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TRIUMPHS AND FAILURES OF QUANTUM ELECTRODYNAMICS*

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Quantum electrodynamics, after more than sixty years since its discovery, still presents challenges and offers rewards to inquiring minds. This presentation describes some theoretical intricacies of this beautiful theory.

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1. Introduction

The whole information about the external world comes to us via electromagnetic interactions and most of the physics at the human scale is dominated by electromagnetic phenomena and is describable, at least in principle, by quantum electrodynamics. Only the nuclear and subnuclear phenomena that dominate at very small distances and the phenomenon of gravity that dominates at the other end of the scale do not fall into the range of applicability of quantum electrodynamics. The theoretical applicability, however, does not translate directly into practical applications since the description of realistic experiments involving charged particles within the framework of quantum electrodynamics is exceedingly complicated. Moreover, the simplifications commonly employed in ordinary quantum mechanics lead very often to meaningless conclusions. As a result, highly accurate predictions of quantum electrodynamics that made this theory so famous are limited almost exclusively to calculations of energy spectra. All other predictions are tainted with the uncertainties in the description of the experimental setup and the necessary approximations in the theoretical analysis.

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Due to high complexity and sophistication, full and direct application of quantum electrodynamics to atomic and molecular physics, quantum optics, solid state and condensed matter physics and plasma physics is not possible. Various approximate models developed to describe all these branches of physics have not, so far, been systematically derived from the first principles of quantum electrodynamics. In my opinion, the main reason for that failure is the lack of a consistent canonical Hamiltonian description in quantum electrodynamics. Such a description forms the foundation of all approximate models but in full quantum electrodynamics a combination of infrared and ultraviolet divergences makes an effective construction of a Hamiltonian operator virtually impossible. This construction is hampered by the following problems.

Operators in any quantum theory are defined by their action on families of state vectors whose members are labeled with some "quantum numbers". The paradigm of such labeling is the position representation in quantum mechanics. All operators in the Schrödinger wave mechanics are usually defined by their action on the wave function in the position representation. Unfortunately, all the traditional labelings of state vectors do not work, in general, in quantum electrodynamics. They can be used systematically only in the relativistic theory of *noninteracting* particles. While for short range interactions a labeling based on the quantum numbers of free particles is often very useful, the infinite range of electromagnetic interactions introduces serious difficulties. A moving charge always feels the Coulomb force from all other charges, its motion is accelerated and it keeps radiating photons all the time. This is the physical cause of the infrared divergences. In actual calculations they make their appearance in the following way. The probability for a scattered charge to emit any finite number of photons is zero of the form: $\exp(-e^2\infty)$. A perturbative expansion of this expression in e^2 gives infinite results in each order. To obtain a meaningful result one has to include an infinite number of photon emissions but this, in turn, leads to a cumbersome formalism in which the states of the system must be described by density operators. Density operators always appear when one does not distinguish between states with a different content of very soft photons. Such states cannot be distinguished with a laboratory equipment of finite extent and in an experiment of finite duration. Mixed states must be used since the information about very soft photons is always missing and every loss of information calls for density operators. Mathematically, one may distinguish all states with a different content of soft photons but the collection of appropriate state vectors is nondenumerable; these vectors cannot be accommodated in a standard Hilbert space with a denumerable basis.

Of course, there are no infrared divergences in the real world; they are introduced by abstractions — by the assumed infinite extension of scattering processes in space and their infinite duration. These abstractions, however, are crucial for setting up the S-matrix formalism and it is only within this formalism that one can avoid the ultraviolet divergences. Transition amplitudes for finite times cannot be renormalized. The physical reason for this difficulty is the impossibility to localize events in time. A sharp time variable is again a convenient abstraction but it leads to infinite expressions. A smeared, diffused time would be more realistic but it would lead to severe complications.

An additional source of difficulties is the requirement of local charge conservation and gauge invariance. They are closely connected: gauge invariance implies local charge conservation. It is very convenient to introduce, as the main theoretical tool in quantum field theory, the particle propagators. Many formulations of quantum field theory (Feynman path-integral, Dyson-Schwinger formulation, LSZ formalism) use the propagators as the main tool. However, the very notion of an electron propagator in quantum electrodynamics violates charge conservation. The function of spacetime variables, called the electron propagator, describes in essence the probability amplitude for a process in which an electron originates at a certain spacetime point x and ends at a spacetime point y. Such processes obviosly violate local charge conservation; an electron cannot appear from nowhere. In reality, electrons are only separated from atoms, molecules, heated wires, etc., and are reunited again with positive charges after the completion of an experiment. In a realistic description of experiments problems with charge conservation and gauge invariance can be avoided but the formalism becomes very complicated.

What are then the quantities that can be calculated in quantum electrodynamics with high accuracy without any ambiguities? In addition to the energy spectrum that has already been mentioned, there are also transition amplitudes for pure photon processes. If there are no charged particles present, infrared divergences do not occur since there are no Coulomb forces and the S-matrix limit does exist. A very simple process of this type is a photon-photon scattering. Its scattering cross section can be precisely calculated but it is so small that it has, so far, not been directly measured. There is an even simpler process of the pure photon category, namely the one that does not involve any photons. This might be the vacuum to vacuum transition or a transition involving any other stable state, for example the ground state of an isolated atom. While in empty space the transition probability of such a transition is trivially equal to one, in the presence of external fields things become much more exciting. As a matter of fact, many realistic experiments can be ultimately described in terms of such transitions.

2. Realistic description of an experiment

To explain how realistic treatment evades difficulties, let us consider an operational description of electron scattering experiments. As I have already discussed, a simplified picture of the scattering process involving an electron, prepared in a state with a fixed momentum and observed in the final state also with a well defined momentum, leads to infrared divergences. This simplification has gone too far! In a realistic situation, the electron is not alone — its preparation and detection is necessarily accompanied by emission of soft photons. In order to prepare a beam of electrons we might, for example take a piece of metal and apply a pulse of electric field to detach electrons and form a nearly monochromatic beam. The theoretical description of such a preparation process is simple, at least in principle. Let us denote the initial state of the metal by $|M\rangle$. For simplicity, I shall assume that it is a stationary ground state. It might be visualized, for example, as a shallow potential well produced by a distribution of positive charges. It must be shallow to reduce the momentum uncertainty of the initial electrons resulting in a small momentum spread in the beam. The state of the system after the application of the pulse is given by $U[V]\,|M\rangle$. The unitary evolution operator $U[V]$ describes the evolution of the system in the presence of a uniform, but time dependent, electric field $\vec{E}(t)$ represented by a scalar potential $V(\vec{r},t) = -\vec{r}\cdot\vec{E}(t)$. One can write the following formula for the evolution operator in the interaction picture (also known as the Furry picture) with respect to the external field

$$U[V] = T\exp\left[(-i/\hbar)\int dt \int d^3r\, j^0(\vec{r},t)V(\vec{r},t)\right] . \tag{1}$$

In this picture, the state vector changes only on account of the applied field. All remaining interactions are included in the evolution of the operators. The operator appearing under the time integral in the exponent of (1) can also be written in a more transparent form as $-\vec{d}\cdot\vec{E}$, where the dipole moment operator $\vec{d}$ is defined as

$$\vec{d}(t) = \int d^3r\, \vec{r} j^0(\vec{r},t) , \tag{2}$$

confirming the fact that a uniform electric field couples to the total electric dipole moment of the system. The state vector $U[V]\,|M\rangle$ is a good candidate to describe a beam of electrons with a well-defined momentum $\vec{p}$ when the electric field is suddenly turned on and off, *i.e.* when the electric field is of the form $\vec{E} = \vec{p}/ef(t - t_0)$, where $f(t)$ is a peaked function. In a similar fashion, one can describe the state of the electrons that are being detected by the measuring device. In between the state preparation and detection we will have, in general, some interaction, say with an external potential, described by a unitary operator $U\left[V_{\text{int}}\right]$. Thus, the full transition amplitude A_{if} will have the form

$$A_{if} = \langle M'|U\left[V_f\right]\, U\left[V_{\text{int}}\right]\, U\left[V_i\right]|M\rangle\,, \tag{3}$$

where V_i and V_f refer to different values of the electric field and, therefore, to different electron momenta. It is worth noting that the only operators sandwiched between two state vectors $|M\rangle$ and $|M'\rangle$ are the electromagnetic current operators. Such expressions describe pure photon processes (current operator can be defined as the source of the electromagnetic field) and are always free of infrared divergences. All possible emissions of soft photons, accompanying the acceleration of electrons by the action of the electric field, are included. There is no information about the soft-photon emissions in the matrix element (3) and charged particles never travel to infinity as in the standard S-matrix formulation. Also, the ultraviolet divergences are avoided by smearing out the interaction in time, due to presence of the function $f(t)$. The description of the electron scattering in this manner is realistic and free from divergences but prohibitively complicated. These difficulties are due to the fact that we cannot use perturbation expansion to evaluate the initial and final state vectors. What is always done in practice is to forget about realism and apply a great dose of abstraction. The initial and the final states are described as eigenstates of momenta with all the dire consequences that I have mentioned. Let us note one more attractive feature of the formalism based on the formulas like (3) for the scattering amplitude. The states are labeled with the help of macroscopically controlled parameters: the form of the external electromagnetic field. This description of the states of the system is very close to the Bohr's idea that the preparation and detection processes should be described in terms of macroscopic quantities.

3. Quantum electrodynamics — science and art

Extracting experimentally measurable quantities from theoretical constructs, based on many simplifications, one must be extremely careful. Theoretical physics of quantum electrodynamics is both science and art. A great deal of experience is necessary to avoid pitfalls. Still, as I have said in the Introduction, not too many processes and characteristics involving charged particles and photons can be handled in a precise manner in quantum electrodynamics. On the other hand, not too many results are known from experiments with very high precision. The best known and the easiest to calculate (at least conceptually) are the energy spectra. It is the fundamental tenet of quantum theory that the energy spectrum can be obtained by the Fourier analysis of the time evolution. In this manner one can avoid one of the most intricate problems in quantum electrodynamics: the precise identification of the states of the system. In other words, one may use a rather crude description of the state of the system and still obtain a highly accurate value for the energy. The main idea is the following. Let us consider matrix elements of the form $\langle 0|A(t)\,B(0)|0\rangle$, where $A(t)$ and $B(t)$ are some operators in the Heisenberg picture with known general properties (quantum numbers, transformation properties). The spectrum of energies E_n can be determined from the formula

$$\langle 0|A(t)\,B(0)|0\rangle = \sum_n e^{-iE_n t}\langle 0|A|\psi_n\rangle\langle\psi_n|B|0\rangle \tag{4}$$

even when a precise identification of the states $|\psi_n\rangle$ is missing provided one knows precisely the left hand side as a function of t.

The calculation of the radiative energy shift — the Lamb shift — is a good illustration of this point. It involves a calculation of the energy of the $2P$ state of the hydrogen atom. Since this is an unstable, decaying state it is hard to give its precise characteristic; we do not have a convenient labeling to describe such states. The energy of this state is, however, almost unambiguously defined within the formalism of quantum electrodynamics because the lifetime of this state is very long on the QED scale. Calculation of other quantities is not so straightforward. For example, the lifetime of positronium is not very long on the QED scale. Here the ambiguities in the proper definition of the states of positronium lead to much larger uncertainties. It is much harder to make a distinction between the undecayed state of positronium and the states of its decay products.

In conclusion, it is conceptually quite simple to perform perturbative calculations based on Feynman diagrams in quantum electrodynamics. After renormalization one obtains well defined functions and numbers. The hard task is to connect these functions and numbers with realistic experiments and to understand the limitations of our abstractions. Many detailed calculations of a few selected processes have been made but we are far from understanding the relation between the theory and the real world. It might also be true that some of the puzzles connected with the interpretation of quantum theory might be resolved by a more careful analysis of the relation between the theoretically calculated and the measured quantities.

The same is true, to an even larger extent, in more complicated field theories. Quantum chromodynamics is a good case in point. Here the relation to the real world is much more obscure since there is no understanding, whatsoever, how physical particles are made from the fundamental building blocks — quarks and gluons.