

SECOND QUANTIZATION OF ANTIPARTICLES WITH NEGATIVE FREQUENCIES: A ROUTE TO SOLVE THE CP-VIOLATION PUZZLE?*

L. HANNIBAL

Fachbereich Physik, Carl v. Ossietzky Universität Oldenburg
D-26111 Oldenburg, Germany
e-mail: hannibal@caesar.physik.uni-oldenburg.de

(Received July 3, 1996)

A fully covariant construction of quantum field theory with antiparticles realized by negative frequency states, but positive energy density is shown to provide a qualitative explanation for the experiments on the neutral K mesons so far interpreted as symmetry violation.

PACS numbers: 11.30. Er, 13.20. Eb

1. Introduction

The existence of the positron was predicted on the basis of the negative energy solutions of the Dirac equation, but then a physical interpretation of negative energy states was not found in the framework of single-particle theory. So, in the usual construction of quantum field theory antiparticle states have positive frequencies, and we have an axiomatic positivity condition on the free antiparticle spectrum [1–3].

But there is an alternative. In 1941 Stueckelberg [4] gave a classical picture of pair creation and annihilation where the electron-positron pair is described by a single world line which is reflected in time by electromagnetic interaction. The electron part of the world line has $dt/d\tau > 0$, where τ is the proper time, the positron part has $dt/d\tau < 0$. For particles the classical energy variable p_0 is positive, for antiparticles negative. Stueckelberg also devised a quantum theory which is in correspondence to this

* Presented at the II German–Polish Symposium “New Ideas in the Theory of Fundamental Interactions”, Zakopane, Poland, September 1995.

classical picture by introducing the proper time as an evolution parameter into quantum theory, with the mass as the conjugate invariant quantity, masses become the eigenvalues of the operator $i\partial_\tau$. Particles are described by positive frequency solutions, antiparticles have negative frequencies. It is well known [5] that this manifest covariant quantum theory solves the problems of localization and zitterbewegung which arise when we consider only the time-development in a 3×1 decomposition.

But Stueckelberg's classical and quantum picture, further extended by Feynman [6] to spin $1/2$, could not solve the negative energy problem. The idea we present here is a solution to this problem, in the way that, negative frequency states are interpreted in a consistent way with positive energy density, with corresponding pictures in classical theory, quantum mechanics and quantum field theory. This theory is used to analyse neutral K-meson decay with the result that no symmetry violation needs be assumed in order to explain the experimental observations.

2. Positivity of the energy density

We start with classical mechanics [4]. The constraint

$$\dot{x}^\mu(\tau)g_{\mu\nu}(x(\tau))\dot{x}^\nu(\tau) = c^2 \quad (1)$$

on the trajectories of classical point particles allows for solutions with negative $\dot{x}^0 = cdt/d\tau$, *i.e.* $t = -\tau$ for a particle at rest. These solutions were usually discarded, postulating $t = \tau$ for a particle at rest, until Stueckelberg showed that these solutions may be interpreted as antiparticles. With respect to the physical interpretation it is important to note that the solutions of the canonical equations with $d\tau/dt < 0$, which also have negative canonical energy $p_0 < 0$, possess a positive energy density [7]

$$T_0^0 = \int d\tau p_0 \dot{x}^0 \delta^4(x - x(\tau)), \quad (2)$$

which is positive due to the two negative signs of p_0 and \dot{x}^0 . (We use the sign convention $+- - -$ for the metric.)

With respect to quantum theory, Klein-Gordon theory has an energy density which is positive irrespective of the sign of the eigenvalues of the operator $\hat{p}_0 = \frac{i\hbar}{c} \frac{\partial}{\partial t}$. A solution to the positivity problem for spin $1/2$ was first indicated by Arshansky and Horwitz [9]. The argument is from representation theory. Since the energy density operators [8]

$$\hat{T}_0^0 = \hat{v}^0 \hat{p}_0, \quad \hat{p}_0 = \frac{i\hbar}{c} \frac{\partial}{\partial t}, \quad \hat{v}^0 = \begin{cases} \frac{i\hbar}{mc} & \text{for spin } 0 \\ \gamma^0 & \text{for spin } 1/2 \end{cases} \quad (3)$$

always have a positive spectrum the question of a positive energy density is a matter of choosing a positive definite scalar product which preserves the sign of the eigenvalues when expectation values are taken. The idea by Arshansky and Horwitz [9] was to start with two-component spinors which transform under Wigner's unitary induced representation or its conjugate representation, for which we have a manifest positive norm [10]. Then the embedding of the two-component spinors into a space of four-component spinors is done in a norm-preserving way, leading to the positive norm

$$N = \int \frac{d^3p}{p_0} \psi^* \gamma^0 \psi \quad (4)$$

on the embedded subspace [11]. We note that the functional form (4) of the norm, with a sign included for antiparticles, is the same as in standard quantum field theory, where antiparticle states are realized with positive frequencies [1] p. 188.

3. Second quantization, discrete symmetries

The second quantization of negative frequency states can be carried out just as that for positive frequency states, with the antiparticle sector related to the particle sector by an antilinear, antiunitary charge conjugation transformation, which is precisely the one from usual Dirac and Klein-Gordon theory extended to Fock space [12, 13]. The decisive point is the construction of the energy density operator, which is done in the following different way compared to standard theory. In standard theory the free Dirac quantum field $\hat{\psi}(x^i)$ is constructed as a time-zero field with Hermitian conjugate $\hat{\psi}^\dagger(x^i)$, so that $\hat{\psi}^\dagger\hat{\psi}$ is a positive operator. The time dependence is generated in a Heisenberg picture by $\hat{\psi}(x^i, t) = e^{iHt}\hat{\psi}(x^i)e^{-iHt}$. The energy density operator : $\hat{\psi}^\dagger i\partial_t \hat{\psi}$ + h.c. : then is positive definite if and only if the Hamiltonian is positive. In the 4×1 decomposition the adjoint $\bar{\hat{\psi}}(x^\mu)$ of the field $\hat{\psi}(x^\mu)$ is constructed with respect to the four-dimensional, Lorentz invariant scalar product on Fock space, such that $\bar{\hat{\psi}}\hat{\psi}$ is positive, and the dependence on proper time is generated by a positive definite mass operator M , $\bar{\hat{\psi}}(x^\mu, \tau) = e^{iM\tau}\bar{\hat{\psi}}(x^\mu)e^{-iM\tau}$. The energy density operator is given by $\bar{\hat{\psi}}\gamma^0 i\partial_t \hat{\psi}$ and now the negative entries of the matrix γ^0 allow for positive definiteness only if the operator $\hat{p}_0 = H$, generator of infinitesimal translations in time is negative on antiparticle states. This construction was carried out rigorously [13]. The use of negative frequencies is not in contradiction to the axiomatic approach, since it is well known that positivity is independent of all other axioms [2, 3].

In original Dirac and Klein-Gordon theories the charge conjugation is an antilinear transformation. In the parametrized theory this charge conjugation transformation is extended to parameter-dependent wave-equations, whereby it retains its antilinearity, and, due to a positive definite scalar product, is also antiunitary. For clarity we denote this antiunitary charge conjugation by \tilde{C} . \tilde{C} transforms τ into $-\tau$ and p_μ into $-p_\mu$ with x invariant. The parity transformation P remains unitary and the time inversion transformation T antiunitary, since both can first be defined on the particle sector alone and then carried over to the antiparticle sector with help of the charge conjugation transformation. The time-inversion carries t into $-t$ and τ into $-\tau$, so that particles remain particles. The combined \tilde{CPT} -transformation, which leaves τ invariant, is unitary. Since antiunitary transformations do not have eigenvalues and eigenstates we cannot define \tilde{C} or $\tilde{C}P$ eigenstates in our theory, but we can define $\tilde{C}T$ and \tilde{CPT} eigenstates. The action of the symmetry transformations on spin 0 and spin 1/2 wave functions is identical with that of quantum mechanics. The infinitesimal generator $i\partial_\tau$ of translations of the invariant parameter is invariant under all these transformations, whereas the infinitesimal generator $i\partial_t$ of time translations is invariant only under P and T , changing sign under \tilde{C} and \tilde{CPT} . As a consequence any Hamiltonian must be indefinite in a \tilde{CPT} -symmetric theory.

We consider some relevant subsystem Ψ_i of a physical system, with a finite number $i = 1, \dots, n$ of states, for which an effective evolution equation is assumed. The development of Ψ_i in time is governed by an effective Hamiltonian matrix \tilde{H}_{ij} ,

$$i\partial_t \Psi_i = \tilde{H}_{ij} \Psi_j, \quad (5)$$

whereas the evolution in proper time τ is described by an effective mass matrix M_{ij} with

$$i\partial_\tau \Psi_i = M_{ij} \Psi_j. \quad (6)$$

If the invariance of the subsystem under \tilde{CPT} is assumed, the mass matrix is invariant under \tilde{CPT} , as $i\partial_\tau$ is, whereas the Hamiltonian \tilde{H}_{ij} is transformed into $-\tilde{H}_{ij}$ under \tilde{CPT} , as is $i\partial_t \rightarrow -i\partial_t$. The mass matrix may be chosen to be positive definite, the Hamiltonian necessarily is indefinite.

4. The neutral K-meson system

For all definitions which are not standard we use a tilde.

The K^0 meson is a spin 0 particle consisting of a d and a \bar{s} quark, which have spin 1/2. Since $\tilde{C}P$ is not unitary, we use \tilde{CPT} to define the antiparticle to the K^0 meson, which we denote by $\tilde{\bar{K}}^0$, by

$$\tilde{CPT} | K^0 \rangle = | \tilde{\bar{K}}^0 \rangle, \quad \tilde{CPT} | \tilde{\bar{K}}^0 \rangle = | K^0 \rangle, \quad (7)$$

so that we have \tilde{CPT} eigenstates

$$\begin{aligned} |\tilde{K}_1\rangle &= \frac{1}{2} \left(|K^0\rangle - |\tilde{K}^0\rangle \right), \quad \tilde{CPT} = -1, \\ |\tilde{K}_2\rangle &= \frac{1}{2} \left(|K^0\rangle + |\tilde{K}^0\rangle \right), \quad \tilde{CPT} = +1. \end{aligned} \quad (8)$$

We describe the K^0 and \tilde{K}^0 mesons by a two-state vector $\Psi = (|K^0\rangle, |\tilde{K}^0\rangle)$, where \tilde{CPT} interchanges both components through the multiplication by the 2×2 matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Assuming a \tilde{CPT} -invariant evolution equation the 2×2 mass matrix is invariant under \tilde{CPT} , which implies

$$M_{11} = M_{22} \quad \text{and} \quad M_{12} = M_{21}. \quad (9)$$

It follows that $|\tilde{K}_1\rangle$ and $|\tilde{K}_2\rangle$ are eigenstates of M , just as in standard theory. Now in standard theory the assumption that both Hamiltonian and mass matrix are invariant under the CP -transformation leads to the conclusion that $|\tilde{K}_1\rangle$ and $|\tilde{K}_2\rangle$ are also eigenstates of the Hamiltonian, and cannot mix in time. The experimentally observed decay of the longest-living state into both CP -eigenstates hence implies that the physical states K_L and K_S are distinct from $|\tilde{K}_1\rangle$ and $|\tilde{K}_2\rangle$ and thus CP -symmetry must be violated [14]. This is different in our theory. \tilde{CPT} -invariance in our theory implies that our Hamiltonian \tilde{H} changes sign under \tilde{CPT} , so we have

$$\tilde{H}_{11} = -\tilde{H}_{22} \quad \text{and} \quad \tilde{H}_{12} = -\tilde{H}_{21}. \quad (10)$$

Transforming \tilde{H} into the basis $\Psi' = (|\tilde{K}_1\rangle, |\tilde{K}_2\rangle)$ yields

$$\tilde{H}' = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tilde{H} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \tilde{H}_{11} + \tilde{H}_{12} \\ \tilde{H}_{11} - \tilde{H}_{12} & 0 \end{pmatrix}. \quad (11)$$

The structure of \tilde{H}' is a general consequence of the Hamiltonian being \tilde{CPT} -odd. If A and B are any two \tilde{CPT} -eigenstates with eigenvalues $a, b \in \{+1, -1\}$ then

$$\langle A | \tilde{H} | B \rangle = \langle \tilde{CPT} A | \tilde{CPT} \tilde{H} (\tilde{CPT})^{-1} \tilde{CPT} | B \rangle = -ab \langle A | \tilde{H} | B \rangle, \quad (12)$$

so that only states with different \tilde{CPT} -eigenvalues yield nonzero matrix elements. From (11) we see that the states $|\tilde{K}_1\rangle$ and $|\tilde{K}_2\rangle$ always mix

in time if \tilde{H} is nonzero. The \tilde{CPT} -eigenstates are not preserved in time, the eigenstates of the Hamiltonian are not \tilde{CPT} -eigenstates. Hence, if we start with an arbitrary initial state and wait until only the slowest-decaying eigenstate has survived, we will always observe that this state decays into both channels of \tilde{CPT} -eigenstates. So we are not forced to assume \tilde{CPT} - or \tilde{CP} - violation in order to explain *qualitatively*, from first principles, the experimentally observed phenomenon of the long-living K_L decaying into different \tilde{CPT} -eigenstates. The dynamical mixing of the $|\tilde{K}_1\rangle$ and $|\tilde{K}_2\rangle$ also offers an explanation for the fact that CP -violation practically is not seen in the decay of the physical K_S [14]: The short decay time provides no time for the mixing process to happen. A numerical study of a simple model system confirms the mechanism as described [11].

REFERENCES

- [1] N.N. Bogolubov, A.A. Logunov, I.T. Todorov, *Introduction to Axiomatic Quantum Field Theory*, Benjamin, London 1975.
- [2] R.F. Streater, A.S. Wightman, *PCT, Spin & Statistics, and All That*, Benjamin, New York 1963.
- [3] M. Reed, B. Simon, *Methods of Modern Mathematical Physics Vol II: Fourier Analysis, Self-Adjointness*, Academic Press, New York 1975.
- [4] E.C.G. Stueckelberg, *Helv. Phys. Acta* **14**, 322 and 588 (1941), and *Helv. Phys. Acta* **15**, 23 (1942).
- [5] J.R. Fanchi, *Found. Phys.* **23**, 487 (1993); J. R. Fanchi, *Parametrized Relativistic Quantum Theory*, Kluwer, Dordrecht 1993 and references therein.
- [6] R.P. Feynman, *Phys. Rev.* **74**, 939 (1948) and *Phys. Rev.* **80**, 440 (1950).
- [7] L. Hannibal, *Int. J. Theor. Phys.* **30**, 1431 (1991).
- [8] L. Hannibal, *Int. J. Theor. Phys.* **30**, 1445 (1991).
- [9] L.P. Horwitz, R. Arshansky, *J. Phys. A: Math. Gen.* **15**, L659 (1982).
- [10] E. Wigner, *Ann. Math.* **40**, 149 (1949); M.A. Naimark, *Linear Representations of the Lorentz Group*, Pergamon, New York 1964.
- [11] L. Hannibal, *Found. Phys. Lett.* **8**, (1995), in print.
- [12] L. Hannibal, *Rep. Math. Phys.* **33**, 77 (1993).
- [13] L. Hannibal, *Found. Phys. Lett.* **7**, 551 (1994).
- [14] J.H. Christensen, J. W. Cronin, V.L. Fitch, R. Turlay, *Phys. Rev. Lett.* **13**, 138 (1964); L. K. Gibbons *et al.*, *Phys. Rev. Lett.* **70**, 1199 and 1203 (1993); Particle Data Group, L. Montanet *et al.*, *Phys. Rev.* **D50**, 1173 (1994) and references therein; T.T. Wu, C.N. Yang, *Phys. Rev. Lett.* **13**, 380 (1964); V. Barmin *et al.*, *Nucl. Phys.* **B247**, 293 (1984); L. Wolfenstein, *Ann. Rev. Nuc. Sci.* **36**, 137 (1986).