SELF-DUAL HOMOGENEOUS GLUON VACUUM AND MESON SPECTRUM*

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The model of induced nonlocal quark currents based on the hypothesis that the QCD vacuum is realized by the (anti-)self-dual homogeneous gluon field is suggested. The field produces quark confinement and chiral symmetry breaking. Nonlocal extension of the bosonization procedure of quark currents is developed, which leads to the ultraviolet finite unitary S-matrix on the space of meson states. The model has a minimal set of parameters: quark masses, vacuum field strength and the quark-gluon coupling constant. The vacuum field provides qualitative regimes in the meson spectrum: mass splitting between pseudoscalar and vector mesons, Regge trajectories, masses of heavy quarkonia and heavy-light mesons in the heavy quark limit. The masses and weak decay constants of mesons from all qualitatively different regions of the spectrum are described to within ten per cent inaccuracy.

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1. Introduction

In my talk I would like to present the model of induced quark currents formulated in our recent papers [1].

Achievements of the Nambu–Jona–Lasinio (NJL) model in description of meson masses, decay constants and so on are well-known [2–4]. This success can be explained by the bosonization procedure which makes possible to extract collective modes and dynamical breaking of $\mathrm{SU_L(3)}\times\mathrm{SU_R(3)}$ and $\mathrm{U_A(1)}$ symmetries. At the same time, an incorporation of quark confinement into consideration, description of heavy quarkonia and heavy-light mesons,

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radial and angular excitations of mesons, as well as different form-factors require an essential modification of the NJL model (*eg*, see [5]). Another general disadvantage is the nonrenormalizability of the local four-fermion interaction.

In [1] we have suggested a model that in some sense can be considered as an extension of the standard NJL model. There are two crucial modifications. First, our model is based on the hypothesis that the QCD vacuum to be realized by the (anti-)self-dual homogeneous background gluon field. Second, the effective quark-quark coupling is described by the nonlocal four-quark interaction induced by the one-gluon exchange in presence of the (anti-)self-dual homogeneous gluon vacuum field which is a stable configuration [7]. This vacuum field ensures the analytical quark confinement and breaks the chiral symmetry. The model of induced quark currents gives a basis for investigating of all the above-mentioned problems from a general point of view. The main features of the model are as follows [1].

— There is the quark confinement. The quark propagator being an entire analytical function in the complex momentum plane [6] has the standard local ultraviolet behavior in the Euclidean region, and is modified essentially in the physical, *i.e.*, Minkowski region.

— The one-gluon exchange is decomposed into an infinite sum of current-current interaction terms, in which the quark currents are nonlocal, colorless and carry a complete set of quantum numbers including the orbital and radial ones. This effective quark-quark interaction generates a superrenormalizable perturbation expansion.

— The bosonization of the nonlocal four-quark interaction leads to ultraviolet finite effective meson theory. Mesons are treated as extended nonlocal objects.

— The model contains the minimal number of parameters: the quark masses, quark-gluon coupling constant and the tension of the background gluon field.

The model describes all qualitatively different regions of meson spectrum to within ten percent inaccuracy, breaks the chiral symmetry and gives rise to the splitting between masses of the pseudoscalar and vector mesons with identical quark structure ($\rho - \pi$, $K - K^*$). This spin-field interaction drives also the weak decay of pion and kaon.

Furthermore, the vacuum field produces three rigid asymptotic regimes for the spectrum of collective modes. The spectra of radial and orbital excitations of light mesons are equidistant for $\ell \gg 1$ or $n \gg 1$, *i.e.*, they have Regge character. Localization of meson field at the center of masses of a quark system provides other two asymptotic regimes. In the limit of infinitely heavy quark, a mass of quarkonium tends to be equal to a sum of the masses of constituent quarks, while a mass of heavy-light meson approaches the mass of a heavy quark: $M_{Q\bar{Q}} \to 2m_Q - \Delta_{Q\bar{Q}}$, $M_{Q\bar{q}} \to m_Q + \Delta_{Q\bar{q}}$. The next-to-leading terms $\Delta_{Q\bar{Q}}$ and $\Delta_{Q\bar{q}}$ do not depend on the heavy quark mass. The same reasons provide the correct asymptotic behavior of the weak decay constant for the heavy-light pseudoscalar mesons: $\frac{f_P \sim 1}{\sqrt{m_Q}}$.

2. The model of induced nonlocal quark currents

2.1. Basic assumptions, approximations and notation

The representation of the Euclidean generating functional for QCD, in which the gluon and ghost fields are integrated out, serves as a starting point for many models of hadronization. This starting representation for the case of nontrivial vacuum gluon field [8](see also [9]) looks like

$$Z = \int d\sigma_{\rm vac} \int Dq D\bar{q} \exp\left\{ \int d^4x \sum_f^{N_F} \bar{q}_f(x)\left(i\gamma_\mu \hat{\nabla}_\mu - m_f\right) q_f(x) + L_2 \right\},$$

$$L_2 = \frac{g^2}{2} \iint dx dy \; j^a_\mu(x) G^{ab}_{\mu\nu}(x, y \mid B) j^b_\nu(y)\,, \tag{1}$$

where N_F is the number of flavors corresponding to the SU(N_F) flavor group and

$$j^a_\mu(y) = \sum_f^{N_F} \bar{q}_f(x)\gamma_\mu t^a q_f(x), \qquad \hat{\nabla}_\mu = \partial_\mu - it^a B^a_\mu\,.$$

The function $G^{ab}_{\mu\nu}(x, y \mid B)$ is the exact (up to the quark loops) 2-point gluon Green function in the external field B^a_μ. This function is unknown, and some approximation has to be introduced. For instance, the local NJL model corresponds to the choice $G^{ab}_{\mu\nu} = \delta^{ab}\delta_{\mu\nu}\delta(x - y)$. Our aim is to investigate the mesonic $(q\bar{q})$-collective modes.

Representation (1) implies, that there exists some vacuum (classical) gluon field $B^a_\mu(x)$, which minimizes the effective action or effective potential of the Euclidean QCD. In the general case, the vacuum field depends on a set of parameters $\{\sigma_{\rm vac}\}$, and the measure $d\sigma_{\rm vac}$ averages all physical amplitudes over a subset of $\{\sigma_{\rm vac}\}$, in respect to which the vacuum state is degenerate (for more details see [1] and references therein).

The quark-gluon interaction in Eqs. (1) is local, and a decomposition over degrees of g^2 generates a renormalizable perturbation theory. It means, that an appropriate regularization should be implied. This point has to be stressed here, since our final technical aim is a transformation of the interaction term in Eq. (1), which generates completely new superrenormalizable perturbation expansion of the functional integral (1).

The homogeneous (anti-)self-dual vacuum field looks like

$$\hat{B}_\mu(x) = \hat{n}B_\mu(x), \qquad B_\mu(x) = B_{\mu\nu}x_\nu, \qquad \hat{n} = n^a t^a, \qquad n^2 = n^a n^a = 1,$$

$$B_{\mu\nu} = -B_{\nu\mu}, \qquad B_{\mu\rho}B_{\rho\nu} = -B^2\delta_{\mu\nu}, \qquad \tilde{B}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}B_{\alpha\beta} = \pm B_{\mu\nu}\,.$$

Since the chromomagnetic **H** and chromoelectric **E** fields relate to each other like **H**$= \pm$**E**, two spherical angles (φ, θ) define a direction of the field in the Euclidean space.

In the diagonal representation of $\hat{n} = n^a t^a$, an additional angle ξ is needed to fix a direction of the field in the color space $\hat{n} = t^3\cos\xi + t^8\sin\xi$, for $0 \le \xi < 2\pi$. The one-loop calculations and some nonperturbative estimations of the effective potential for the homogeneous gluon field argue that the potential could have a minimum at nonzero value of the field tension $B \neq 0$ (*e.g.*, see [6, 7, 10]) and for $\hat{n} = t^8$.

The vacuum is degenerated with respect to the directions of the field in the Euclidean space and anti-self- and self-dual configurations. Therefore the measure $d\sigma_{\rm vac}$ has the form

$$\int d\sigma_{\rm vac} = \frac{1}{8\pi}\int\limits_0^\pi \mathrm{d}\theta\sin\theta\int\limits_0^{2\pi}\mathrm{d}\varphi\sum_\pm, \tag{2}$$

where the sign $\sum_\pm$ denotes averaging over the self- and anti-self-dual configurations.

2.2. *Quark and gluon propagators*

The quark propagator $S_f(x, y \mid B)$ satisfies the equation:

$$(i\gamma_\mu\hat{\nabla}_\mu - m_f)S_f(x, y \mid B) = -\delta(x - y)\,,$$

and can be written in the form

$$\begin{aligned} S_f(x, y \mid B) =& e^{\frac{i}{2}(x\hat{B}y)}H_f(x - y \mid B)e^{\frac{i}{2}(x\hat{B}y)}\,, \\ H_f(z) =& \frac{i\hat{\nabla}_\mu(z)\gamma_\mu + m_f}{\hat{\nabla}^2(z) + m_f^2 + (\sigma\hat{B})}\delta(z)\,, \end{aligned} \tag{3}$$

$$\begin{aligned} \tilde{H}_f(p \mid B) =& \frac{1}{2v\Lambda}\int\limits_0^1 dt e^{-\frac{p^2}{2v\Lambda^2}t}\left(\frac{1-t}{1+t}\right)^{\frac{\alpha_f^2}{4v}}\left[\alpha_f + \frac{1}{\Lambda}p_\mu\gamma_\mu + it\frac{1}{\Lambda}(\gamma f p)\right] \\ &\times\left[P_\pm + P_\mp\frac{1+t^2}{1-t^2} - \frac{i}{2}(\gamma f\gamma)\frac{t}{1-t^2}\right], \end{aligned} \tag{4}$$

where

$$P_\pm = \tfrac{1}{2}(1 \pm \gamma_5)\,, \quad \alpha_f = \frac{m_f}{\Lambda}\,, \quad (xBy) = x_\mu B_{\mu\nu} y_\nu\,,$$

$$(pf\gamma) = p_\mu f_{\mu\nu}\gamma_\nu\,, \quad f_{\mu\nu} = \frac{t^8}{v\Lambda^2} B_{\mu\nu}\,, \quad f_{\mu\rho}f_{\rho\nu} = -\delta_{\mu\nu}\,,$$

$$\frac{1}{v} = \mathrm{diag}\left(3, 3, \frac{3}{2}\right)\,, \quad \Lambda^2 = \sqrt{3}B\,.$$

The upper (lower) sign in the matrix $P_\pm$ corresponds to the self-dual (anti-self-dual) field.

The term $(\sigma\hat{B}) = \sigma_{\alpha\beta}\hat{B}_{\alpha\beta}$ in Eq. (3) describes an interaction of a quark spin with the background field. This spin-field interaction leads to the singularity $1/m_f$ for $m_f \to 0$, which is a manifestation of the zero mode (the lowest Landau level) of the massless Dirac equation in the external (anti-)self-dual homogeneous field. The mathematical point is that the spectrum of the operator $\gamma_\mu\partial_\mu$ is continuous, whereas the spectrum of the operator $\gamma_\mu\hat{\nabla}_\mu(x)$ is discrete and the lowest eigen number is equal to zero. Simple calculations give for $m_f \to 0$

$$\lim_{\varepsilon\to 0}\lim_{m_f\to 0} m_f\langle\bar{q}_f(x)q_f(x+\varepsilon)\rangle_B = -\lim_{\varepsilon\to 0}\lim_{m_f\to 0} m_f \mathrm{Tr}H_f(\varepsilon|B)$$
$$= -\int\frac{d^4p}{(2\pi)^4}\lim_{m_f\to 0} m_f\mathrm{Tr}\tilde{H}_f(p\mid B) = -\frac{4}{3\pi^2}\Lambda^4\,. \tag{5}$$

Due to the spin-field interaction the quark condensate is nonzero in the limit of vanishing quark mass. Thus the chiral symmetry is broken by the vacuum field in the limit $m_f \to 0$ (see also [6]). Namely the spin-field interaction gives rise to the splitting between the masses of the pseudoscalar and vector mesons and provides a smallness of pion mass.

In terms of the variable $\zeta = p_\mu\gamma_\mu$ the propagator $\tilde{H}_f(\zeta\mid B)$ is an entire analytical function in the complex ζ-plane. There are no poles corresponding to the free quarks, which is treated as the confinement of quarks. The following asymptotic behavior takes place:

$$\tilde{H}_f(\zeta\mid B) \to \begin{cases} \frac{m_f+\zeta}{-\zeta^2} = \frac{m_f+\gamma_\nu p_\nu}{p^2} \text{ if } \zeta\to\pm\infty\ (p^2\to\infty) \\ O\big(\exp(\frac{\zeta^2}{2v\Lambda^2})\big) = O\big(\exp(\frac{-p^2}{2v\Lambda^2})\big) \text{ if } \zeta\to\pm i\infty\ (p^2\to-\infty)\,. \end{cases} \tag{6}$$

Equations (6) shows the standard local behavior of the fermion propagator in the Euclidean region ($p^2 \to \infty$), while in the physical region ($p^2 \to -\infty$) we see the exponential increase typical for nonlocal theories (for more details about the general theory of nonlocal interactions of quantized fields see [9, 11]).

The gluon propagator $G^{ab}_{\mu\nu}(x,y|B)$ satisfies the equation:

$$(\breve{\nabla}^2\delta_{\mu\nu}+4i\breve{B}_{\mu\nu})G_{\nu\rho}(x,y\mid B)=-\delta_{\mu\rho}\delta(x-y)\,.$$

The solution of this equation (for details see [1]) can be represented in the form

$$G^{ab}_{\mu\nu}(x,y\mid B)=D^{ab}_{\mu\nu}(x,y\mid B)+R^{ab}_{\mu\nu}(x,y\mid B)\,,$$

where the function $D^{ab}_{\mu\nu}(x,y|B)$ is the confined part of the gluon proparator

$$D_{\mu\nu}(x,y\mid B)=\delta_{\mu\nu}K^2e^{\frac{i}{2}(x\breve{B}y)}D(x-y\mid\Lambda^2)e^{\frac{i}{2}(x\breve{B}y)}\,,$$
$$D(z\mid\Lambda^2)=\frac{1}{(2\pi)^2z^2}\exp\Big\{-\frac{\Lambda^2z^2}{4}\Big\}\quad K^2=\mathrm{diag}(0,0,0,1,1,1,1,0)\,. \tag{7}$$

The Fourier transform of the function $D(z\mid\Lambda^2)$ is an entire analytical function in the momentum space. It has the local behavior in the Euclidean region, but increases exponentially in the physical region. This function describes a propagation of the confined modes of the gluon field. Other terms $R^{ab}_{\mu\nu}$, that contain a contribution of the zero modes and an antisymmetric part, will be omitted.

Thus, the central point of our extension of the NJL-model consists in taking into account the confining influence of the background field both on the quark and gluon propagators.

2.3. Color singlet bilocal quark currents

Substituting gluon propagator (7) to the interaction term in representation (1), using the Fierz transformation of the color, flavor and Dirac matrices, and keeping only the scalar J^{aS}, pseudoscalar J^{aP}, vector J^{aV} and axial-vector J^{aA} colorless currents, we arrive at the expression [1]

$$L_2=\frac{g^2}{2}\sum_{bJ}C_J\iint \mathrm{d}^4x\mathrm{d}^4y J^{bJ}(x,y)D(x-y\mid\Lambda^2)J^{bJ}(y,x)\,, \tag{8}$$

$$J^{bJ}(x,y)=\bar{q}_f(x)M^b_{ff'}\Gamma^Je^{i(x\hat{B}y)}q_{f'}(y)\,, \tag{9}$$
$$\Gamma^J\ \{J=S,\ P,\ V,\ A\}\,,\quad C_S=C_P=\tfrac{1}{9}\,,\quad C_V=C_A=\tfrac{1}{18}\,.$$

Here $M^b_{ff'}$ are the flavor mixing matrices ($b=0,\ldots,N_F^2-1$) corresponding to the SU(N_F) flavor group. Due to the phase factor $\exp\{i(x\hat{B}y)\}$, bilocal quark currents (9) are the scalars under the local gauge transformations.

Let us transform integration variables x and y in Eq. (8) to the coordinate system corresponding to the center of masses of quarks $q_f(x)$ and $q_{f'}(y)$

$$x \to x + \xi_f y, \quad y \to x - \xi_{f'} y, \quad \xi_f = \frac{m_f}{m_f + m_{f'}}, \quad \xi_{f'} = \frac{m_{f'}}{m_f + m_{f'}}. \quad (10)$$

This transformation turns out to be crucial for simultaneous description of the light-light, heavy-light and heavy-heavy mesons. Corresponding transformation of the quark currents looks as

$$\begin{aligned} J^{bJ}(x,y) &\to \bar{q}_f(x + \xi_f y) M^b_{ff'} \Gamma^J e^{i(x\hat{B}y)} q_{f'}(x - \xi_{f'} y) \\ &= \bar{q}_f(x) M^b_{ff'} \Gamma^J e^{\frac{1}{2} y \overleftrightarrow{\nabla}_{ff'}(x)} q_{f'}(x), \\ \overleftrightarrow{\nabla}_{ff'}(x) &= \xi_f(\overleftarrow{\partial} + i\hat{B}(x)) - \xi_{f'}(\overrightarrow{\partial} - i\hat{B}(x)). \end{aligned} \quad (11)$$

The currents (11) are nonlocal and colorless. Interaction term (8) takes the form

$$L_2 = \frac{g^2}{8\pi^2} \sum_{bJ} C_J \iint d^4x d^4y \frac{1}{y^2} \exp\left\{-\frac{\Lambda^2 y^2}{4}\right\} J^{bJ}(x,y) J^{bJ}(y,x). \quad (12)$$

2.4. Decomposition of bilocal currents

An idea of our next step consists in a decomposition of the bilocal currents (11) over some complete set of orthonormalized polynomials in such a way, that the relative coordinate of two quarks y in Eq. (12) would be integrated out. The particular form of this set is determined by the form of the gluon propagator which plays the role of a weight function in the orthogonality condition. The physical meaning of the decomposition consists in classifying a relative motion of two quarks in the bilocal currents over a set of radial n and angular ℓ quantum numbers. Thus, we are looking for a decomposition of the form

$$\begin{aligned} J^{bJ}(x,y) &= \sum_{n\ell} (y^2)^{\ell/2} f^{n\ell}_{\mu_1\ldots\mu_\ell}(y) \mathcal{J}^{bJ\ell n}_{\mu_1\ldots\mu_\ell}(x), \\ f^{\ell n}_{\mu_1\ldots\mu_\ell}(y) &= L_{n\ell}(y^2) T^{(\ell)}_{\mu_1\ldots\mu_\ell}(n_y), \quad n_y = y/\sqrt{y^2}. \end{aligned} \quad (13)$$

Here the irreducible tensors $T^{(\ell)}_{\mu_1\ldots\mu_\ell}$ satisfy the conditions:

$$\begin{aligned} &T^{(\ell)}_{\mu_1\ldots\mu\ldots\nu\ldots\mu_\ell}(n_y) = T^{(\ell)}_{\mu_1\ldots\nu\ldots\mu\ldots\mu_\ell}(n_y), \quad T^{(\ell)}_{\mu\ldots\mu\ldots\mu_\ell}(n_y) = 0, \\ &T^{(\ell)}_{\mu_1\ldots\mu_\ell}(n_y) T^{(\ell)}_{\mu_1\ldots\mu_\ell}(n_{y'}) = \frac{1}{2^\ell} C^{(1)}_\ell(n_y n_{y'}), \end{aligned} \quad (14)$$

where $C_\ell^{(1)}$ are the Gegenbauer's (ultraspherical) polynomials. The polynomials $L_{n\ell}(u)$ are the generalized Laguerre's polynomials. The details of calculation of the currents $\mathcal{J}^{bJ\ell n}_{\mu_1\ldots\mu_\ell}(x)$ can be found in paper [1]. As a result, the interaction term L_2 takes the form

$$L_2 = \frac{1}{2}\sum_{bJ\ell n}\left(\frac{G_{J\ell n}}{\Lambda}\right)^2\int \mathrm{d}^4x\left[\mathcal{J}^{bJ\ell n}_{\mu_1\ldots\mu_\ell}(x)\right]^2,$$

$$G^2_{J\ell n} = C_J\, g^2\,\frac{(\ell+1)}{2^\ell n!(\ell+n)!}, \tag{15}$$

$$\mathcal{J}^{bJ\ell n}_{\mu_1\ldots\mu_\ell}(x) = \bar{q}(x)V^{bJ\ell n}_{\mu_1\ldots\mu_\ell}(x)q(x), \tag{16}$$

$$V^{bJ\ell n}_{\mu_1\ldots\mu_\ell}(x) \equiv V^{bJ\ell n}_{\mu_1\ldots\mu_\ell}\left(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right)$$

$$= M^b\Gamma^J\left\{\!\!\left\{F_{n\ell}\left(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2}\right)T^{(\ell)}_{\mu_1\ldots\mu_\ell}\left(\frac{1}{i}\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right)\right\}\!\!\right\}, \tag{17}$$

$$F_{n\ell}(4s) = s^n\int_0^1 dt t^{\ell+n}e^{st}. \tag{18}$$

The doubled brackets in Eq. (17) mean that the covariant derivatives commute inside these brackets. Form-factors $F_{n\ell}(s)$ are entire analytical functions in the complex s-plane, which is a manifestation of the gluon confinement.

The classification of the currents will be complete if we will decompose $\mathcal{J}^{aJ\ell n}_{\alpha,\mu_1\ldots\mu_\ell}$ with $J = V, A$ and $\ell > 0$ into a sum of orthogonal currents $\mathcal{I}^{bJ\ell nj}_{\alpha\mu_1\ldots\mu_\ell}$ with the different total angular momentum $j = \ell-1, \ell, \ell+1$ (for detaials see [1]).

For large Euclidean momentum the vertices $\tilde{V}^{aJln}$ behave as $1/(p^2)^{1+\ell/2}$. Therefore, only the "bubble" diagrams are divergent. These divergencies can be removed by the counter-terms of the form $-2\mathcal{I}(x)\mathrm{Tr}VS$. The renormalized vacuum amplitude Z takes the form

$$Z = \int d\sigma_{\rm vac}\int DqD\bar{q}\exp\left\{\iint \mathrm{d}^4x\mathrm{d}^4y\bar{q}(x)S^{-1}(x,y|B)q(y)\right.$$

$$\left. +\sum_{\mathcal{N}}\frac{1}{2\Lambda^2}G^2_{\mathcal{N}}\int \mathrm{d}^4x\left[\mathcal{I}_{\mathcal{N}}(x) - \mathrm{Tr}V_{\mathcal{N}}S\right]^2\right\}. \tag{19}$$

where the condensed index $\mathcal{N} = \{a, J, \ell, n, j\}$ is introduced.

2.5. Bosonization

By means of the standard bosonization procedure [2, 3] applied to Eq. (19) the amplitude Z can be represented in terms of the composite meson fields $\Phi_{\mathcal{N}}$ [1]:

$$Z = \int \prod_{\mathcal{N}} D\Phi_{\mathcal{N}} \exp\left\{\frac{1}{2}\int dx \Phi_{\mathcal{N}}(x)\left(\Box - M_{\mathcal{N}}^2\right)\Phi_{\mathcal{N}}(x) + I_{\rm int}\left[\Phi\right]\right\}, \tag{20}$$

$$I_{\rm int} = -\frac{1}{2}\int d^4x_1 \int d^4x_2 h_{\mathcal{N}} h_{\mathcal{N}'} \Phi_{\mathcal{N}}(x_1)$$
$$\times \left[\Gamma_{\mathcal{N}\mathcal{N}'}(x_1, x_2) - \delta_{\mathcal{N}\mathcal{N}'} \Pi^R_{\mathcal{N}}(x_1 - x_2)\right] \Phi_{\mathcal{N}'}(x_2)$$
$$- \sum_{m=3} \frac{1}{m} \int d^4x_1 ... \int d^4x_m \prod_{k=1}^{m} h_{\mathcal{N}_k} \Phi_{\mathcal{N}_k}(x_k) \Gamma_{\mathcal{N}_1...\mathcal{N}_m}(x_1, ..., x_m),$$
$$\Gamma_{\mathcal{N}_1...\mathcal{N}_m} = \int d\sigma_{\rm vac} {\rm Tr}\left\{V_{\mathcal{N}_1}(x_1) S(x_1, x_2 \,|\, B) ... V_{\mathcal{N}_m}(x_m) S(x_m, x_1 \,|\, B)\right\}.$$

Meson masses $M_{\mathcal{N}}$ are defined by the equations

$$\Lambda^2 + G^2_{J\ell n} \tilde{\Pi}_{bJ\ell nj}(-M^2_{bJ\ell nj}; m_f, m_{f'}; \Lambda) = 0, \tag{21}$$

where the function $\tilde{\Pi}_{bJ\ell nj}$ is given by the diagonal part in the momentum representation of the tensor

$$\Pi^{bJ\ell nj}_{\mu_1 ..., \nu_1 ...}(x - y; m_f, m_{f'}; \Lambda)$$
$$= \int d\sigma_{\rm vac} {\rm Tr}\left[V^{bJ\ell nj}_{\mu_1 ...}(x) S(x, y \mid B) V^{bJ\ell nj}_{\nu_1 ...}(y) S(y, x \mid B)\right]. \tag{22}$$

Relation (21) is the master equation for meson masses. The function $\tilde{\Pi}$ can be calculated using the representations (3) and (4) for the quark propagator and (17) for the vertices.

The fields $\Phi_{\mathcal{N}}$ ($\mathcal{N} = \{a, J, \ell, n, j\}$) with $j > 0$ satisfy the on-shell condition

$$p_\mu \tilde{\Phi}^{...\mu ...}_{\mathcal{N}}(p) = 0, \quad \text{for} \quad p^2 = -M^2_{\mathcal{N}},$$

which excludes all extra degrees of freedom of the field, so that the numbers ℓ and j can be treated as the O(3) orbital momentum and total momentum.

The constants

$$h_{\mathcal{N}} = 1/\sqrt{\tilde{\Pi}'_{\mathcal{N}}(-M^2_{\mathcal{N}})} \tag{23}$$

play the role of the effective coupling constants of the meson-quark interaction.

The quark masses m_f, the scale Λ (strength of the background field) and the quark-gluon coupling constant g are the free parameters of the effective meson theory (20)–(23).

In the lowest approximation the interactions between mesons with given quantum numbers $\mathcal{N} = \{b, J, \ell, n, j\}$ are described by the two-point quark loops the structure of which is the same as in the standard NJL-model, but in our case the quarks propagate in the vacuum gluon field, and the meson-quark vertices are nonlocal. Due to this nonlocality the quark loops are ultraviolet finite. The whole diagram should be averaged by integration over the measure $d\sigma_{\text{vac}}$.

Thus the effective four-quark interaction is represented as an infinite series of interactions between the nonlocal quark currents characterized by the complete set of quantum numbers $\{b, J, \ell, n, j\}$. The form of the currents is induced by a particular form of gluon propagator. This new representation of the four-quark interaction generates an expansion of any amplitudes into series of partial amplitudes with a particular value of the quantum numbers. Each partial amplitude is ultraviolet finite. The composite meson fields in Eq. (20) are nothing else but the "elementary" collective excitations, that are classified according to the complete set of quantum numbers of the relativistic two-quark system.

It should be stressed, that the model (20) satisfies all demands of the general theory of nonlocal interactions of quantum fields [11], which means that Eq. (20) defines a nonlocal, relativistic, unitary and ultraviolet finite quark model of meson-meson interactions.

3. Meson spectrum and weak decay constants

3.1. Light pseudoscalar and vector mesons

The parameters of the model are fixed by the fit of the masses of π, ρ, K and K^* mesons as the basic quantities. Ther results are shown in the Tables I and II.

TABLE I

Parameters of the model

m_u (MeV)	m_d (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)	Λ (MeV)	g
198.3	198.3	413	1650	4840	319.5	9.96

TABLE II

The masses(MeV), weak decay constants (MeV) and meson-quark coupling constants h of the light mesons. M^* — calculation without taking into account the spin-field interaction.

Meson	π	ρ	K	K^*	ω	ϕ
M	140	770	496	890	770	1034
$M^{\rm exp}$	140	770	496	890	786	1020
f_P	126	-	145	-	-	-
$f_P^{\rm exp}$	132	-	157	-	-	-
h	6.51	4.16	7.25	4.48	4.16	4.94
M^*	630	864	743	970	864	1087

It is well-known, that there should be a special reason, which provides a small pion mass and splits the masses of pseudoscalar and vector mesons. Breaking of chiral symmetry due to the four-quark interaction and two independent coupling constants for pseudoscalar and vector mesons ($g_P \neq g_V$ instead of our parameter g) play the role of such reason in the local NJL-model. In our model the interaction of quark spin with the vacuum field leads to the singular behavior of the quark propagator in the massless limit and generates a non-zero quark condensate, which indicates breaking of the chiral symmetry by the vacuum gluon field. In our case the same spin-field interaction is responsible for small pion mass and for the mass-splitting between P- and V-mesons. Let us compare a behavior of pion and ρ-meson polarization functions $\tilde{\Pi}_J$ ($\ell = 0$, $n = 0$, $J = P, V$) in the case $m_f = m_{f'} = m_u \ll \Lambda$. Calculations give

$$\tilde{\Pi}_\pi(-M^2; m_u, m_u; \Lambda) = O\left(\frac{1}{m_u^2}\right), \quad \tilde{\Pi}_\rho(-M^2; m_u, m_u; \Lambda) = O(1) \quad (24)$$

and means that the masses of pion and ρ-meson are strongly splitted and $m_\rho \gg m_\pi$ when the quark mass goes to zero. The samer picture takes place for K and K^* mesons, but the effect is more smooth since the strange quark mass is not so small.

The scalar polarization function $\tilde{\Pi}_S$ is positive for a wide range of parameter values. As a result, Eq. (21) has no real solutions for scalar mesons. It looks interesting, that the scalar $(q\bar{q})$ bound states do not appear due to the same spin-field interaction.

Consideration of the $\mathrm{SU}_F(3)$ singlet and the eighth octet states shows an ideal mixing both for vector and pseudoscalar mesons. The masses of ω and ϕ are in a good agreement with the experimental values (see Table II). An ideal mixing of the pseudoscalar states is not the case, that can provide an appropriate description of η and η' mesons. It is well known, that the problem of η and η' masses can be solved by taking into account another Euclidean gluon configuration — the instanton vacuum field [12]. The instantons can be incorporated into our formalism without any principal problems, and we hope to realize this idea in forthcoming publications.

We have considered the weak decays of π and K mesons. Results are shown in the Table II. One could get a definite impression, that simultaneous description of the masses of π, K, ρ, K^*, ω and ϕ mesons, and quite accurate values of f_π and f_K are obtained mostly due to the breakdown of chiral symmetry by the spin-field interaction.

In order to clarify a status of this impression, one needs to investigate the chiral limit of the model. It is a quite interesting problem, and we leave it for further investigations.

3.2. Regge trajectories

It has been shown in [1], that the spectrum of radial and orbital excitations of the light mesons is asymptotically equidistant:

$$M^2_{aJ\ell n} = \tfrac{8}{3}\ln\left(\tfrac{5}{2}\right)\cdot\Lambda^2\cdot n \ + O(\ln n)\ , \ \text{for} \ \ n \gg \ell\,, \tag{25}$$

$$M^2_{aJ\ell n} = \tfrac{4}{3}\ln 5\cdot\Lambda^2\cdot\ell \ + O(\ln\ell)\ , \ \text{for} \ \ \ell \gg n\,. \tag{26}$$

Technically this result is based on the exponential behavior of the quark propagator (6) and vertex function $F_{n\ell}$ (18) in the Minkowski region ($p^2/\Lambda^2 \to -\infty$) and on the specific dependence of the coupling constant $G_{J\ell n}$ (see Eq. (15)) on the orbital and radial quantum numbers ℓ, n. In general words, the Regge character of the spectrum is determined in our model by the confining properties of the vacuum field.

Numerical calculation of masses of the first orbital excitations of π, K, ρ and K^* mesons by means of Eq. (21) gives the masses shown in Table III. The super-fine structure of the excited states of ρ and K^* mesons coming from classification of currents over total momentum is qualitatively correct. Super-fine splitting of the levels with $\ell = 1$ is not very large.

TABLE III

The masses (MeV) of orbital excitations of π, K, ρ and K^* mesons. Super-fine structure of the $\ell = 1$ excitation of ρ and K^* is shown (ℓ is the orbital momentum and j is the total momentum (an observable spin) of a state).

Meson	ℓ	j	M	$M^{\exp}$
π	0	0	140	140
b_1	1	1	1252	1235
K	0	0	496	496
$K_1(1270)$	1	1	1263	1270
ρ	0	1	770	770
	1	0	1238	
a_1	1	1	1311	1260
a_2	1	2	1364	1320
K^*	0	1	890	890
	1	0	1274	
$K_1(1400)$	1	1	1342	1400
K_2^*	1	2	1388	1430

3.3. Heavy quarkonia

Exponential behavior of the quark propagator and vertices is responsible for the following relation between the masses of heavy quarkonia $M_{Q\bar{Q}}$ and heavy quark m_Q in the leading approximation [1]:

$$M_{Q\bar{Q}} = 2m_Q, \qquad \text{for} \qquad m_Q \gg \Lambda.$$

The next-to-leading term in the mass formula can be computed

$$\frac{\Delta^{(J)}_{Q\bar{Q}}}{\Lambda} = \frac{2(\sqrt{2}-1)}{\pi\sqrt{\pi}} C_J g^2 + O\left(\frac{\Lambda}{m_Q}\right), \tag{27}$$

where $C_P = 1/9$, $C_V = 1/18$. It should be stressed, that the difference in the constants

$$\Delta^{(P)}_{Q\bar{Q}} = 2\Delta^{(V)}_{Q\bar{Q}} \tag{28}$$

originates from the Fierz transformation of the Dirac matrices in the interaction term L_2 in representation (1). Relation (28) means that the vector quarkonium state is always heavier than the pseudoscalar one.

The results of numerical calculation of the masses of different heavy quarkonia states are summarized in Tables IV and V. The fit of the heavy meson masses gives $m_c = 1650$ MeV, $m_b = 4840$ MeV. An agreement with the experimental values is rather satisfactory. The super-fine splitting (χ_{c0}, χ_{c1}, χ_{c2} and so on) is very small, since it is regulated by the terms $O(1/m_Q)$ in Eq. (21). Its description is qualitatively correct.

TABLE IV

The spectrum of charmonium

Meson	η_c	J/ψ	χ_{c_0}	χ_{c_1}	χ_{c_2}	ψ'	ψ''
n	0	0	0	0	0	1	2
ℓ	0	0	1	1	1	0	0
j	0	1	0	1	2	1	1
M (MeV)	3000	3161	3452	3529	3531	3817	4120
$M^{\exp}$ (MeV)	2980	3096	3415	3510	3556	3770	4040

TABLE V

The spectrum of bottomonium

Meson	Υ	χ_{b_0}	χ_{b_1}	χ_{b_2}	Υ'	χ'_{b_0}	χ'_{b_1}	χ'_{b_2}	Υ''
n	0	0	0	0	1	1	1	1	2
ℓ	0	1	1	1	0	1	1	1	0
j	1	0	1	2	1	0	1	2	1
M (MeV)	9490	9767	9780	9780	10052	10212	10215	10215	10292
$M^{\exp}$ (MeV)	9460	9860	9892	9913	10230	10235	10255	10269	10355

We conclude that the correct description of the heavy quarkonia in our model is provided by the specific form of nonlocality of the quark and gluon propagators induced by the vacuum field, localization of meson field at the center of masses of constituent quarks and by a separation of the nonlocal currents with different total momentum.

3.4. Heavy-light mesons

Another interesting sector of meson spectrum is the heavy-light mesons, characterized by a rich physics [13, 14]. Our calculations show that in the limit $m_Q \gg \Lambda$ and $m_q \sim \Lambda$ the leading and next-to-leading terms for the heavy-light mesons are

$$M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q)\,, \qquad (29)$$

where the next-to-leading term $\Delta_{Q\bar{q}}^{(J)}$ does not depend on the heavy quark mass m_Q. In particular, we get

$$\Delta_{Q\bar{u}}^{(P)} = 20\ \text{MeV}\,, \qquad \Delta_{Q\bar{u}}^{(V)} = 155\ \text{MeV}\,,$$
$$\Delta_{Q\bar{s}}^{(P)} = 63\ \text{MeV}\,, \qquad \Delta_{Q\bar{s}}^{(V)} = 191\ \text{MeV}\,.$$

Results of numerical calculation of the masses and the weak decay constants for different pseudoscalar mesons are given in Tables V and VI.

TABLE VI

The masses and weak decay constants (MeV) of heavy-light mesons

Meson	D	D^*	D_s	D_s^*	B	B^*	B_s	B_s^*
M	1766	1991	1910	2142	4965	5143	5092	5292
M^{exp}	1869	2010	1969	2110	5278	5324	5375	5422
f_P	149	-	177	-	123	-	150	-

4. Discussion

For conclusion, we would like to point out several problems, that require more profound studying.

In order to clarify the basic assumption of this paper, one needs to get a reliable nonperturbative estimation of the free energy density or effective potential of QCD for the background field under consideration.

More detailed consideration of the chiral symmetry breaking by the background field is needed.

The coupling constant g in Table I is rather large. This point also has to be investigated carefully.

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