PION AND ρ MESON MASSES FROM QUARK AND GLUON CONDENSATES*

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Nonzero quark and gluon condensates generate nonzero value of pion mass, even in the zero limit of current quark mass. In turn, the nonzero ρ meson width is due to the nonzero pion mass. Nonzero quark and gluon condensates are also necessary conditions for permanent confinement of quarks and gluons. The notion of permanent confinement means: *(i)* the nonexistence of any asymptotic quark or gluon states, *(ii)* the nonexistence of any asymptotic continuum partonic states, and *(iii)* the nonexistence of any colourfull bound systems. Only colourless hadrons composed of permanently confined partons are present. These hadrons must be calculated as solutions of truly relativistic bound state equations. Masses of constituent quarks are defined in hadrons, and crucially depend on the magnitude of a space-like Wightman–Garding relative momentum. The dominating binding potential of constituents in hadrons is the QCD analog of Coulomb interaction. There is no place for any interaction between constituents which could increase indefinitely with a space-like separation of constituents. For example, a linear interaction, with singularity $(q-q')^{-4}$ in momentum space, is ruled out on two grounds: *(i)* as contradicting Dyson–Schwinger equations, and *(ii)* as being in conflict with the cluster property of local QCD, if there is a nonzero mass gap. For QCD with quark condensate for up and down quarks the Goldstone theorem fails. If it would hold it would require massless pion for massless quarks, and the possibility of approximating the pion field by a local field. Nonperturbative QCD with permanently confined quarks says "no" to both of these claims.

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1. Introduction

$\Lambda_{\rm QCD}$ is truly nonperturbative QCD mass scale. It arrises from dimensional transmutation when proving asymptotic freedom in the domain of large space-like virtualities. For these virtualities there are absent any physical singularities, and perturbative QCD calculations are appropriate. In the nonperturbative domain of QCD beside $\Lambda_{\rm QCD}$ there are other mass scales such as the quark and gluon condensate mass scales. These mass scales are of the same order of magnitude as $\Lambda_{\rm QCD}$.

The mass scales of quark and gluon condensates are present in the trace of the QCD energy momentum tensor

$$\Theta^{\mu}_{\mu} = -\langle| \frac{\alpha}{\pi} : G^{a}_{\mu\nu} G^{\mu\nu}_{a} :|\rangle + (1+\gamma_m) \Sigma_f m_f \langle|: \overline{\psi_f} \psi_f :|\rangle , \quad (1)$$

where $\langle| \frac{\alpha}{\pi} : G^{a}_{\mu\nu} G^{\mu\nu}_{a} :|\rangle \equiv \nu^4$ is the gluon condensate with Shifman-Veinshtein -Zakharov [1] mass scale $\nu = 331$ MeV, γ_m is the mass anomalous dimension, m_f is the current quark mass of the QCD Lagrangian density, and Σ_f is sum over all flavours. The quark condensate is $\langle| \sqrt{\alpha} : \overline{\psi}\psi :|\rangle \equiv -\chi^3$, with $\sqrt{\alpha}$ to account for the same logarithmic factor as provided by the variation of m_f under the renormalization group equation, to make $\langle| \sqrt{\alpha} : \overline{\psi}\psi :|\rangle$ independendent of any momentum scale. For u (d), s, c, b, and t quarks the mass scale χ is approximately equal to the following fractions of GeV: 1/4, 1/5, 1/10, 0.06, and 0.02, respectively. More precisely, the mass scale χ is: 240 MeV for u and d quarks, 222 MeV for s quark, 90 MeV for c quark, 60 MeV for b quark, and 20 MeV for t quark.

The dynamical origin of χ and ν is still unknown except for their presence in Θ^{μ}_{μ}, and at this moment the numerical values of χ and ν must be taken from phenomenological fits such as the QCD sum rules. However, once these condensates are recognized as being nonzero, then we can self-consistently reproduce their values from nonlinear equations which arrise if we calculate a vacuum-to-vacuum transition by closing up in the position space either quark or gluon line. In these closed loops we put nonperturbative propagators, and for renormalization purpose we have to subtract loops corresponding to perturbative propagators.

The condensate mass scales χ and ν are also present in the residua of physical singularities, in inverses of quark and gluon propagators [2],

$$S^{-1}(p) = \not p - m + i\varepsilon + \frac{\chi^3/M}{\not p - M + i\varepsilon}, \quad D_T^{-1}(p) = p^2 + i\varepsilon + \frac{\nu^4}{p^2 + i\varepsilon}, \quad (2)$$

where m is the current quark mass, and M is the mass of the corresponding pseudoscalar meson: π for u, d; K for s; D for c; and B for b.

In the above expression for $S^{-1}(p)$ beside the mass scale χ there is also the mass M of pseudoscalar meson. The value of M is uniquelly determined from truly relativistic bound state equation as the position of the physical pole for this hadron which is the pseudoscalar meson.

M is also the physical pole, in the sense of the Feynman pole, in the quark- gluon-quark three-point vertex function. This is demanded by Slavnov–Taylor identities, and the Feynman singularity at $p^2 = M^2$ produces a nonzero absorptive part and results in the nonzero cross section measured in the experimental counters after quarks and gluons undergo a soft hadronization. In the language of high energy physics the soft hadronization is manifestation of the "Local Parton - Hadron Duality", with "Local" refering to momentum space, and "Duality" meaning that after soft hadronization (also known as the "soft blanching"), hadrons follow colourless clusters of parent partons in momentum space.

Physical singularities present in Eq. (2), together with corresponding singularities in three-point vertex functions, allow us to solve exactly at these singularities a decoupled finite subset of Dyson-Schwinger equations. The mathematical basis for this solution are nonperturbative logarithms of Stingl [3] directly connected with ultra-violet divergences of loops and their renormalization. The square of unrenormalized coupling constant, present in Dyson-Schwinger equations, is canceled out by a sum of a nonperturbative logarithm and a divergent term in dimensional regularization. For such cancelation to take place it is essential that the remormalized running coupling constant is small for an arbitrarly small parton virtuality.

In three-point vertex functions there are two sets of singularities: *(i)* singularities demanded by Slavnov-Tylor identities, *i.e.* by algebraic constraints in momentum space on the longitūdinal projection of three-point vertex functions and inverses of propagators, and *(ii)* physical singularities in the transverse part of three-point vertex functions which are beyond control of Slavnov–Taylor identities. We do not have any infrared singularities in loops because the nonzero mass scales χ and ν protect us from infrared problems.

A very important bonus of physical singularities in inverses of quark and gluon propagators in Eq. (2) is the immediate fulfilment of the necessary condition for permanent confinement of quarks and gluons in hadrons. Inverting algebraically Eq. (2) we find following expressions for gluon and quark propagators [2]

$$D_T(p) = \frac{p^2}{p^4 + \nu^4} = \frac{1}{2}\left(\frac{1}{p^2 + i\nu^2} + \frac{1}{p^2 - i\nu^2}\right). \tag{3}$$

$$S(p) = \frac{Z(\not{p}+\rho)}{p^2-\rho^2} + \frac{Z^*(\not{p}+\rho^*)}{p^2-\rho^{*2}}\,, \tag{4}$$

where two complex quantities Z and ρ are uniquely determined by the pseudoscalar meson mass M, the current quark mass m, and the quark condensate mass scale χ

$$\mathrm{Re}\,Z = \tfrac{1}{2}\,, \qquad \mathrm{Im}\,Z = \frac{(M-m)/4}{\sqrt{\chi^3/M - \frac{1}{4}\,(M-m)^2}}\,,$$

$$\mathrm{Re}\,\rho = \tfrac{1}{2}(m+M)\,, \quad \mathrm{Im}\rho = \sqrt{\chi^3/M - \tfrac{1}{4}(M-m)^2}\,. \tag{5}$$

In Eqs (3) and (4) we see that any asymptotic states either for gluons or for quarks are absent, *i.e.* we do not have any real momentum poles in Eqs (3) and (4). The presence of complex poles in these equations has nothing to do either with the breaking of causality, or with the breaking of unitarity. We must remember that causality restricts commutators (anticommutators) of the fundamental QCD fields and this restriction is very well respected both by Dyson–Schwinger equations and by Slavnov–Taylor identities.

The unitarity condition is on the physical S-matrix in the hadronic world, with hadrons being the asymptotic states. In contrast to hadrons the fundamental QCD fields do not have any asymptotic states. The asymptotic hadrons are represented by the physical poles in many-point Green functions, with their residua expressed in terms of relativistic wave functions. These wave functions are calculated in momentum space. If we Fourier transform hadronic poles to position space, then we find that hadrons must be represented by nonlocal fields. The nonlocality of an effective hadronic field in position space is one of the basic reasons for the absence of the Goldstone theorem for the physical pion. Our pion is a bound system of permanently confined quarks and gluons, and as such it can not be approximated by a local field in position space.

For large parton virtualities both quark and gluon propagators in Eqs (4) and (3) reduce trivially to the well known perturbative (asymptotic freedom) propagators. Then, and only then, the perturbative QCD S-matrix makes sense and unitarity condition is obviously satisfied.

2. Constituent quark mass $\mathcal{M}(q^2)$

Relativistic, relative four-momentum q of two constituents of masses m_1 and m_2 must be a space-like four-vector to insure two basic properties of the relativistic relative-motion:

1. The space-like character of $q(q^2 < 0)$ which is the necessary condition for the proper relativistic definition of angles between various relative momenta during relativistic relative-motion with cosine of angles in the interval $[-1.0, 1.0]$. This restriction on the cosine of an angle is necessary for any sensible angular momentum (partial wave) analysis. It is also necessary for defining the orbital angular momentum in Minkowski space, and consecutively the total spin of a hadron as the bound system of constituent quarks.
2. The cluster decomposition property in the sense of decoupling the relative motion dynamics, described by three degrees of freedom of constrained momentum $q(qP = 0)$ from the overall motion of the whole hadron with the total momentum P on its mass-shell $P^2 = M^2$.

The space-like character of q is guaranteed by the condition $qP = 0$, with P the time-like hadron momentum. The condition $qP = 0$ is kept as a constraint on the truly relativistic hadronic wave function. Such q is the Weightman–Garding vector [4]

$$q = \frac{1}{2}\left(1 - \frac{m_1^2 - m_2^2}{P^2}\right)p_1 - \frac{1}{2}\left(1 + \frac{m_1^2 - m_2^2}{P^2}\right)p_2\,,\quad P = p_1 + p_2\,, \tag{6}$$

where $\overline{P}^2$ is a positive quantity determined by the negative value of q^2, *i.e.* we have

$$\overline{P}^2 \equiv \left(\sqrt{m_1^2 - q^2} + \sqrt{m_2^2 - q^2}\right)^2\,. \tag{7}$$

Note, that if we make an off-shell continuation in the relative motion of two constituents in a hadron the quantity q^2 is unconstrained and it is always non-positive.

We have to demand the orthogonality condition $qP = 0$ of the space-like relative momentum q and the time-like total momentum $P = p_1 + p_2$. For the time-like total momentum P, constrained by $P^2 = M^2$, we can go to the bound state rest-frame $\vec{P} = 0$, and there $qP = 0$ means, that the time-component of the Wightman–Garding relative momentum is zero ($q^0 = 0$), what proves the space-like character of q ($q^2 < 0$). It also shows that the off-shell continuation in total energy of this system is done only in the time-component of total momentum, without changing reflection symmetry properties under the change of the sign of the off-energy-shell continued time-component of total momentum.

In the case of two constituent quarks in a hadron the product of two Wheeler quark propagators $S(p_1)S(p_2)$ is equal to four terms with various combinations of complex quantities ρ^2 and ρ^{*2}. For four combinations of

ρ^2 and ρ^{*2} we introduce four analogs of the Wightman–Garding relative momentum q, and four accompanying quantities $\overline{P}^2$:

$$q = \tfrac{1}{2}(p_1 - p_2)\,, \qquad \overline{P}^2 \equiv 4(\rho^2 - q^2)\,, \tag{8a}$$

$$q = \tfrac{1}{2}(p_1 - p_2)\,, \qquad \overline{P}^{*2} \equiv 4(\rho^{*2} - q^2)\,, \tag{8b}$$

$$q_a = \frac{1}{2}\left(1 - \frac{\rho^2 - \rho^{*2}}{\overline{P_a}^2}\right)p_1 - \frac{1}{2}\left(1 + \frac{\rho^2 - \rho^{*2}}{\overline{P_a}^2}\right)p_2\,,$$

$$\overline{P_a}^2 \equiv \left(\sqrt{\rho^2 - q_a^2} + \sqrt{\rho^{*2} - q_a^2}\right)^2\,, \tag{8c}$$

$$q_b = \frac{1}{2}\left(1 - \frac{\rho^{*2} - \rho^2}{\overline{P_b}^2}\right)p_1 - \frac{1}{2}\left(1 + \frac{\rho^{*2} - \rho^2}{\overline{P_b}^2}\right)p_2\,,$$

$$\overline{P_b}^2 \equiv \left(\sqrt{\rho^2 - q_b^2} + \sqrt{\rho^{*2} - q_b^2}\right)^2\,. \tag{8d}$$

The real relative momentum q corresponds to two terms with two pairs: $(\rho^2,\ \rho^2)$ and $(\rho^{*2},\ \rho^{*2})$. Only for these pairs we can demand the crucial constraint $qP = 0$. For the remaining two pairs: $(\rho^2,\ \rho^{*2})$ and $(\rho^{*2},\ \rho^2)$ to which correspond q_a and q_b, respectively, it is impossible to set equal to zero either $q_a P$, or $q_b P$. The easiest way to see it is in the bound state rest-frame ($\vec{P} = 0$), where the vanishing of either Im (q_a^0), or Im (q_b^0) is prevented by the nonzero value of Im ρ.

The Wheeler propagator for the relative motion of a confined quark and antiquark which form a bound system of mass M must depend on the relative momentum q obeying $qP = 0$ with $P^2 = M^2$. This propagator is a sum of two terms, corresponding to two pairs of terms: $(\rho^2,\ \rho^2)$, and $(\rho^{*2},\ \rho^{*2})$. The remaining two pairs: $(\rho^2,\ \rho^{*2})$, and $(\rho^{*2},\ \rho^2)$ are absent because Im $\rho \neq 0$. Therefore, the Wheeler propagator of the relative-motion of confined constituents is

$$\left[\frac{Z^2(\not{p}_1 + \rho)(\not{p}_2 + \rho)}{(p_1^2 - \rho^2)(p_2^2 - \rho^2)}\right]_{|qP=0} + \left[\frac{Z^{*2}(\not{p}_1 + \rho^*)(\not{p}_2 + \rho^*)}{(p_1^2 - \rho^{*2})(p_2^2 - \rho^{*2})}\right]_{|qP=0} \tag{9}$$

Here, the individual momenta of constituents are p_1 and p_2, and they have to be replaced by the space-like relative momentum q and the time-like total momentum P. The dependence on P in Eq. (9) must be extracted, and then in the relativistic relative-motion propagator we find in the denominator of two complex conjugate terms of Eq. (9) the expression $\mid \rho^2 - q^2 \mid$ which is the QCD analog of a similar factor $(\vec{q}^2 + \mathrm{m}^2)$, known in the center of mass system when each constituent has the rest mass equal to m.

For a hadron of momentum P, with $P^2 = M^2$ and $Pq = 0$, we introduce the notion of a constituent quark mass $\mathcal{M}(q^2)$ by writing the identity

$$| \rho^2 - q^2 | \equiv \mathcal{M}^2(q^2) - q^2 , \tag{10}$$

and the expression for $\mathcal{M}$ follows from Eq. (10)

$$\begin{aligned} \mathcal{M}(q^2) &= \sqrt{| \rho^2 - q^2 | + q^2} \\ &= \sqrt{\sqrt{(mM + \chi^3/M)^2 - q^2(m^2 + M^2 - 2\chi^3/M) + q^4} + q^2} . \end{aligned} \tag{11}$$

It is interesting to note, that in the non-QCD limiting case, when $\chi = 0$ and $M = m$, *i.e.* $\rho = \rho^* \equiv m$, the constituent mass $\mathcal{M}(q^2) \equiv m$ is independent of q^2 and coincides with the mass m of the equal mass constituents. On the other hand in QCD, *i.e.* when $\chi \neq 0$ and $M \neq m$, there are two other limiting cases for the value of $\mathcal{M}(q^2)$: a) $q^2 \to 0$, and b) $q^2 \to -\infty$. Respectively, we have

$$\mathcal{M}(0) = | \rho | = \sqrt{mM + \chi^3/M}, \qquad \text{and} \qquad \mathcal{M}(-\infty) = 0 . \tag{12}$$

The constituent quark mass $\mathcal{M}(q^2)$ takes its maximum value at the mimnimal value of $-q^2$, *i.e.* when both quark constituents are at rest in their relative motion in the bound-state rest-frame. The constituent quark mass $\mathcal{M}(q^2)$ decreases to zero at the largest value of $-q^2$, independently of the quark flavour.

Numerical values of $\mathcal{M}(0) = | \rho |$ depend on the value of M which in turn can be adjusted by chosing a particular value of the angle ϕ_q which determines the complex quantity ρ. There is a cubic equation for M with three parameters: m, χ, and ϕ_q

$$\mathrm{M}^3 - 2\mathrm{m}\mathrm{M}^2\cos(2\phi_q) + \mathrm{m}^2\mathrm{M} - 4\chi^3\cos^2(\phi_q) = 0 , \tag{13}$$

and for the u (d) quarks in the limit $\mathrm{m}_{\mathrm{u,d}} \to 0$ we find $\mathrm{M} = \chi(4\cos^2(\phi_{u,d}))^{1/3}$ and $\mathcal{M}(0) = | \rho | = \chi/(2\cos(\phi_q))^{1/3}$. For $\chi = 240$ MeV and $\phi_{u,d} = 79^o$ we have M = 126 MeV and $\mathcal{M}(0) = | \rho | = 331$ MeV, and for $\chi = 250$ MeV, $\phi_{u,d} = 78^o$ we get M = 139 MeV and $\mathcal{M}(0) = 335$ MeV. For heavy quarks (c, b, t) we have: $\mathrm{M} \approx \mathrm{m}(1{+}2\sin(\phi_{c,b})) \approx \mathrm{m}(1{+}\ \nu^2/\ \sqrt{3}\ \mathrm{m}^2) \approx \mathrm{m}$, and $\mathcal{M}(0) = | \rho | \approx$ m (the current mass).

Although the u (d) quarks have negligably small current mass m, nevertheless their constituent quark mass $\mathcal{M}(q^2)$ at zero relative momentum

($q^2 = 0$) is $\mathcal{M}(0)$= 0.33 GeV, *i.e.* $\mathcal{M}(0)$ is of the order of Λ_{QCD} for light quarks u and d. In many models of hadrons composed of the light valence quarks u and d it is crucial to have sizable constituent quark mass $\mathcal{M}(q^2)$ in the description of static properties of hadrons, while for the deep inelastic structure functions of the same hadrons, when constituent virtualities are very large, the constituent quark mass $\mathcal{M}(q^2)$ must be very close to zero to agree with negligably small helicity spin-flip amplitudes.

3. Pion mass and rho meson width both $\neq 0$

Nonzero pion mass M and nonzero ρ meson width follow from solutions of truly relativistic bound state equations for constituent quark and antiquark. To write these equations we must assume nonzero values of the mass scales χ and ν. They are explicitly in inverses of quark and gluon propagators in Eq. (2), in the relative-motion propagator in Eq. (9), and in the quark-gluon-quark vertex function. Transverse part of the quark-gluon-quark vertex function contains many terms with real constants determined from simultaneous solutions of a decoupled subset of Dyson-Schwinger equations.

The quark-gluon-quark vertex function is given below, and for ilustration we write only one transverse term which explicitly generates the QCD analog of Coulomb interaction between quark constituents exchanging a nonperturbative gluon

$$\Gamma^{\mu,c}(p_1, q-q', p_1') = \frac{\lambda^c}{2}\left\{\gamma^\mu - \frac{\chi^3/M}{p_1.\gamma - M + i\varepsilon}\,\gamma^\mu \frac{1}{p_1'.\gamma - M + i\varepsilon} + \frac{A\chi^3}{M}\,\frac{p_1.\gamma\gamma_T^\mu p_1'.\gamma}{(q-q')^2 + i\varepsilon}\left(\frac{1}{p_1^2 - M^2 + i\varepsilon} + \frac{1}{p_1'^2 - M^2 + i\varepsilon}\right)\right\}, \tag{14}$$

where p_1, and p'_1 denote the incoming, and the outgoing quark momentum, respectively, $q - q'$ is the outgoing gluon momentum, A is one of real constants determined from solutions of a decoupled subset of Dyson–Schwinger equations, and $\gamma_T^\mu \equiv (q-q')^\mu (q.\gamma - q'.\gamma)/(q-q')^2 - \gamma^\mu$.

The QCD analog of Coulomb interaction arrises from two singular factors $1/(q-q')^2$ in two quark-gluon-quark vertex functions of the nonperturbative one-gluon exchange, and one factor of the gluon momentum squared $(q-q')^2$ of the exchanged gluon. After canceling two factors $(q-q')^2$ in the numerator and in the denominator, the remaining singular factor $1/(q-q')^2$ of the one-gluon exchange interaction becomes $1/(\vec{q} - \vec{q'})^2$, because both relative momenta q and q' obey the Wightman-Garding orthogonality condition: $qP = 0$ and $q'P = 0$, *i.e.* $q^0 = q'^0 = 0$ in the hadron rest frame ($\vec{P} = 0$).

Singular terms in the gluon momentum generate the QCD-Coulomb interaction $1/(\vec{q} - \vec{q'})^2$ in the hadron rest frame. Remaining terms of the quark-gluon-quark vertex which are nonsingular in the gluon momentum, for example, first two terms of Eq. (14) required by Slavnov–Taylor identity, give rise to non-Coulomb corrections. However, all terms in Eq. (14), except for the perturbative part of the quark-gluon-quark vertex contain singular terms in quark momenta. These factors, depending on the individual quark momenta and not on their differences, are the main source of a nonlocal dependence in position space of the quark-antiquark interaction. Also these terms are responsible for the nonlocality of an effective hadronic field in position space.

It is totally impossible in nonperturbative QCD with permanently confined quarks to generate quark-antiquark interaction with the $(q - q')^{-4}$ singularity. In gluon momentum, the highest singularity of the quark-gluon-quark vertex function is only $(q - q')^{-2}$, and in the numerator of nonperturbative gluon propagator there is always the factor $(q - q')^2$. The absence of the $(q - q')^{-4}$ singularity, in the context of a nonperturbative gluon propagator, is shown in Ref. [5] as producing even more singular terms than $(q - q')^{-4}$ in Dyson–Schwinger equations, if extended beyond one loop level.

If one would allow for the singularity $(q - q')^{-4}$, then the Fourier transform of it in four dimensions gives a logarithmically rising potential in position space, while for the case of $q^0 = q'^0 = 0$ the Fourier transform in three dimensions leads to a linerly rising quark-antiquark interaction. For the case of QCD with nonzero values of quark and gluon condesates which generate nonzero hadron masses, *i.e.* a nonzero mass gap in the hadronic spectrum, it is possible to prove that the nonzero mass gap necessairly leads to the cluster property [6] and excludes the possibility of any rising potential either logarithmically, or linearly.

The QCD-Coulomb interaction between constituent quark and antiquark is proportional to the mass scales χ and ν, therefore it can only act if $\chi \neq 0$. In turn, the nonzero value of the pion mass M which is mainly generated by the QCD-Coulomb binding is due to $\chi \neq 0$ and $\nu \neq 0$. For the QCD-Coulomb interaction it is possible to extend the well known results of Fock and Schwinger [7] for the exact solution of hydrogen, or positronium, to the domain of complex numbers. The S-wave positronium wave function, in units of the electron mass set equal to unity, is

$$\sqrt{2\pi\alpha_e^5}(\vec{q}^2 + \alpha_e^2/4)^{-2}\,, \tag{15}$$

where α_e is the QED fine structure constant, and $\vec{q}$ is the electron-positron relative momentum in the positronium center of mass system.

For pion the dominating QCD-Coulomb interaction allows us to approximate the relativistic pion wave function by a difference of two complex conjugated terms

$$\frac{(\rho/A)^2}{(\vec{q}^2+\rho^2)^2} - \frac{(\rho^*/A)^2}{(\vec{q}^2+\rho^{*2})^2}, \tag{16}$$

where $\vec{q}$ is the space component of real Wightman–Garding relative momentum q of a constituent quark with respect to a constituent antiquark in the pion rest frame, and real constant A is the same one which appears in the transverse part of the quark-gluon-quark vertex function.

In finding Eq.(16) we simplified spinor factors in the relativistic bound state equation, replacing them by effective scalar factors, to get an estimate of the order of magnitude of M. Within this approximation the pion mass M, as the eigenvalue $P^2 = M^2$ in the bound state equation, is solution of a complex algebraic equation

$$AZ^2 = \sqrt{\rho^2/M^2 - 1/4}, \tag{17}$$

where Z and ρ are complex quantities specified in Eq. (5).

The imaginary part of Eq. (17), in the limit of zero current quark mass of u and d quarks, gives for A the result

$$A = (4\chi^3/M^3 - 1)/\sqrt{2(2\chi^3/M^3 - 1)}. \tag{18}$$

The real part of Eq. (17), also in the limit of the zero current quark mass of u and d quarks, gives quadratic equation for the ratio χ^3/M^3

$$6\frac{\chi^6}{M^6} - 7\frac{\chi^3}{M^3} + \frac{3}{2} = 0. \tag{19}$$

Eq. (19) has two solutions: *(i)* $M = 1.04\chi$, and *(ii)* $M = 1.52\chi$. This result means, that the pion mass M is nonzero if, and only if, the quark condensate mass scale χ is not equal to zero. Of course, it is only an estimate of the pion mass, and the remaining QCD-Coulomb terms which are not written in Eq. (14) as well as the non-Coulomb interactions correct this result, but they exclude the possibility of the zero value for M if $\chi \neq 0$.

The ρ meson is a pole in the transverse part of the vector three-point vertex function. The position of the ρ meson pole has both real and imaginary part. The nonzero value of the imaginary part of the position of the ρ meson pole is due to singular factors in quark (antiquark) constituent momenta which are present in two quark-gluon-quark vertex functions of the one-gluon exchange. Singular factors of two quark-gluon-quark vertex functions, appearing in the kernel of integral equation for the ρ meson wave function, produce a nonzero absorptive part of this kernel. This absorptive part gives a nonzero decay width of ρ meson into two pions.

For ρ meson, considered as the relativistic system of u and d constituent quark and antiquark, the essential new feature in comparison with pion is a new domain of the variation of two factors $1/(p_{1,2}^2 - M^2)$ in denominators of two quark-gluon-quark vertex functions in the nonperturbative one-gluon exchange interaction. Here M is the pion mass, while the mass of ρ meson denoted as M_ρ, and the width of ρ meson, denoted as Γ_ρ, correspond to the ρ meson pole at $P_\rho^2 = M_\rho^2 - i\Gamma_\rho M_\rho$.

In the equation for ρ meson both factors are equal to $1/(M_\rho^2/4 - i\Gamma_\rho M_\rho/4 - M^2 + q^2)$, and their real part is : either positive, or zero, or negative, since q^2 is non-positive, and the orthgonality condition $qP_\rho = 0$ is enforced as a constraint. The resulting factor $1/(M_\rho^2/4 - i\Gamma_\rho M_\rho/4 - M^2 + q^2)$ appears in the ρ meson kernel and is responsible for generating the nonzero width of the ρ meson. Also this factor, in the kernel of the integral equation for the ρ meson wave function, makes it impossible to apply the Fock–Schwinger method to estimate the ρ meson mass. Note, that in the ρ meson CMS ($\vec{P}_\rho = 0$) the constraint $P_\rho q = 0$ still means $q^0 = 0$ in spite of $\mathrm{Im}\,(P_\rho^0) \neq 0$, and excludes the ($\rho$, ρ^*) and (ρ^*, ρ) terms in the Wheeler relative motion propagator in Eq. (9), *i.e.* excludes the possibility of the continuum asymptotic partonic states, if $\mathrm{Im}\,(\rho) \neq 0$.

The kernel of integral equation for the ρ meson wave function is dramatically different from the corresponding kernel of the integral equation for the pion wave function, in spite of the fact that in both cases the dominating binding interaction is the QCD-Coulomb interaction. For pion, both of two factors $1/(p_{1,2}^2 - M^2)$ in two quark-gluon-quark vertex functions in the nonperturbative one-gluon exchange interaction are always negative for any value of the space-like Wightman–Garding momentum and the Fock–Schwinger method is applicable for the estimate of the pion mass M.

4. Absence of the QCD Goldstone theorem

In QCD with confined quarks it is impossible to have massless pion in the limit of zero mass of the current quark (u, d). The QCD Lagrangian in this limit is explicitly chirally symmetric. The nonzero quark condensate mass scale $\chi_{u,d}$ means spontaneous chiral symmetry breaking, however, in contrast to the common believe, it prevents the pion from becoming massless. Of course our pion is the relativistic bound system of the confined, constituent quark and antiquark. Such pion is necessairly described in position space as truly nonlocal field.

The nonzero values of χ and ν are both necessary and sufficient conditions for permanent confinement of quarks and gluons and for the nonzero value of pion mass. The sufficient condition for the nonzero pion mass we discussed in the former Section, and the sufficient condtion for the permanent confinement in Introduction. Here, we present the necessary condition for the pion mass to be different from zero, if $\chi \neq 0$ is to represent the qurak condensate mass scale.

The QCD Lagrangian, and the inverse of the quark propagator given in Eq. (2) are equivalent to the following local Meissner [8] Lagrangian density in which beside the quark field ψ there is an extra ghost-quark field ξ

$$L = \overline{\psi}(iD \cdot \gamma - m)\psi - \overline{\xi}(iD \cdot \gamma - M)\xi + \chi\sqrt{\chi/M}(\overline{\xi}\psi + \overline{\psi}\xi)\,. \qquad (20)$$

From this Lagrangian follows Eq. (2) for the inverse of quark propagator. To find Eq. (2) we must integrate out the quark-ghost field ξ in the action corresponding to this Lagrangian. The easiest way to do this integration is to make the following shift of the quark-ghost field ξ

$$\xi \to \xi + \frac{\chi\sqrt{\chi/M}}{p.\gamma - M}\psi\,. \qquad (21)$$

It should be noted, that the pion mass M appears in the Meissner Lagrangian in Eq. (20) in two places: *(i)* as the mass of the quark-ghost field, and *(ii)* in the coupling constant of the quark-ghost field and the quark field. Here M is in the denominator (under the square root) and such M can not be sent to zero. The appearence of M in Eq. (20) is of course the consequence of the form of Eq. (2) for the inverse of quark propagator.

In the zero limit of the current mass $m_{u,d} \to 0$, and the nonzero value of the quark condensate mass scale χ, we have on the level of the QCD Lagrangian the spontaneous chiral symmetry breaking, while on the level of the Meissner Lagrangian the explicit chjral symmetry breaking. Moreover, if we would consider a possibility that in the quark-ghost mass term we set $M = 0$, and in the term in Eq. (20) which couples the quark-ghost field ξ with the quark field ψ we set $M = \chi$, then we get an explicitly chirally symmetric Meissner Lagrangian. However, such mass scale χ which is the only one non-perturbative mass scale left in the chirally symmetric Meissner Lagrangian, has nothing to do with the magnitude of quark condensate. This is so, because in this case the effective quark propagator, resulting after we integrate out the quark-ghost field ξ, is

$$S(p) \;=\; \frac{p \cdot \gamma}{p^2 + \chi^2}\,, \qquad (22)$$

and the quark condensate mass scale corresponding to this non-perturbative quark propagator, in Eq. (22), is trivially zero because the trace of such $S(p)$ is equal to zero.

To get a nonzero value of the quark condensate mass scale χ from the vacuum-to-vacuum transition, by closing up in position space the Fourier transform of the nonperturbative quark propagator, it is necessary to keep in the Meissner Lagrangian the pion mass M not equal to zero, and have the explicit chiral symmetry breaking in the Meissner Lagrangian, corresponding to the spontaneous symmetry breaking of the QCD Lagrangian.

In field theories without permanent confinement of fundamental fields it is possible to prove the Goldstone theorem if we assume the existence of a local current (Swieca [9], Strocchi [10]), and we take for granted that the zero Goldstone mode has something to do with pion as the bound system of a constituent quark and antiquark. Such massless pion field necessairly must be a local field (for example the divergence of an axial current), or at most it can be a composite field, but still it must be a local field. However, pion which is the solution of relativistic bound state equation with confined constituent quark and antiquark, is necessairly a non-local field in position space, as explained in former Section.

Sometimes, a massless pion shows up as a pole in the longitudinal part of the axial three-point vertex function. However, from the axial Ward identity it follows, that the longitudinal part of the axial current must have in momentum space the particular factors which depend individually on the incoming and outgoing quark momenta. If we Fourier transform the axial three-point vertex function to position space, then these factors depend on the individual quark momenta, and they make the residuum of such pion pole necessairly nonlocal in the position space of the pion field, in conflict with the assumption that the pion field can be approximated by a local field. Our pion is a non-local pole term in the transverse part of the axial vector three-point vertex function.

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