$H = H_0 + V_{ls} = H_0 + \kappa(\vec{l}\cdot\vec{\sigma}). \tag{1}$$

In the spin–orbit part a constant form factor κ has been assumed. This model is a simplified version of the Nilsson single-particle model [6] extensively used in nuclear physics to describe the properties of deformed nuclei.

For pedagogic reasons we discuss here the motion of a special family of states, corresponding to classical particles moving on circular orbits. The general case of elliptic orbits is discussed in [4]. As the initial condition we assume the coherent state of the harmonic oscillator multiplied by the spin state, the eigenstate of s_u operator ($\vec{u}$ — arbitrary axis). More precisely, we focus our attention on the particular case in which the $\vec{u}$ direction lies in orbit's plane, *i.e.* $\vec{u}$ is perpendicular to orbital momentum. In this particular case the effect we want to discuss is most pronounced and not obscured by additional details. Such initial states are pure in both subspaces, ordinary and spin ones. Explicitly, we choose (without any loss of generality) Oxy plane as the orbit plane and Ox as the initial spin direction. Then the initial states take the following explicit form

$$|\Psi(t=0)\rangle = |N,\vec{x}\rangle = |N\rangle\,\frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right), \tag{2}$$

where the eigenstate of s_x is expressed explicitly by the eigenstates of s_z operator ($|+\rangle$ and $|-\rangle$) and $|N\rangle$ is the coherent state of spherical harmonic oscillator corresponding to a circular orbit. In configuration space it has the form

$$\langle\vec{r}|N\rangle = \pi^{-\frac{3}{4}}\,\exp\left(-\tfrac{1}{2}[(x-x_0)^2+y^2+z^2]\right)\,\exp(ip_0y). \tag{3}$$

As operators H_0 and V_{ls} commute, the evolution operator connected with the Hamiltonian (1) can be factorized as

$$U(t) = U_0(t)\,U_{ls}(t) = \mathrm{e}^{-itH_0}\,\mathrm{e}^{-it(\vec{l}\cdot\vec{\sigma})}, \tag{4}$$

where the appropriate time units are chosen to absorb κ and $\hbar$.

2. Motion in spin subspace

Equations (7–9) of [5] describe time evolution of the system in the Heisenberg picture. During this evolution the state (2), initially pure in both spin and ordinary subspaces, becomes mixed. Analytical calculations of the expectation values of spin operator given in [3] show that they undergo fast collapse, then stay close to zero and are approximately restored (with spin reversed) at $t = T_{ls}/2$. The evolution in the second half of the spin–orbit period is symmetrical to that in the first half, with exact revival at $t = T_{ls}$. At revivals the purity of the state in both subspaces is also restored. As the total angular momentum is conserved, the above oscillations must be accompanied by the corresponding oscillations of orbital angular momentum. This phenomenon, periodic transfer of the average angular momentum between spin and orbital subspaces, is called by us *spin–orbit pendulum*. The exact motion of $\langle\vec{s}\rangle$ and $\langle\vec{l}\rangle$ for the particular case $N{=}8$ (N stands for the average value of n quantum number) and time range $[0, {}^1\!/\!_2 T_{ls}]$ is given in Fig. 1 (periodicity makes the motion in the second half of the period symmetric to the shown one).

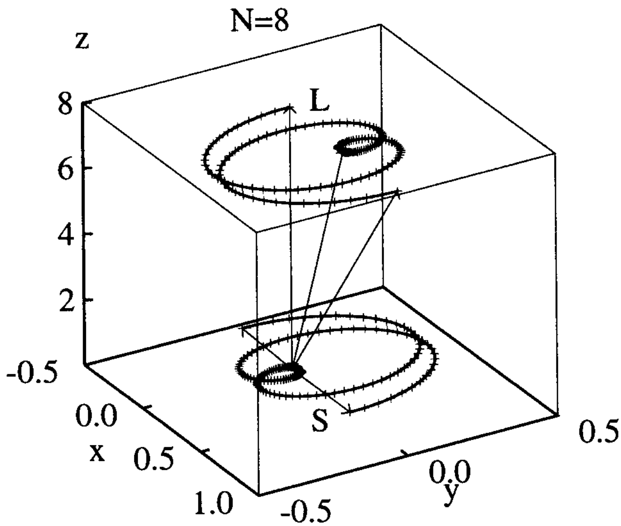

Fig. 1. Motion of $\langle\vec{s}\rangle$ and $\langle\vec{l}\rangle$ during $t \in [0, T_{ls}/2]$ illustrating oscillations and exchange of angular momenta (*spin-orbit pendulum*). Symbols show the equidistant time steps $T_{ls}/500$.

3. Motion in configuration subspace

The full understanding of this phenomenon needs the complementary description in terms of motion in the configuration subspace. The separation (4) which is exact if H_0 commutes with the spin-orbit potential allows to consider the full motion of the system as a superposition of two independent, periodic motions. The motion induced by the U_0 operator is analogous to the motion of a classical particle, U_0 shifts the wave packet along the classical orbit without any change of its form. On the top of this 'classical' motion there is another one due to U_{ls} operator. Briefly speaking this motion consists in the opposite motions of the spin-up and spin-down subpackets. The spin collapse is explained in this picture as loosing the overlap of the subpackets, what leads to vanishing of average values of all components of $\vec{s}$ operator. Revivals are understood as meetings of the subpackets on the orbit at times $T_{\rm rev} = n\,T_{ls}/2$. During the motion subpackets do not remain identical, some components of one of them are periodically partially shifted to the other and backwards. The projection of the full motion of both subpackets onto the plane of the classical orbit is shown in Fig. 2 for $N = 8$ case. This representation of the time evolution fails, however, in showing the motion in the third coordinate. To see it we display the 3–dimensional trajectories of the subpackets' maxima in Fig. 3. It is clear from this figure that the spin-up subpacket changes it's orbit plane two times within $t \in [0, T_{ls}/2]$, which reflects changes of $\langle\vec{l}\rangle$ shown in Fig. 1 when $\langle\vec{s}\rangle$ vanishes and revives with reversion. For more details see [5].

4. Perspectives for experimental verification

Nuclear systems, although suitable because of the strong spin–orbit interaction, are too weekly bounded to allow for an excitation of a required wave packet (which must be built from several states with diferent n quantum number). The unconveniency of atomic systems is due to the week spin–orbit interaction which makes $T_{ls}/T_{\rm Kepler}$ very large. This ratio can be strongly reduced, however, for atoms (ions) with high Z number. For such systems one can apply our approach in the first order perturbation theory at least for circular orbits [4]. Our estimations show that such systems could be promising candidates for experimental attemps to detect effect predicted by us.

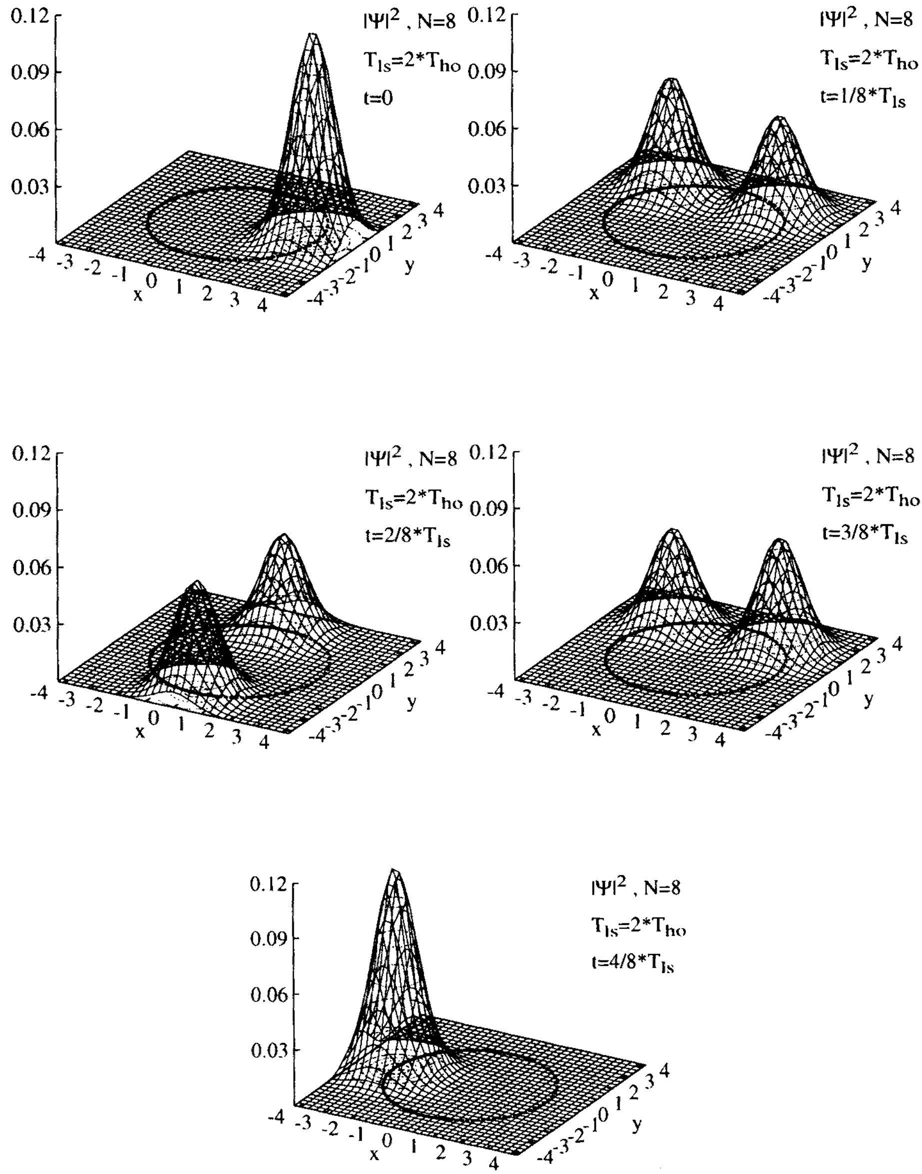

Fig. 2. Motion of the wave packet with $N{=}8$ in time range $[0, T_{ls}/2]$. Shown is $|\Psi(t)|^2 = |\Psi_+(t)|^2 + |\Psi_-(t)|^2$ integrated over θ as the function of coordinates on the plane of the classical orbit (marked by the thick circle). The strength of the spin-orbit interaction is chosen to ensure $T_{ls} = 2\,T_0$.

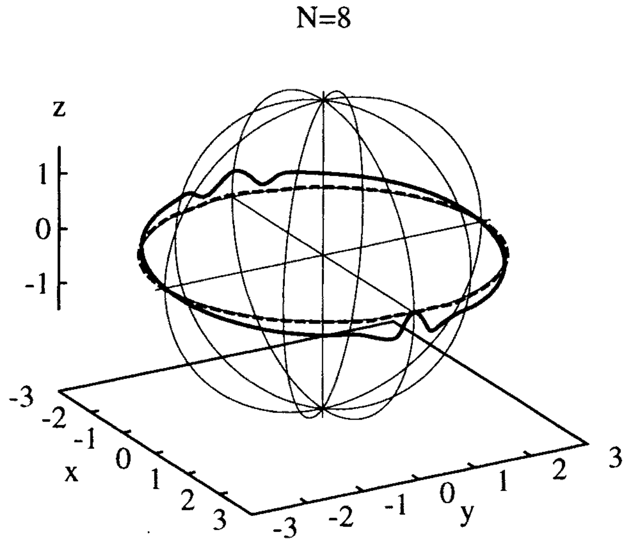

Fig. 3. Trajectories of the maxima of the spin-up subpacket (solid line) and the spin-down one (dashed line) on the sphere (thin circles) with the radius equal to the radius of classical orbit for $N = 8$ case.

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