PRESENT STATUS OF THEORETICAL PREDICTIONS OF ELECTROMAGNETIC FORM FACTOR BEHAVIOURS*

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The present status of the electromagnetic form factors of strongly interacting particles is very roughly reviewed. First, the concept of the electromagnetic form factor is introduced, then a connection of its asymptotic behaviour with the quark structure of hadrons is specified and different approaches in a theoretical prediction of form factor behaviours are mentioned. A phenomenological approach, based on a synthesis of the analyticity with an experimental fact of a creation of vector-meson in the electron-positron annihilation processes and the asymptotic behaviour of the electromagnetic form factors, is discussed in more detail.

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The elastic electron scattering $e^-H \to e^-H$ and the annihilations $e^+e^- \leftrightarrow H\bar{H}$, where H means an arbitrary strongly interacting object (including also the atomic nucleus), are the most usual processes, in which the concept of the electromagnetic (EM) form factor (FF) appears. The cross-sections of these processes are proportional to the absolute value squared of the corresponding scattering amplitudes, which one knows formally to calculate in the framework of the quantum electrodynamics (QED) perturbation expansion according to the fine structure constant $\alpha \simeq 1/137$. Since the value of the fine structure constant $\alpha \ll 1$, those scattering amplitudes are considered practically in the one-photon-exchange approximation as follows

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$$M(e^-H \to e^-H) \simeq M^{(\gamma)}(s,t) = e^2\bar{u}(k_2)\gamma_\mu u(k_1)\frac{g_{\mu\nu}}{q^2}\langle H|J_\nu^{\rm EM}|H\rangle \quad (1)$$

and

$$M(e^+e^- \to \bar{H}H) \simeq M^{(\gamma)}(t,s) = e^2\bar{v}(k_2)\gamma_\mu u(k_1)\frac{g_{\mu\nu}}{q^2}\langle 0|J_\nu^{\rm EM}|H\bar{H}\rangle\,, \quad (2)$$

where $g_{\mu\nu}/q^2$ is the photon propagator and $\langle H|J_\nu^{\rm EM}|H\rangle$ (resp.$\langle 0|J_\nu^{\rm EM}|H\bar{H}\rangle$) is a matrix element of the hadron EM current, which, however, due to the non-point-like nature of the hadron H is unknown. Therefore, in practice it is decomposed according to a maximal number of linearly independent relativistic covariants constructed from the four-momenta and spin parameters of H as follows

$$\langle H|J_\mu^{\rm EM}|H\rangle = \sum_i R_\mu^i F^i(t)\,, \quad (3)$$

or

$$\langle H\bar{H}|J_\mu^{\rm EM}|0\rangle = \sum_i X_\mu^i F^i(t)\,, \quad (4)$$

where the scalar coefficients $F_i(t)$ are the EM FF's of the hadron H as functions of one invariant variable t- the momentum squared to be transferred by a virtual photon.

The number of $F^i(t)$ depends essentially on the spin S of H.

If $S = 0$ (*e.g.* $\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$, He^4, C^{12}, O^{16}, *etc.*) there is only one EM FF defined by

$$\langle H|J_\mu^{\rm EM}|H\rangle = (p+p')_\mu F_C(t). \quad (5)$$

In principle, there is no problem of obtaining of the experimental information on $|F_C(t)|$ in $t<0$ and $t>0$ regions as $d\sigma/d\Omega \sim |F_C(t)|^2$. However, the data on nuclei exist only for $t<0$ up to now and there is no concept of the EM FF of nucleus for $t>0$ to be known by nuclear physicist.

As a consequence of a compound nature of nuclei so-called diffraction minima of $|F_C(t)|$ appear in $t<0$ region, which are interpreted as zeros of $F_C(t)$ on the real axis of the complex t-plane. It is observed, that if more compound nucleus is investigated then the number of diffraction minima is increased in the same range of momentum transfer values.

In the case of H with $S = 1/2$ (*e.g.* p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, He^3, H^3, *etc.*) there are two EM FF's, usually chosen to be the electric $G_E(t)$ and magnetic $G_M(t)$ FF's, and defined as follows

$$\langle H|J_\mu^{\rm EM}|H\rangle = \frac{1}{2\pi^3}\bar{u}(p_2)\{\gamma_\mu F_1(t) + \frac{1}{2m_H}\sigma_{\mu\nu}(p_2-p_1)_\nu F_2(t)\}u(p_1), \quad (6)$$

$$G_E(t) = F_1(t) + \frac{t}{4m_H^2}F_2(t);\;\; G_M(t) = F_1(t) + F_2(t). \quad (7)$$

The experimental information on $G_E(t)$, $G_M(t)$ in $t < 0$ region can be easily determined from parameters of the straight-line (so-called Rosenbluth) plot of

$$\frac{d\sigma(e^-H \to e^-H)}{d\Omega}\left\{\frac{\alpha^2\cos^2(\vartheta/2)}{4E^2\sin^4(\vartheta/2)[1+(2E/M)\sin^2(\vartheta/2)]}\right\}^{-1} \tag{8}$$

in the laboratory system versus $\tan^2(\vartheta/2)$ at fixed t and for nuclei again diffraction minima appear.

All existing data on $G_E(t)$, $G_M(t)$ in $t > 0$ region are obtained from $\sigma_{\rm tot}(e^+e^- \leftrightarrow H\bar{H})$ under the assumption that $|G_E(t)| = |G_M(t)|$.

If $S = 1$ (*e.g.* vector mesons, deuteron, etc.) then there are three EM FF's, usually chosen to be the charge $G_C(t)$, the magnetic $G_M(t)$ and the quadrupole $G_Q(t)$ FF's, defined by

$$\langle H|J_\mu^{\rm EM}|H\rangle = F_1(t)(\xi'^*\cdot\xi)d_\mu + F_2(t)[\xi_\mu(\xi'^*\cdot q) - \xi'^*_\mu(\xi\cdot q)] - F_3\frac{(\xi\cdot q)(\xi'^*\cdot q)}{2m_H^2}d_\mu\,, \tag{9}$$

where ξ and ξ' are polarization vectors for incoming and outgoing H of four-momenta p and p', respectively

$$\xi'\cdot p' = 0;\ \xi\cdot p = 0;\ \xi'^2 = -1;\ \xi^2 = -1;\ d_\mu = (p'+p)_\mu;\ q_\mu = (p'-p)_\mu$$

and

$$G_C(t) = F_1(t) - \frac{t}{6m_H^2}G_Q(t);\ G_M(t) = F_2(t);\ G_Q(t) = F_1(t) - F_2(t) + (1-\frac{t}{4m_H^2})F_3(t). \tag{10}$$

One can determine all $G_C(t)$, $G_M(t)$, $G_Q(t)$ FF's from $d\sigma/d\Omega$ of the $e^-H \to e^-H$ process provided that polarized particles are used in the corresponding experiments. Otherwise only the structure functions $A(t)$ and $B(t)$ can be drawn out from

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2E'\cos^2(\vartheta/2)}{4E^3\sin^4(\vartheta/2)}[A(t) + B(t)\tan^2(\vartheta/2)]\,, \tag{11}$$

where

$$A(t) = -\frac{t}{6m_H^2}(1-\frac{t}{4m_H^2})G_M^2(t) + G_C^2(t) + \frac{t^2}{18m_H^4}G_Q^2(t)\,,$$
$$B(t) = -\frac{t}{3m_H^2}(1-\frac{t}{4m_H^2})^2G_M^2(t). \tag{12}$$

On the other hand, from

$$\sigma_{\rm tot}(e^+e^- \to H\bar{H}) = \frac{\pi\alpha^2\beta_H^3}{3t}\left\{\frac{t}{m_H^2}|G_C(t)+G_M(t)+G_Q(t)|^2+\right.$$
$$\left.+\left[2|G_C(t)+\frac{t}{2m_H^2}G_Q(t)|^2+|G_C(t)+\frac{t}{2m_H^2}G_M(t)|^2\right]\right\} \qquad (13)$$

one can see immediately that it is not a single task to obtain any experimental information on the corresponding EM FF's in $t>0$ region.

For strongly interacting particles H with $S>1$ a situation is even more complicated and generally it is not solved up to now.

Summarizing our knowledge about the experimental behaviour of EM FF's we come to a conclusion, that all of them have a similar behaviour in the shape (see Fig. 1). But they differ in the asymptotic behaviour, normalization, number of bumps corresponding to vector-meson resonances and also in the shape and height of those bumps.

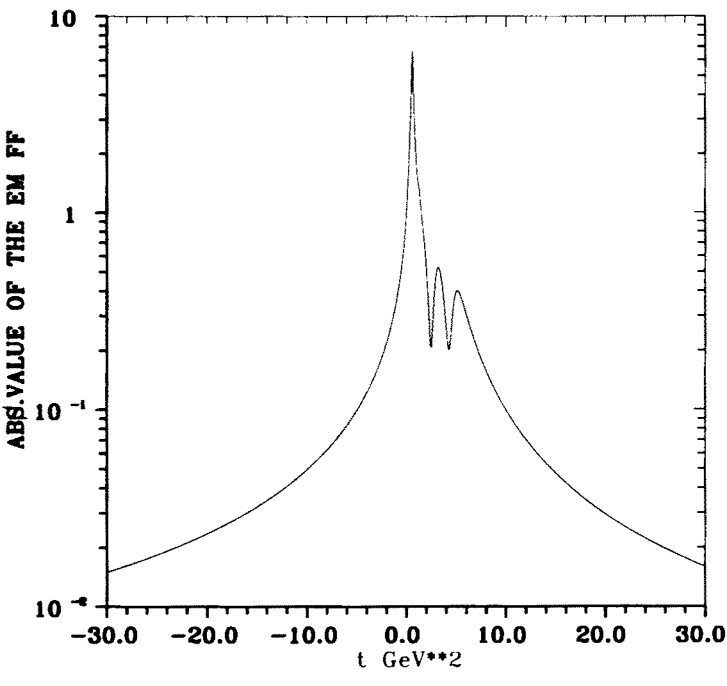

Fig. 1. A universal shape of EM FF's of all strongly interacting particles.

A behaviour of EM FF's, as shown in Fig. 1, is a matter of predictions of a strong interaction dynamical theory. However, there is no such theory able to predict a correct behaviour of $|F(t)|$ for $-\infty < t < +\infty$ up to now and only partial successes were achieved in this direction.

The great discovery in the elementary particle physics was a revelation of the quark-gluon structure of hadrons and a direct relation [1, 2] of the asymptotic behaviour of EM FF's with a number of constituent quarks n_q of the hadron H as follows

$$F(t)_{|t|\to\infty} \sim t^{1-n_q} \tag{14}$$

which is in a qualitative agreement with existing experimental data.

On the other hand, it is well known that on the role of a true dynamical theory of strong interactions the quantum chromodynamics (QCD), the gauge-invariant local quantum field theory of interactions of quarks and gluons, is pretending. But, as a consequence of the asymptotic freedom of QCD, in the framework of the perturbation theory, the latter is able to reproduce [3, 5] just the asymptotic behaviour (14) up to logarithmic corrections.

Not even the nonperturbative QCD sum rules [6] by means of which a prediction [7, 8] of a behaviour of EM FF's in a restricted $t < 0$ region is achieved, solve the problem of a reconstruction of EM FF's in the framework of QCD completely.

For a completeness we mention also the chiral perturbation approach [9], in the framework of which a correct behaviour of EM FF's of hadrons around the point $t = 0$ is predicted. This is very important to be mentioned as the chiral perturbation approach is equivalent to QCD at low energies where the running coupling constant $\alpha_s(t)$ takes large values and the PQCD is nonapplicable.

Summarizing, QCD (not even its equivalent form) gives no quantitative predictions in the most important part of the time-like ($4m_\pi^2 < t \leq 4$ GeV^2) region, where EM FF's are already complex functions of t and the e^+e^- experiments exhibit a nontrivial behaviour of measured cross-sections caused by a creation of various unstable vector-meson states.

Therefore for the present an appropriate phenomenological approach based on the synthesis of the experimental fact of a creation of vector-mesons in e^+e^-- annihilation processes into hadrons, the asymptotic behaviour (14) and the well-established analytic properties is still the most successful way in a global theoretical reconstruction of EM FF's of hadrons.

The vector-meson creation in e^+e^-- annihilation processes is taken into account by means of the vector-meson-dominance (VMD) model given for isoscalar and isovector parts of EM FF's by the relation

$$F(t) = \sum_v \frac{f_{vH\bar{H}}}{f_v} \frac{m_v^2}{m_v^2 - t}, \tag{15}$$

where $f_{vH\bar{H}}$ and f_v are the vector-meson-hadron and the universal vector-meson coupling constants, respectively, and m_v is the vector-meson mass.

The analyticity consists in a hypothesis that EM FF's are analytic functions in the whole complex t- plane besides infinite number of branch points on the positive real axis corresponding to normal and anomalous thresholds. Practically we realize two-cut approximation of the latter analytic structure by means of the nonlinear transformation

$$t = t_0 - \frac{4(t_{\rm in} - t_0)}{[1/W - W]^2} \tag{16}$$

to be applied in (15), where t_0 is the lowest branch point and t_{in} is an effective branch point approximating all higher contributions. As a result a factorization of (15) into pure asymptotic and finite-energy terms is obtained as follows

$$F(t) = \sum_v \left(\frac{f_{vH\bar{H}}}{f_v}\right)\left(\frac{1-W^2}{1-W_N^2}\right)^2 \times \frac{(W_N - W_{v_0})(W_N + W_{v_0})(W_N - 1/W_{v_0})(W_N + 1/W_{v_0})}{(W - W_{v_0})(W + W_{v_0})(W - 1/W_{v_0})(W + 1/W_{v_0})} \tag{17}$$

where W_{v_0} are the zero-width (that is why they have a subindex 0) VMD poles and W_N are the normalization points (corresponding to $t = 0$) in the W- plane.

Now, provided that there are i and j vector-mesons $(i + j = v)$ with masses remaining the inequalities $t_0 < m_i^2 < t_{\rm in}$ and $m_j^2 > t_{\rm in}$ to be true, one can derive relations

$$W_{i_0} = -W_{i_0}^* \quad \text{and} \quad W_{j_0} = 1/W_{j_0}^*, \tag{18}$$

which together with a subsequent incorporation of the nonzero values of vector-meson widths $\Gamma_v \neq 0$ and a change of the power $2 \to 2(n_q - 1)$ in the asymptotic term, give the following unitary and analytic model of EM FF's

$$F[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^{2(n_q-1)} \times \left\{\sum_i \frac{(W_N - W_i)(W_N - W_i^*)(W_N - 1/W_i)(W_N - 1/W_i^*)}{(W - W_i)(W - W_i^*)(W - 1/W_i)(W - 1/W_i^*)}\left(\frac{f_{iH\bar{H}}}{f_i}\right) + \sum_j \frac{(W_N - W_j)(W_N - W_j^*)(W_N + W_j)(W_N + W_j^*)}{(W - W_j)(W - W_j^*)(W + W_j)(W + W_j^*)}\left(\frac{f_{jH\bar{H}}}{f_j}\right)\right\}$$

to be defined on the four-sheeted Riemann surface with complex conjugate vector-meson poles on unphysical sheets and the asymptotic behaviour (14). It depends only on $(f_{vH\bar{H}}/f_v)$ and t_{in} as free parameters, provided m_v are fixed at the world averaged values.

The model was successfully applied to a description of the EM structure of pions [10, 11], kaons [11, 12], nucleons [13–15], Λ-hyperon [16], deuteron [17] and He^4 [18].

However, for H with $n_q > 2$ one finds two possibilities in a construction of the unitary and analytic EM FF models. The first one was just demonstrated above and it can be used practically, if a plentiful experimental information on EM FF's exists in order to determine free parameters of the model reliably. Otherwise correlations among those parameters appear. In this case the second approach is suitable, which consists first in a modification of VMD model parametrization (15) to be normalized automatically and to govern the asymptotic behaviour (14), and only then a two-cut approximation of the FF analytic properties is incorporated. The latter restrictions on VMD model lead to a system of algebraic equations which reduce a number of free parameters $(f_{vH\bar{H}}/f_v)$ to be expressed through the vector-meson masses m_v.

The second approach was demonstrated practically [19] on the nucleon EM FF's and a perfect reproduction of existing data was achieved with only 8 (to be compared with 14 in the previous formulation) free parameters.

Finally, what about the EM FF's on which there is no experimental information up to now. Is there any chance to predict their behaviour?

A natural solution of the latter problem is offered in the framework of the second formulation of the unitary and analytic model of EM FF's as it was clearly demonstrated [20] on the case of the $1/2^+$ octet baryons.

The idea was to use in (15) only such number of vector mesons that, requiring parametrization (15) to be automatically normalized and to have the asymptotic behaviour (14), a number of terms in the corresponding algebraic equations is obtained to be equal to number of equations. As a result all $(f_{vH\bar{H}}/f_v)$ are expressed through m_v and the model contains only t_{in} to be free parameters on which, however, the latter is very weakly dependent. Therefore t_{in} are fixed always at $B\bar{B}$- threshold.

The predicted behaviours [20] of $1/2^+$ octet baryon EM FF's are quite encouraging.

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