CAVITY QED*

ZOFIA BIAŁYNICKA-BIRULA

Institute of Physics, Polish Academy of Sciences
Al. Lotników 32/46, 02-668 Warsaw, Poland

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The presence of macroscopic objects, like mirrors or cavity walls, changes the mode structure of the electromagnetic field. As a consequence, the vacuum fluctuations are also changed and they become position-dependent. This effect manifests itself in the appearance of the Casimir force, Casimir-Polder force, position-dependent energy shift and modified spontaneous emission. Also, the measurement of the electron magnetic moment is influenced by the cavity formed by the electrodes of the Penning trap.

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1. Introduction

Cavity QED is a theory of electromagnetic interactions between charged particles and electromagnetic fields in a presence of some macroscopic bodies, like mirrors (metallic or dielectric), cavity walls or waveguides. In this theory, the electromagnetic radiation field is quantized and the charges (usually, the electrons that are free or bound in atoms or molecules) are described according to nonrelativistic quantum mechanics. Macroscopic dielectric or metallic bodies are taken into account by assuming proper boundary conditions that should be satisfied at their surfaces by electromagnetic fields.

The most striking features of quantum electrodynamics (QED) are the properties of vacuum. In the process of electromagnetic field quantization the notion of vacuum is redefined. QED vacuum is no longer the state devoid of any fields. It is a state of the lowest energy with all field expectation values equal to zero. Nevertheless, the nonvanishing field fluctuations in the vacuum are the source of observable effects. These fluctuations depend on

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the boundary conditions. In the presence of macroscopic objects, called mirrors in what follows, the values of these fluctuations are position-dependent since they depend on the configuration of such mirrors. Moreover, in the presence of mirrors the vacuum state does not have the additional properties that are usually satisfied in a free space. Namely, one cannot demand that the vacuum is invariant under the transformations of the Poincaré group, since the homogeneity and isotropy of space-time are already destroyed by the presence of mirrors. As a consequence, the energy, the momentum, and the angular momentum of the vacuum are not necessarily equal to zero and moreover, they depend on the configuration of mirrors. The nonvanishing of the vacuum energy leads to the appearance of the Casimir force between uncharged macroscopic bodies.

In my lecture I will restrict myself to the discussion of those effects that are strongly influenced by the properties of the vacuum.

2. Preliminaries

According to quantum electrodynamics [1] vectors representing electric field and magnetic induction, $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$, are operators. These operators may be decomposed into any complete, orthonormal set of complex solutions of the Maxwell equations,

$$\vec{E}(\vec{r},t) = \sum_i \left[\vec{\mathcal{E}}^i(\vec{r},t)a_i + \vec{\mathcal{E}}^{i*}(\vec{r},t)a_i^\dagger\right], \tag{1}$$

$$\vec{B}(\vec{r},t) = \sum_i \left[\vec{\mathcal{B}}^i(\vec{r},t)a_i + \vec{\mathcal{B}}^{i*}(\vec{r},t)a_i^\dagger\right], \tag{2}$$

where all pairs of vectors $\vec{\mathcal{E}}^i(\vec{r},t)$ and $\vec{\mathcal{B}}^i(\vec{r},t)$ (and also $\vec{\mathcal{E}}^{i*}(\vec{r},t)$ and $\vec{\mathcal{B}}^{i*}(\vec{r},t)$) form c-number solutions of the Maxwell equations. The members of the complete set are labelled here by an index i.

As a consequence of the canonical commutation relations for field operators, (where $\vec{D}(\vec{r},t)$ is the operator of an electric induction),

$$\left[B_n(\vec{r},t), D_m(\vec{r}',t)\right] = i\hbar\epsilon_{nmk}\partial_k\delta(\vec{r}-\vec{r}'), \tag{3}$$

$$\left[D_n(\vec{r},t), D_m(\vec{r}',t)\right] = 0 = \left[B_n(\vec{r},t), B_m(\vec{r}',t)\right], \tag{4}$$

the operators a_i and $a_i^\dagger$ satisfy the bosonic commutation relations,

$$\left[a_i, a_j^\dagger\right] = \delta_{ij}, \tag{5}$$

$$\left[a_i, a_j\right] = 0 = \left[a_i^\dagger, a_j^\dagger\right]. \tag{6}$$

The operators a_i and $a_i^\dagger$ are called annihilation operators and creation operators, respectively.

If the mirrors are at rest, it is convenient to choose harmonic functions of time as mode functions $\vec{\mathcal{E}}^i(\vec{r},t)$ and $\vec{\mathcal{B}}^i(\vec{r},t)$,

$$\vec{\mathcal{E}}^i(\vec{r},t) = e^{-i\omega_i t}\vec{\mathcal{E}}^i(\vec{r})\,, \tag{7}$$

$$\vec{\mathcal{B}}^i(\vec{r},t) = e^{-i\omega_i t}\vec{\mathcal{B}}^i(\vec{r})\,. \tag{8}$$

In such a case, the energy operator (Hamiltonian) for the electromagnetic field has the form,

$$H = \hbar\sum_i \omega_i(a_i^\dagger a_i + \tfrac{1}{2})\,. \tag{9}$$

The vector functions $\vec{\mathcal{E}}^i(\vec{r})$ and $\vec{\mathcal{B}}^i(\vec{r})$ should satisfy the proper boundary conditions at the surfaces of the mirrors. For example, in the space bounded by two flat, parallel, infinite, metallic mirrors the proper mode functions have the form,

$$\vec{\mathcal{E}}^E_{\vec{\kappa},n}(\vec{r}) = \mathcal{N}^E \sin(k_z z)\hat{\kappa}\times\hat{z}\,e^{i\vec{\kappa}\cdot\vec{\rho}}\,, \tag{10}$$

$$\vec{\mathcal{B}}^E_{\vec{\kappa},n}(\vec{r}) = \mathcal{N}^E(\omega_{\vec{\kappa},n})^{-1}\left[-ik_z\cos(k_z z)\hat{\kappa} - \kappa\sin(k_z z)\hat{z}\right]e^{i\vec{\kappa}\cdot\vec{\rho}}\,, \tag{11}$$

$$\vec{\mathcal{E}}^M_{\vec{\kappa},n}(\vec{r}) = \mathcal{N}^M c(\omega_{\vec{\kappa},n})^{-1}\left[-ik_z\sin(k_z z)\hat{\kappa} + \kappa\cos(k_z z)\hat{z}\right]e^{i\vec{\kappa}\cdot\vec{\rho}}\,, \tag{12}$$

$$\vec{\mathcal{B}}^M_{\vec{\kappa},n}(\vec{r}) = \mathcal{N}^M c\cos(k_z z)\hat{\kappa}\times\hat{z}\,e^{i\vec{\kappa}\cdot\vec{\rho}}\,. \tag{13}$$

Here, the mode functions are labelled by a two-dimensional wave-vector $\vec{\kappa}$ lying in the plane of the mirrors and by a natural number n, since the component of the wave-vector in the direction z perpendicular to the mirrors is quantized,

$$k_z = n\pi/L\,. \tag{14}$$

The distance between mirrors is denoted by L. There are two sets of modes: transverse electric modes labelled by a superscript E and transverse magnetic modes labelled by a superscript M. The characteristic frequencies are

$$\omega_{\vec{\kappa},n} = c\sqrt{\vec{\kappa}^2 + (n\pi/L)^2}\,. \tag{15}$$

$\mathcal{N}^E$ and $\mathcal{N}^M$ are the normalization constants.

The annihilation and creation operators act in the Hilbert space of state vectors. The vacuum state is defined as a state represented by a vector $|0\rangle$ that is annihilated by every annihilation operator a_i,

$$a_i|0\rangle = 0\,. \tag{16}$$

The vacuum expectation values of all components of electric and magnetic fields vanish,

$$\langle 0|\vec{E}(\vec{r},t)|0\rangle = 0 = \langle 0|\vec{B}(\vec{r},t)|0\rangle . \tag{17}$$

However, the vacuum expectation values of the quadratic forms built from the components of electromagnetic field are, in general, different from zero,

$$\langle 0|E_n(\vec{r},t)E_m(\vec{r}',t)|0\rangle = \sum_i \mathcal{E}_n^i(\vec{r})\mathcal{E}_m^{i*}(\vec{r}') , \tag{18}$$

and they express the vacuum fluctuations. Vacuum fluctuations depend on the boundary conditions, since the mode functions have to satisfy those conditions. In the presence of mirrors, the vacuum fluctuations are position-dependent, contrary to the case of a free infinite space.

3. Casimir force

In 1948 Casimir [2] calculated the force that acts between two plane, parallel, infinite, perfectly conducting mirrors. He showed that it is an attractive force directed perpendicularly to the plane of the mirrors. It results from a change in the vacuum energy (the zero point energy) caused by a change of the boundary conditions as compared to free space.

It follows from (9) that the vacuum energy may be expressed as half of the sum of all eigenfrequencies multiplied by the Planck's constant,

$$\langle 0|H|0\rangle = \hbar/2 \sum_i \omega_i . \tag{19}$$

In the case of two parallel mirrors, it depends on the separation L between the mirrors. Although the expression (19) is infinite, the force acting on the unit of the surface is finite. The force was calculated as a gradient of the difference ΔE between the vacuum energies with the mirrors and in free space,

$$\vec{F} = \nabla \Delta E/\mathcal{A} . \tag{20}$$

The Casimir force, when expressed in terms of fundamental constants and the mirror separation L, has the form,

$$F = \frac{\pi^2}{240}\frac{\hbar c}{L^4} = 1.3 \times 10^{-27} N m^2/L^4 . \tag{21}$$

The Casimir force is, of course, very weak. For $L = 1\mu m$, it is equal to $1.3 \times 10^{-3} N/m^2$. Nevertheless, there were several attempts to measure the Casimir force, starting by a Russian experiment on the dielectric samples performed by Deryagin and Abrikosova [3] that gave a reasonable qualitative agreement with the Casimir expression (21). Later measurements are described in a review publication edited by Levin and Miche [4]. Recently, Onofrio and Carugno [5] proposed to use a tunneling electromechanical transducer to detect Casimir forces between two conducting surfaces separated by 1μm and less (down to 0.1μm). They plan to measure the modulations of the Casimir force caused by small oscillations of both surfaces.

4. The Casimir-polder force and the energy shift

An attractive force, similar to the Casimir force, acts between an atom (or a molecule) and the metallic mirror and also between two atoms. Casimir and Polder [6] predicted this force which results from the position-dependent shift of the atomic energies. They calculated the retardation corrections to the static van der Waals forces. Thus, Casimir-Polder force is closely related to the energy shifts, that are, in turn, caused by the vacuum fluctuations.

The shifts of eigenenergies of an atom placed between two parallel, perfectly conducting mirrors were calculated by Barton [7] in the lowest order of perturbation theory. He used the nonrelativistic Hamiltonian,

$$H = H_0 + H_I\,, \tag{22}$$

where H_0 denotes the energy of a free atom,

$$H_0 = \frac{p^2}{2m} + V(r)\,, \tag{23}$$

$$H_0|i\rangle = E_i|i\rangle\,, \tag{24}$$

and H_I denotes the interaction Hamiltonian, which follows from the minimal electromagnetic coupling in the dipole approximation,

$$H_I = H_{\rm es} - e\vec{A}(\vec{r})\cdot\vec{p}/m + e^2\vec{A}^2(\vec{r})/2m\,. \tag{25}$$

Here, $H_{\rm es}$ describes electrostatic interactions, and $\vec{A}(\vec{r})$ is an operator of an electromagnetic potential taken at the position of the center of the atom. The potential $\vec{A}(\vec{r})$ satisfies proper boundary conditions at the surfaces of the mirrors.

Under the influence of the interaction between the atom and the electromagnetic field in the vacuum state, the eigenenergies E_i of the atom are shifted by δE_i. For the ground state of an alkali atom, the position-dependent part of such a shift as calculated by Barton [7] is

$$\delta E_g(z) = -\pi \sum_e \frac{\overline{|\langle e|\vec{d}|g\rangle|^2}}{6\epsilon_o L^3} \int_0^\infty d\rho \frac{\rho^2\cosh(2\pi\rho z/L)}{\sinh(\pi\rho)} \tan^{-1}\left[\frac{\rho\lambda_{eg}}{2L}\right]\,, \tag{26}$$

where the summation is over all excited states of the atom, $\overline{|\langle e|\vec{d}|g\rangle|^2}$ denotes the modulus squared of the matrix element of the dipole averaged over all directions, z is the distance of the atom from the center of the two-mirror cavity, L is the distance between the mirrors and λ_{eg} is the wavelength of the $|e\rangle \rightarrow |g\rangle$ transition. For a sodium atom, almost entire (more than 98%) position-dependent shift of the ground level 3s is due to the virtual transitions to the first excited state 3p.

The Casimir-Polder shift can be recovered from the formula (26) in the limit when $L \gg \lambda_{eg}$ and the atom is not too close to the mirrors. In Fig. 1, the ground state energy shift calculated from Eq. (26) for a sodium atom placed between two mirrors separated by $1\mu m$ is depicted together with its Casimir-Polder limit and the instantaneous static van der Waals potential.

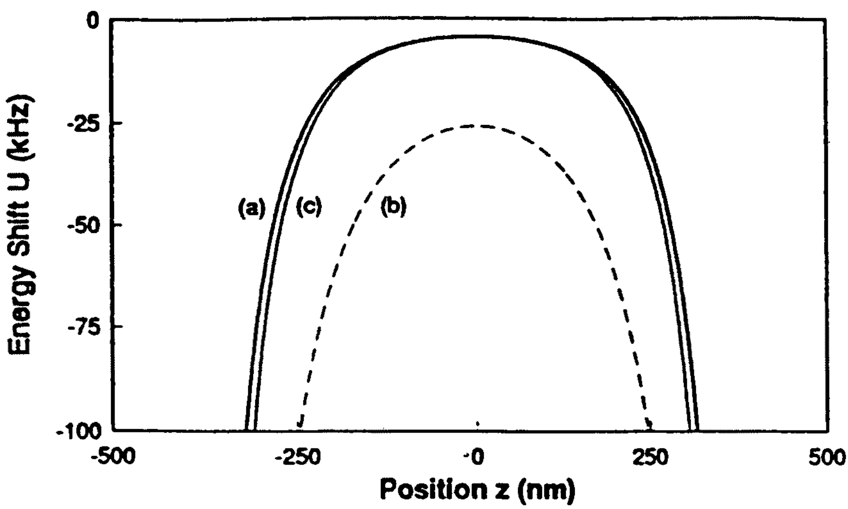

Fig. 1. Ground-state energy shift calculated for a sodium atom, for $L = 1\mu$m. (a) — energy according to Eq. (18); (b) — instantaneous van der Waals potential; (c) — asymptotic Casimir–Polder potential. From Ref. [8].

The position-dependent energy shift results in a force, given by the gradient $\nabla\delta E(z)$, that pulls the atom towards the nearest mirror. An experiment to detect this force was performed at Yale University by Sukenik *et al.* [8] with a beam of sodium atoms in their ground state, passing between two gold-platted mirrors. This experiment not only demonstrated the existence of the Casimir-Polder force, but it verified for the first time the L^{-4} dependence of the retarded QED potential and it discriminated between the van der Waals and the QED potentials. The results obtained by the Yale group are presented in Fig. 2 and Fig. 3, where the opacity of the cavity is plotted versus the cavity width. The opacity, an inverse of the relative transmission of the atomic beam, is a measure of the attractive atom-cavity forces, since those atoms that go towards the wall will stick to its surface.

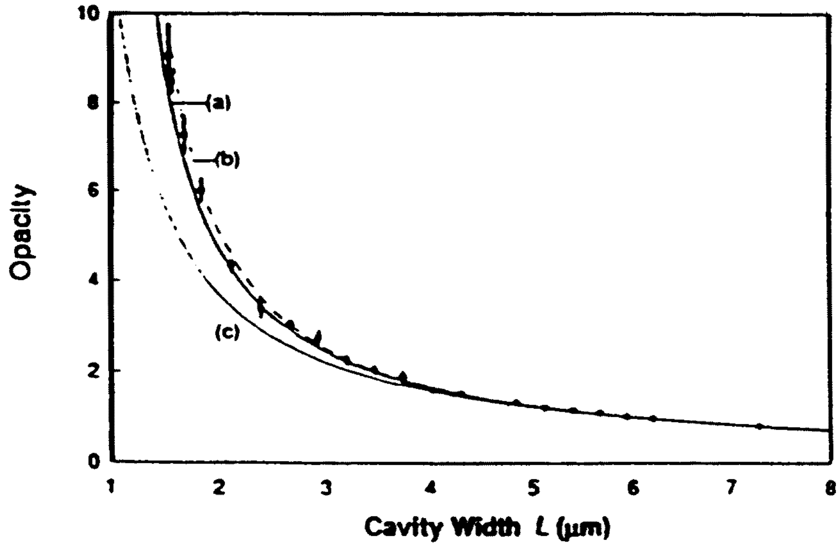

Fig. 2. Measured cavity opacity versus cavity width. (a) — QED interaction; (b) — van der Waals interaction; (c) — no interaction. From Ref. [8].

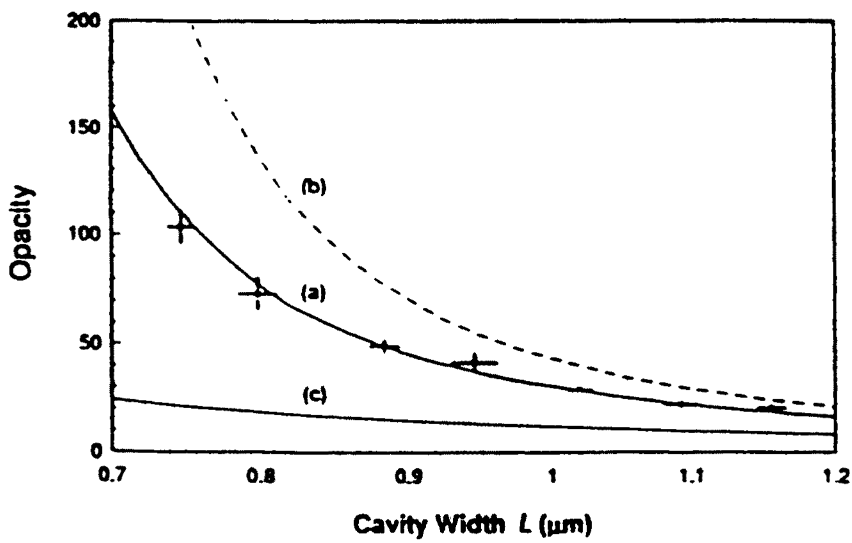

Fig. 3. Measured cavity opacity versus cavity width. (a) — QED interaction; (b) — van der Waals interaction; (c) — no interaction. From Ref. [8].

Recently, the energy shifts of highly excited circular Rydberg atoms of rubidium (such atomic states have high principal quantum number n and m = n-1) with n = 50 were directly measured by Brune *et al.* [9] at l'Ecole Normal Supérieure. The beam of long living Rydberg circular atoms was passing through a microwave cavity made of two superconducting niobium spherical mirrors placed at $L = 2.754$ cm from each other. The atoms experienced also, before entering the cavity and after leaving it, two successive synchronized pulses of resonant microwave radiation. The first pulse produces the coherent superposition of n = 50 and n = 51 states. During the passage through the cavity those component states acquire different energy shifts which later result in the shift of the Ramsey fringe signal produced by the second microwave pulse. This Ramsey interferometry technique is very sensitive and allows for a precise measurement of the energy shift practically without any energy exchange.

5. Spontaneous emission in a cavity

The spontaneous emission of radiation, that accompanies transitions between energy levels in atoms and molecules, is always induced by vacuum fluctuations. Therefore, it may be significantly influenced by the changes in the boundary conditions caused by the presence of mirrors or cavities. This effect for radio-wave frequencies was predicted by Purcell [10] in 1946. The first experimental observation of a modified spontaneous decay of excited molecules placed on the surface of a mirror was performed by Drexhage [11]. Later, for microwave frequencies, an enhanced spontaneous emission from highly excited Na atoms in a cavity was observed [12] and also, an inhibited spontaneous emission from circular Rydberg states of Cs atoms in a waveguide was detected [13]. For visible light, an inhibited spontaneous emission was first observed for Cs atoms passing between two parallel gold-platted mirrors [14], and soon after that both inhibited and enhanced spontaneous decay was observed [15] from Yb atoms in a confocal mirror resonator.

The rate γ of a spontaneous decay of an atomic excitation is given by the Fermi's Golden Rule,

$$\gamma = \frac{1}{\hbar^2}|\langle f|H|i\rangle|^2\rho(\omega_0)\,, \tag{27}$$

where the matrix element of the Hamiltonian is taken between the initial and the final states of the atom and $\rho(\omega_0)$ denotes the density of the photon states with frequency ω_0 corresponding to the energy separation between atomic states. This density function $\rho(\omega)$ depends on the boundary conditions. In an empty space,

$$\rho_f(\omega) = \frac{\omega^2}{\pi^2 c^3}\,. \tag{28}$$

In a lossy cavity, in the vicinity of the characteristic cavity frequency ω_c, the density of states function $\rho_c(\omega)$ may be modelled by the Lorentzian function,

$$\rho_c(\omega) = \frac{\Gamma}{\pi V} \frac{1}{\Gamma^2 + (\omega - \omega_c)^2}, \tag{29}$$

where V is the cavity volume and Γ measures the cavity losses. The rate of cavity loss Γ is related to the cavity quality factor Q,

$$Q = \omega_c / 2\Gamma . \tag{30}$$

The spontaneous decay rate γ_c in a cavity is related to the free space decay rate γ_f by the formula,

$$\gamma_c = \frac{\rho_c(\omega_0)}{\rho_f(\omega_0)} \gamma_f = \frac{\pi c^3 \Gamma}{V \omega_0^2} \frac{1}{\Gamma^2 + (\omega_0 - \omega_c)^2} \gamma_f . \tag{31}$$

The spontaneous emission in a cavity may, therefore, be enhanced or inhibited depending on the distance between the atomic transition frequency ω_0 and the nearest characteristic cavity frequency ω_c. At the resonance $(\omega_0 = \omega_c)$, we get

$$\gamma_c = \frac{1}{(2\pi)^2} \frac{\lambda^3}{V} Q \gamma_f . \tag{32}$$

In a good cavity of the dimensions of the order of the wavelength of the emitted radiation, a very strong enhancement of spontaneous emission is possible, since there exist microwave cavities with the quality factor Q of the order of 10^{10}. On the other hand, if the atomic transition frequency ω_0 is substantially smaller than the smallest cavity eigenfrequency, the spontaneous emission may be drastically inhibited. For example, for $\omega_0 = \omega_c/2$,

$$\gamma_c = \frac{1}{(4\pi)^2} \frac{\lambda^3}{V} \frac{1}{Q} \gamma_f . \tag{33}$$

To describe spontaneous emission in a very good cavity one should not use the Fermi's Golden Rule (27) which results from the lowest order of perturbation theory. The probability that the atom remains in the excited state is no longer given by a simple exponential function, but undergoes several decaying oscillations. In that case, a photon once emitted would not immediately escape from the cavity but may be reabsorbed by an atom several times before eventually leaving the cavity. Nevertheless, the Weisskopf–Wigner method [16] applied to such a case confirms the general features described above, namely, an enhancement or an inhibition depending on the detuning from the resonance condition.

Spontaneous emission from an excited atom placed in a cavity depends on the atomic position with respect to the nodes and antinodes of the cavity mode functions [17]. For moving atoms in an atomic beam it depends also on the velocity of the atom, since the Doppler shift may tune the atom in and out of cavity eigenfrequencies [18], and also because the moving atom probes the spatial variations of the cavity eigenmodes [19].

6. Magnetic moment of the electron

The detailed calculation of the magnetic moment of the electron, confirmed by an experiment, was a great success of quantum electrodynamics. According to relativistic quantum mechanics, the magnetic moment of an electron $\vec{\mu}$ is given by the formula

$$\vec{\mu} = g\frac{e}{2mc}\vec{s}\,, \tag{34}$$

where $\vec{s}$ is the electron spin, $\vec{s} = \hbar\vec{\sigma}/2$, and the gyromagnetic factor g is equal to 2. The interactions of the electron with the QED vacuum result in a change in the value of the gyromagnetic factor g. The parameter a is used as a measure of this change,

$$a = (g - 2)/2\,. \tag{35}$$

When an electron is placed in a static, uniform, homogeneous magnetic field and is induced to undergo spin flip transitions, the parameter a may be related to the difference between the cyclotron frequency ω_c, and the frequency ω_s corresponding to the spin flip,

$$a = (\omega_s - \omega_c)/\omega_c\,. \tag{36}$$

Therefore, the measurement of the magnetic moment of an electron consists of a simultaneous detection of both transition frequencies ω_c and ω_s. Modern measurements of such a kind [20] are performed on an electron kept in the Penning trap. The electrodes of a Penning trap form a microwave cavity of a very complicated shape (see Fig. 4). As every cavity, the Penning trap influences the radiative decay of the cyclotron motion of an electron. The first observation of an inhibited spontaneous emission accompanying the decay of the electron cyclotron motion was reported already in 1985 by Gabrielse and Dehmelt [21]. The slowing down of the cyclotron transitions of an electron helps to achieve an extreme accuracy in the determination of the electronic g factor according to Eq. (36) since it results in a narrowing of the spectral line. The experimental value for the parameter a obtained recently [20] is

$$a_{\rm exp} = 0.001\,159\,652\,188\,4\;(14)(40)\,, \tag{37}$$

where the uncertainties are given in two brackets. The biggest uncertainty (40×10^{-13}) is due to the unknown frequency shift caused by the cavity. For a cavity, such as a Penning trap, it is very difficult to evaluate the eigenfunctions and eigenfrequencies, even numerically. Therefore, the inevitable frequency shift cannot be calculated. This problem was already signalled by Brown *et al.* [22] in 1988 and it is not yet completely resolved.

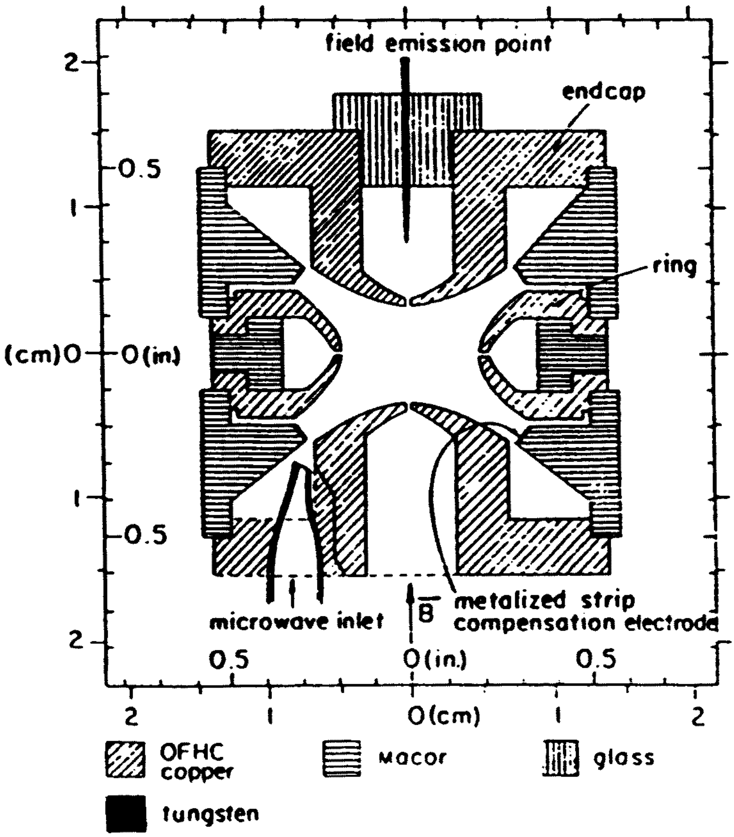

Fig. 4. Cross-section of the Penning trap. From Ref. [21].

The parameter a in free space was calculated by QED methods up to the eight order in perturbation theory by Kinoshita *et al.* [23]. They obtained the value

$$a_{\text{th}} = 0.001\,159\,652\,140\,0\ (53)(41)(271)\,, \tag{38}$$

where the biggest uncertainty is due to the uncertainty in the measurement of the fine structure constant from the Josephson effect.

The magnetic moment of the electron is one of the most accurately known quantities in physics. Its measured value may be used for the determination of the fine structure constant.

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