THE ISOSPIN DEPENDENT *p–n* MULTIPLETS IN THE REGION OF DOUBLY MAGIC ^{100}Sn*

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Experimental data on p–n multiplets appearing in the nuclei near double-magic ^{100}Sn accesible in heavy-ion fusion-evaporation experiments have been analyzed. Proton-rich nuclei in this region have been investigated using the NORDBALL array. A ^{58}Ni beams at energies of 270 Mev on ^{54}Fe and 261 MeV on ^{50}Cr targets were used. Reaction channel separation was achieved with a 4π charged particle multi-detector set-up together with a 1π neutron detector wall placed in the forward direction. On the basis of $\gamma\gamma$-coincidence and angular correlation relations a level schemes of several light Sn, In, Sb, Te and I nuclei were observed for the first time in the present experiment. The observed structure of nuclei are discussed in the framework of the nuclear shell-model. The p–n multiplets in the $A \sim 100$ region corresponding to the nucleon pairs with mixed configurations has been considered as a playground for the tests of the $T = 0$ and $T = 1$ parts of the effective interaction.

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1. Introduction

In this report a special attention has been paid to the structure of the nuclear excited states with mixed configuration with reference to the ideas presented in the early works of Szymański [1] on "Excited levels of nuclei with mixed configurations".

There has been recently a considerable progress in the experimental investigation of nuclei close to the double magic ^{100}Sn. The present status of the shell model structure at around $N = Z = 50$ basing on existing experimental data in the close vicinity of ^{100}Sn was summarized by Grawe *et al.* [2]. The configuration space used, was $\nu(d_{5/2}, g_{7/2}, s_{1/2}, d_{3/2}, h_{11/2})$ and the $\pi(g_{9/2}, p_{1/2}, d_{5/2}, g_{7/2})$ relative to a ^{88}Sr or to ^{100}Sn core. However, there is still a lack of experimental data on the single particle and hole energies relative to the ^{100}Sn core as well as the effective two-body interactions. There are also several other reasons which make the ^{100}Sn region very interesting:

(i) The ^{100}Sn is the heaviest known [3], self-conjugate ($N = Z$) doubly magic nucleus that probably is stable against particle decay.

(ii) It also creates the possibility of studying the shell model and the residual interactions in a full major shell from $^{100}Sn_{50}$ to $^{132}Sn_{82}$ for N/Z ratios varying from 1 to 1.6.

(iii) At last the isospin mixing in nuclear ground state, which results from the Coulomb interaction, increases with Z and is largest for the $N = Z$ nuclei [4–6], what allows on possible studies of isospin admixture in those nuclei.

2. The experimental procedure and results

The γ-rays following the reaction with 261 and 270 MeV ^{58}Ni beam on ^{54}Fe (10 mg/cm^2, enriched up to 99.8%). and ^{50}Cr(3.5 mg/cm^2, enriched up to 97%) targets were recorded by the Nordball detector system [7]. The beam was delivered from the tandem accelerator at the Niels Bohr Institute in Risø, Denmark. The present detector set-up consisted of 15 Ge-BGO spectrometers, one of which was a LEP-detector, a 4π charged particle detector system which comprised 21 ΔE-type Si-detectors [8], a 1π neutron detector assembly consisting of 11 liquid scintillator detectors in the downstream hemisphere [9] and a 2π γ-ray calorimeter composed of 30 BaF_2 crystals covering the upstream hemisphere.

The compound nuclei ^{112}Xe and ^{108}Te obtained in the presently studied reactions are very neutron deficient and the events leading to the evaporation of neutrons are rare. Since in such reactions the most exotic neutron deficient nuclei are produced, a strong emphasis was put on the performance of the neutron detector system. By using both neutron time-of-flight and pulse shape discrimination techniques it was possible to improve the neutron-γ separation by almost an order of magnitude comparing to the case when only pulse shape discrimination was used. The details of the NORDBALL experimental set-up specific for the considered experiment were presented by Persson [10] at this School.

Nuclei in the considered here region have been produced at different bombarding energies of ^{58}Ni reaching quite wide spectrum of spins ($3\hbar \leq I \leq 18\hbar$) and excitation energies up to about $E_x = 8$ MeV. A total amount of about 420 million (in the 1-st experiment) and 1200 milion (in the 2-nd experiment) γ-γ-coincidence events containing information about the detected γ-rays, neutrons, protons and α-particles were collected.

The efficiency of the neutron detector system was estimated to be ~23% in the present experiment. For an α-particle and a proton detection it was ~ 45% and ~ 63% respectively. Since the detection efficiency for a specific type of evaporated particle depends very weakly on the reaction channel, comparison of such ratios with those for γ-rays from previously known nuclei that were populated in the experiment, make the assignments of the final nuclei possible. Using the above method a total of 29 different exit channels were identified including 9 light Cd, Sb, Te and I isotopes not observed before, with no excited states previously known from the "in-beam" experiments, namely ^{99}Cd, ^{101,102}In, ^{106,107,108}Sb, ^{108,109}Te and ^{111}I nuclei.

3. Isospin at low energy excitations

In a specific nucleus the z-component of the total isospin is equal to $M_T = T_z = \frac{1}{2}(N - Z)$. All ground states have minimum possible total isospin $T =| T_z |$, but excited states may have total isospin $T =| T_z |, | T_z + 1 |, \ldots (1/2)A$. According to the assumption of the charge independence the nuclear forces acting between the nucleons do not depend on their electrical charge *i.e.* nuclear forces are equivalent for the pairs of particles (n,p), (n,n), (p,p), when being in the same states. As a result of the charge independence of the nucleon-nucleon interaction, the total isospin T commutes with the nuclear Hamiltonian which implies that T and T_z are good quantum numbers. The above assumption holds only when the Coulomb interaction, which is isospin dependent is neglected. However, the real interaction is not charge-independent as the proton and neutron masses are not equal and the Coulomb forces are acting. The total hamiltonian [4] apart from charge-independent interaction $V_{\rm nuc}$ term, contains also the terms responsible for the rest energy $V_{\rm res}$, kinetic energy $V_{\rm kin}$ and Coulomb interaction $V_{\rm coul}$.

$$H = V_{\rm nuc} + V_{\rm res} + V_{\rm kin} + V_{\rm coul} = V_{\rm nuc} + H' + V_{\rm coul}\,, \tag{1}$$

where

$$H' = \sum_{i=1}^{A}\left[m_n c^2 \frac{1+\tau_z^i}{2} + m_p c^2 \frac{1-\tau_z^i}{2}\right] - \sum_{i=1}^{A}\left[\frac{\hbar^2}{2m_n}\frac{1+\tau_z^i}{2} + \frac{\hbar^2}{2m_p}\frac{1-\tau_z^i}{2}\right]\Delta_i\,, \tag{2}$$

where $((1+\tau_z)/2) = t_n$ and $((1-\tau_z)/2) = t_p$ are neutron- and proton-projection operators, respectively. This equation can be rearranged in the following way:

$$H' = A(\frac{m_n + m_p}{2})c^2 + (m_n - m_p)c^2 T_z - \frac{\hbar^2(m_n + m_p)}{4m_n m_p}\sum_{i=1}^{A}\Delta_i + \frac{\hbar^2(m_n - m_p)}{4m_n m_p}\sum_{i=1}^{A}\tau_z^i\Delta_i\,. \tag{3}$$

The difference of the proton and neutron mass, $m_n - m_p = 786$ keV can be a source of the isospin admixture, however three first terms are isoscalars and therefore do not mix the states with different isospins, though the 2-nd term can cause the energy shift of the states proportionally to T_z. The fourth term — a pseudovector — mixes the states with different isospins — however, is rather small due to the coefficient $(m_n - m_p)/4m_n m_p$, and might be neglected. The Coulomb term $V_{\rm coul}$ gives a more significant isospin admixture:

$$V_{\rm coul} = \sum_{i<k}^{A} e^2 \frac{(1-\tau_z^i)(1-\tau_z^k)}{4r_{ik}}. \tag{4}$$

This relation can be rewritten in the following form:

$$V_{\rm coul} = \sum_{i<k}^{A} \frac{e^2}{4r_{ik}} \left[1 + \tfrac{1}{3}\tau^i\tau^k\right] - \left[\tau_z^i + \tau_z^k\right] + \left[\tau_z^i\tau_z^k - \tfrac{1}{3}\tau^i\tau^k\right]. \tag{5}$$

The 1-st term — isoscalar, does not mix the states with different T. The 2-nd term — isovector can mix the states with $\Delta T = 0, \pm 1$ (with the the exception of $0 \to 0$ transitions). The 3-rd term - isotensor, can mix the states with $\Delta T = 0, \pm 1, \pm 2$ (with the the exception of $0 \to 0$ and $0 \to 1$ transitions). This term introduces the shift proportional to $[T_z^2 - (1/3)T(T-1)]$. Thus, the Coulomb interaction is the main reason causing the mixing of the states with different isospins. The generalized Pauli principle requires that the wave function of the nucleus has to be antisymmetric with respect to the exchange of the spatial, spin and isospin coordinates of any two nucleons. In order to study the consequences of the exclusion principle one can consider a system of 2 nucleons. For such a system the relative motion can be characterized by the orbital angular momentum l and by the total spin S, which can take the value 0 (singlet states: ${}^1s, {}^1p, \ldots$) or 1 (triplet states: ${}^3s, {}^3p, \ldots$). For a system of 2 nucleons, the isospins can couple to the resultant $T = 0$ and $T = 1$ (due to the Pauli principle $S = 0$ implies $T = 1$ and $S = 1$ implies $T = 0$). The space-spin antisymmetric states are the components of the $T = 1$ triplet, while the space-spin symmetric states have $T = 0$. Considering the total wave function of the p–n system one can expect that the state $T = 0$, $T_z = 0$ admixes the state $T = 1$, $T_z = 0$. This admixtures is appearing due to the difference between p and n wave function. The number of possible many-particle states increases dramatically with the number of nucleons. In order to reduce this number one can describe the nucleus as consisting of a core, in which the neutrons and protons fill up several major shells, and valence nucleons are outside of the core. The core particles are assumed to couple to total angular momentum $J = 0$ and to isospin $T_z^{\rm core} = |\ (1/2)(N-Z)\ |$.

In order to consider isospin one can define the effective shell-model residual interaction appropriate to two-body spectra as [11]:

$$H = \sum_{k\tau} \frac{(2\tau+1)\sqrt{((2j_1+1)(2j_2+1)*k)}}{1+\delta(1,2)} \alpha_{k\tau} * (u^{k\tau}(j_1t_1) * u^{k\tau}(j_2t_2)), \tag{6}$$

where $u^{k\tau}$ are the diagonal unit tensors, while $t_1 = t_2 = 1/2$; and $\tau = 0$ or 1 denotes the tensorial rank in the isospin space. The $\alpha_{k\tau}$ coefficient represents the strength of the k-th component of the interaction. The $\delta = 0$ for $j_1 \neq j_2$ and $\delta = 1$ for $j_1 = j_2$. Using the wave function for two nucleons outside the core, which occupy states $(lj)_1$ and $(lj)_2$, one obtains from perturbation theory the binding energy of the nucleus:

$$E_{JT} = E^{J'T'}_{\text{core}} + e_{(lj)_1} + e_{(lj)_2} + E_{J''T''}((lj)_1, (lj)_2) + E(\text{Coulomb}), \tag{7}$$

- where the 1-st term stands for the binding energy of the core;
- the 2-nd term is the sum of the single-particle energies of the two valence nucleons with respect to the core;
- the 3-rd term is the interaction energy of the two valence nucleons, responsible for the energy splitting of the states (forming the multiplets). This term represents the eigenvalue of the shell-model residual interaction (6) ;
- the last term represent the Coulomb energy contribution to the binding energy.

Often several states with the same total angular momentum J and total isospin T have close lying energies. In such a case the residual interaction leads to the mutual interaction between these close lying states and the wave functions corresponding to different configuration mix with each other. Most of the cases for which two-body spectra are available, involve orbits in which it is specified which is the neutron and which the proton, and therefore the isospin of the pair is mixed. For the particle-particle (p–n) multiplets corresponding to nucleon pairs having mixed isospin one can use the simplified (without Coulomb energy contribution) relation:

$$E_J(pn) = \tfrac{1}{2}[1+\delta(1,2)](E_{J,T=0} + E_{J,T=1}). \tag{8}$$

The excitation energies in the multiplets can be calculated [11], using the δ-function force with spin-exchange term for $E_{J,T=0}$ and $E_{J,T=1}$:

$$E_{J,T=0} = \frac{Q}{1+\delta(1,2)}(2j_1+1)(2j_2+1)$$
$$* \left(\tfrac{1}{2}(1+N)\begin{pmatrix} j_1 j_2 J \\ \frac{1}{2}\frac{1}{2}0 \end{pmatrix}^2 + \begin{pmatrix} j_1 j_2 J \\ \frac{1}{2}\frac{1}{2}-1 \end{pmatrix}^2 \right), \tag{9}$$

$$E_{J,T=1} = \frac{Q(1-2\alpha)}{1+\delta(1,2)}(2j_1+1)(2j_2+1) * \tfrac{1}{2}(1-N)\begin{pmatrix} j_1 j_2 J \\ \frac{1}{2}\frac{1}{2}0 \end{pmatrix}^2, \tag{10}$$

where $N = (-1)^{j_1+j_2-J}$ and $\alpha = 0.2$ — parameter appearing in spin-exchange formula;

$$Q = \frac{1}{4\pi}\int_0^\infty R^2_{(nlj)_1} R^2_{(nlj)_2} r^2 dr\,.$$

The behaviour of the excitation energy $E_{J,T}$ as a function of the spin J of the 2-particle states is very characterstic when presented as a function of $\Theta_{1,2}$, *i.e.* the angle between the angular momentum vectors j_1 and j_2 [11].

For large Θ the orbits of two interacting particles, moving in oppsite directions have a large overlap, thus the interaction will be strong. For $\Theta_{1,2} \approx 90°$, the particles have small overlap, which result in a weak interaction. For smaller angles $\Theta_{1,2}$ the Pauli principle becomes important. For $\Theta_{1,2} = 0$ and $j_1 = j_2$, one must distinguish the two possibilities of isospin coupling, *i.e.* $T = 0$ and $T = 1$. For general case, *i.e.* j_1 and j_2 arbitrary, it is decisive whether $j_1 + j_2 + J$=even ($T = 0$, odd J) or $j_1 + j_2 + J$=odd ($T = 1$, even J).

4. Excitation energy of *p–n* multiplets

The experimental spectra for the $\pi g^{-1}_{9/2} \nu d_{5/2}$ (gd) in ^{102,104,106,108}In and for $\pi g^{-1}_{9/2} \nu g_{7/2}$ (gg) multiplets in ^{104,106,108}In nuclei are compared with a δ-function force calculation (Figs 1 and 2). The spacing of the states within mixed isospin components are reproduced more or less correctly in the ^{102}In and ^{104}In (gd) and in the ^{104}In, ^{106}In and ^{108}In (gg) nuclei. One can see that the nearly antiparallel position of j's (with $J = 2$), gives the strongest interaction and highest energy and that the nearly parallel orientation is also execeptionally high. A similar behaviour of excitation energy as a function of Θ (or J) is observed for odd-odd ^{106}Sb and ^{108}Sb (gd) (Fig. 3).

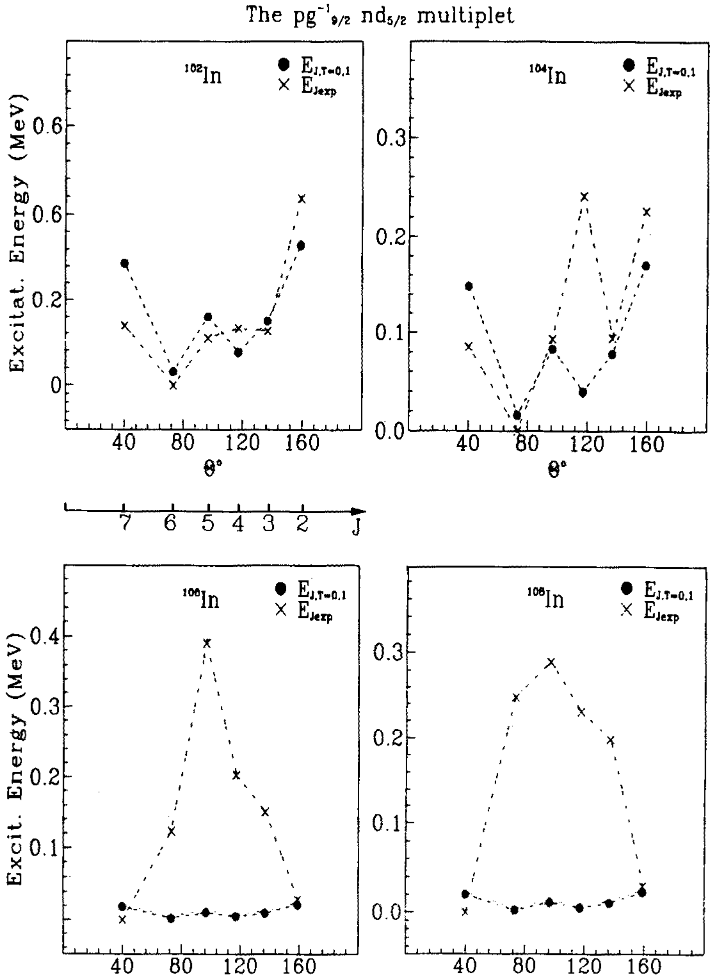

Fig. 1. The experimental spectra for the $\pi g_{9/2}^{-1}\nu d_{5/2}$ multiplet in ^{102,104,106,108}In [12, 13] as a function of angle between the angular momenta of the proton and neutron, compared with a calculation with a δ-function force. The calculation was normalized to the value of the average two-body energy. In ^{102}In the experimental energies are adopted from the shell model calculation with the exception of ground and first excited states.

For the configuration (gd) the shape of energy splitting changes from an open-up parabola in ^{102}In, ^{104}In, ^{106}Sb and ^{108}Sb to an open-down parabola in ^{106}In and ^{108}In. As it was pointed out [12] the inversion can probably be explained as a consequence of the change in the occupation probability

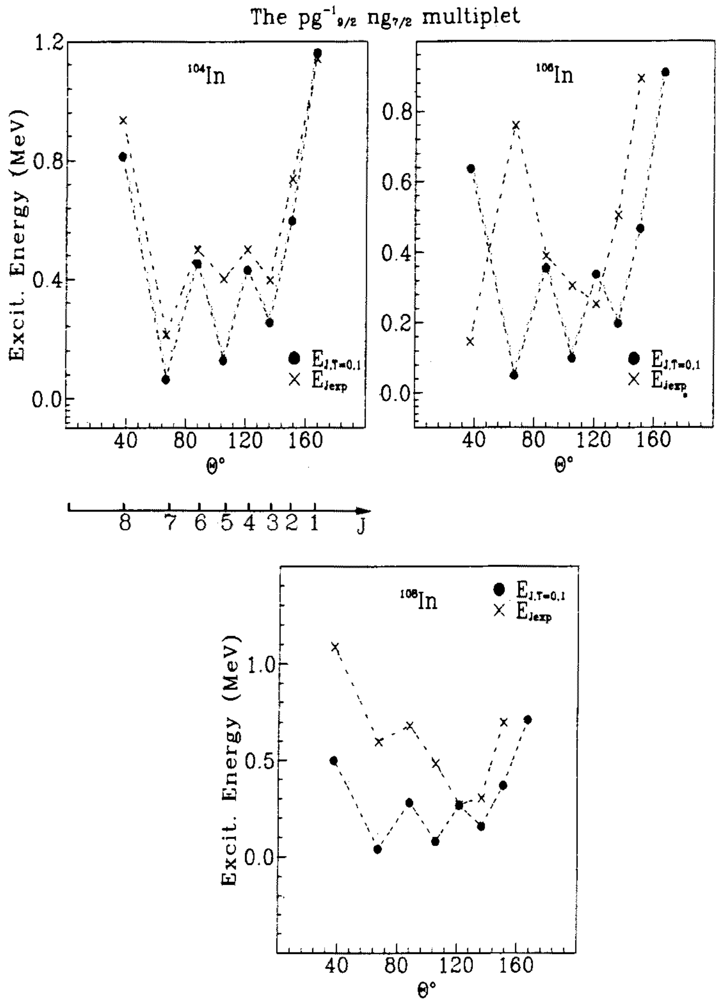

Fig. 2. The experimental spectra for the $\pi g_{9/2}^{-1} \nu g_{7/2}$ multiplet in ^{104,106,108}In [12, 13] as a function of angle between the angular momenta of the proton and neutron, compared with a calculation using a δ-function force. The calculation was normalized to the value of the average two-body energy.

of the $d_{5/2}$ and $g_{7/2}$ orbits. (The further discussion of the observed inversion is under the preparation.) Near the closed shells (or subshell closures), where the core polarization interaction is not strong, the short range interactions may play more important role. A definite energy staggering is observed in considered here multiplets. The p-n multiplets in T_z=1 ,^{102}In and ^{106}Sb and in T_z=3, ^{104}In and ^{108}Sb nuclei for $\pi g_{9/2}^{-1} \nu d_{5/2}$ configuration

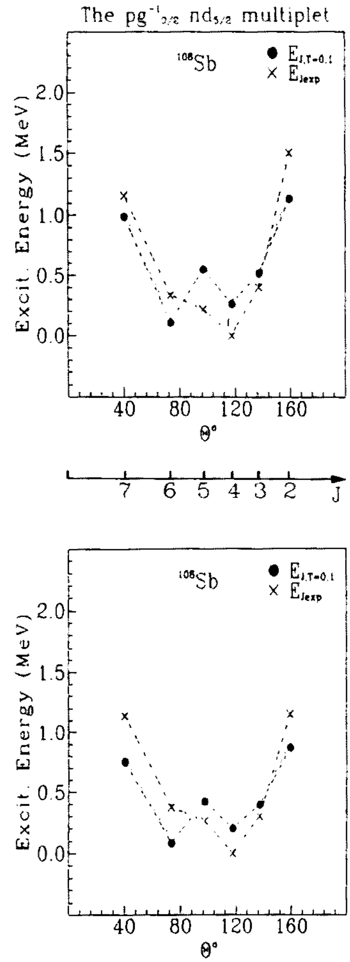

Fig. 3. The experimental spectra for the $\pi g_{9/2}^{-1}\nu d_{5/2}$ multiplet in ^{106,108}Sb [7, 13] as a function of angle between the angular momenta of the proton and neutron, compared with a calculation using a δ-function force. The calculation was normalized to the value of the average two-body energy.

appear to be very similar. The values of excitation energies calculated with a short-range force for mixed isospin follow the observed trend quite well. However, the isospin composition of the members of multiplets is experimentally still unknown. Concluding, — using the ^{58}Ni projectiles and the ^{54}Fe and ^{50}Cr targets, apart from the information concerning above considered isospin-dependent multiplets, a spectroscopic information has been gathered on isotopes in the vicinity of ^{100}Sn nucleus *e.g.* [7, 13]. When moving away from the ^{100}Sn core, collective degrees of freedom play an increasingly important role. In light Te isotopes the regular sequences of vibrational states and quasi-rotational bands have been revealed. A rotational band based on $\pi g_{7/2} \otimes \nu g_{7/2}$ intruder configuration was also found in ^{108}Sb. The above phenomena indicate that nuclei in the region around ^{100}Sn are spherical in their ground states and that collectivity set-in when moving away from double magic nucleus.

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