BOSE EINSTEIN CORRELATIONS AND QUANTUM FIELD THEORY*

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It is shown that there exists an intimate relationship between Bose Einstein correlations and quantum field theory. On the one hand several essential aspects of BEC cannot be understood and even formulated without second quantization. On the other hand BEC can serve as a unique tool in the investigation of modern field theory and in particular of the standard model. Some new developments on this subject related to multiparticle production and squeezed states are also discussed.

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Bose Einstein correlations (BEC) are a topic of high current interest in particle and nuclear physics. This interest has been motivated so far mainly by the fact that they offer a unique possibility to explore the space-time dimensions of sources of particles and this is essential *e.g.* in the search for quark matter. However BEC can in principle offer much more, namely insight into some fundamental aspects of quantum mechanics, as well as the possibility to test important aspects of modern particle physics. Historically BEC came into being when Hanbury–Brown and Twiss invented in the mid fifties the method of photon intensity intereferomentry for the measurement of stellar dimensions (the HBT method). In 1959–1960 G. Goldhaber, S. Goldhaber, W. Lee and A. Pais discovered that identical charged pions produced in $\bar{p} - p$ annihilation are correlated (the GGLP effect). Both the HBT and the GGLP effects are based on Bose–Einstein correlations. Subsequently also Fermi–Dirac correlations for nucleons were observed.

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Loosely speaking both Fermi–Dirac and Bose–Einstein correlations can be viewed as a consequence of the symmetry (antisymmetry) properties of the wave function with respect to permutation of two identical particles with integer (half-integer) spin and are thus intrinsic quantum phenomena. At a higher level, these symmetry properties of identical particles are expressed by the commutation relations of the creation and annihilation operators of particles in the second quantisation (quantum field theory). The quantum field approach is the more general approach as it contains the possibility to deal with creation and annihilation of particles and certain correlation phenomena like the correlation between particles and antiparticles can be properly described only within this formalism. Moreover, at high energies, because of the large number of particles produced, not all particles can be detected in a given reaction and therefore one measures usually only inclusive cross sections. For these reactions the wave function formalism is impracticable. Furthermore, as pointed out quite recently [1] BEC may play an important part in the test of the standard model and in particular in the search for the Higgs particle, because they may affect the W mass. Last but not least BEC can serve for the determination of one of the most characteristic properties of systems made of identical bosons and which is responsable for the phenomenon of *lasing* in quantum optics namely quantum statistical coherence. This feature is also not accessible to a theoretical treatment except in field theory.

BEC, coherent states, and the density matrix

To realise the significance of this topic it is enough to mention that some of the most important developments in particle physics of the last 25 years including the "standard model", are based on spontaneously broken symmetries which imply coherent states. Moreover certain classes of field theories admit classical fields as solutions (*e.g.* solitons) and any classical field is a coherent state. However there is so far no direct experimental evidence for these coherent states. On the other hand it is well known from quantum optics that BEC depend on the amount of coherence [1] in a very characteristic way and therefore one hopes to obtain information about coherence from boson interferometry.

This dependence of BEC on coherence is a particular case of the fact that any probability or cross section in quantum theory is an expectation value and thus depends on the state of the system. This is demonstrated explicitely in the quantum field theoretical treatment of BEC [2]. In general the state of the system is described by the density matrix, which in principle is determined by the theory. For hadron multiproduction this theory is quantum chromodynamics and for processes involving multiphoton production this theory is quantum electrodynamics. However in both cases the use of the fundamental theory is unpractical because of the complexity of the many body problem. That is why one uses in both cases phenomenological approaches. It is interesting to mention that for photonic processes the invention of intensity interferometry by Hanbury-Brown and Twiss led to the development of a new branch of optics, namely quantum optics, with laser physics being one of its major applications. The experience gained in this domain has been instrumental in the analogous problem of hadron multiparticle production and amounts essentially to postulating the form of the density matrix in the *coherent state* representation.

BEC and the notion of identical particles

There are some aspects of principles of quantum mechanics involved in the study of BEC, which have been discovered more recently and which are related to the very concept of *identity* of particles. It is well known that the principle of identity of particles is part of the fundamental postulates of quantum mechanics and states essentially that elementary particles are indistinguishable. The question what means *identical* has not been raised until recently, since it had been considered that the answer to it was obvious. This situation has changed when it was discovered within the classical

[1] In the last decade the conjugated quantity to amount of coherence, *i.e.* the amount of *chaoticity* or just chaoticity, as introduced by the author in Proceedings of LESIP II, Hadronic Matter in Collision, World Scientific 1986, Eds. P. Carruthers and D. Strottman, page 106 is more often used. It is perhaps amusing to mention that the term chaoticity was proposed for the first time in a paper by Fowler, Friedlander, Weiner and Wilk submitted to *Phys. Rev. Lett.*, but the editor of this journal objected to this word, and we had to replace it by "measure of chaos" (*cf.* Fowler, Friedlander, Weiner and Wilk in *Phys. Rev. Lett.* **57**, 2119 (1986). Since then, however, times, preconceptions and editors have changed... At present chaoticity is a universally accepted and used term.

current formalism (*cf.* [3]) that there exists a difference between BEC of neutral and charged pions. While the maximum value of the second order correlation function for charged pions is 2, that for neutral pions is 3. To realise in simpler terms the meaning of this, it is useful to recall that Bose Einstein correlations imply in general a bunching of identical particles and the larger the intercept, the stronger this bunching is. Non identical bosons on the other hand do not show this bunching. Thus the bunching phenomenon can be considered as a signal of the identity of particles. The fact that some identical bosons are more bunched than others implies then in some sense that there exists a "hierarchy" of identity as if some identical particles would be more identical than others. Another aspect of this phenomenon is the fact that the amount of bunching is a manifestation of the state of the system. Thus a conventional coherent state has no bunching (similar to the situation met with non identical particles), a chaotic state has bunching and a squeezed coherent state can have any amount of bunching including negative values, *i.e.* antibunching (*cf.* *e.g.* [4]).

An even more striking aspect of this effect is the fact that there exists a quantum statistical correlation between positive and negative pions [3], although these particles are non identical in the ususal sense of the word. These results appeared so surprising at the moment when they were obtained that some people could not believe them and attempts were made to disprove them. The reason for this reaction of the scientific community lies in the fact that the naive wave function approach to BEC was (is?) still deeply rooted and there is no obvious way to obtain these surprising effects within this approach. Quantum field theory *i.e.* second quantization of which the clasical current formalism is a particular case, is the natural frame for the derivation of these new effects. At present there exist at least three other derivations of these effects [5] and they constitute a definite challenge for experimentalists. The fundamental importance of these effects is such that their experimental observation will compensate by far the efforts necessary for their detection (these effects are quite small and necessitate high statistics to be seen in experiment). It is important to emphasize that the quantum statistical corelation between positive and negative pions mentioned above is just a particular case of quantum statstical correlations between particles and antiparticles in general and is not restricted to isospin one (*e.g.* such a correlation must exist also between positive and negative kaons or W bosons).

The classical current formalism has been used widely in the context of BEC because for this case there exist exact solutions of the corresponding inhomogeneous field equations. On the other hand it has been clear that in particle physics the currents are quantised. Much less clear was how to estimate the quantum corrections to the classical currents. Furthermore one might have wondered whether the new effects discussed above and derived within the classical formalism were not an artifact of this formalism. In Ref. [2] an answer to the above questions is given by formulating explicitely a quantum field theory of BEC in which the quantum nature of the currents is taken into account. Concretely this means considering the currents as operators which implies proper ordering in the corresponding expressions for the physical quantities which are calculated. Besides confirming fully the existence of the new effects quoted above the quantum field theoretical treatment presented in [2] brings another surprise in this saga of BEC. It turns out that these very effects can be used in order to study experimentally the quantum corrections to the classical current approximation.

Finally we will describe a most recent development on the subject of BEC and quantum field theory related to squeezed states.

BEC and squeezed states

Besides ordinary coherent states used as the basis of the representation, squeezed coherent states have been introduced, which are of major interest both from a theoretical point of view as well as because of their application potential. As will be shown below BEC can serve as a tool for the detection of these squeezed states.

Coherent states are the nearest approximation to classical fields because they minimise the product of incertitudes in the Heisenberg indeterminacy relation. They are defined as eigenstates of the one particle annihilation operator. Besides these *ordinary* coherent states there exist however also *squeezed* coherent states which are eigenstates of the two or more particles annihilation operator (cf. *e.g.* [4]). These generalised coherent states, which are a U(1,1) group extension of ordinary coherent states, have been for the last years in the center of interest of several branches of physics. While for ordinary coherent states the fluctuations in momentum and coordinate are equal to the corresponding zero-point vacuum fluctuations, squeezed states allow for even smaller incertitudes (in one canonical variable). Thus the quantum limit can be "beaten" and this is not only of fundamental interest, but may have important applications in communication technology and for measurements of very weak signals (gravitational waves *e.g.*).

Although the effect of squeezed states in BEC has been discussed in the optical and particle physics literature for quite some time, this discussion has been limited so far to the idealised case of pure squeezed states and even for this case only the value of the second order correlation function in the origin was known (*cf.* *e.g.* [6, 7]). Another important issue related to squeezed states is the fact that while in optics squeezed states have been obtained in the last years in several experiments, in particle physics this is apparently not the case. In a recent paper [8] progress along these lines could be reported. In particular it has been shown that squeezed states could be produced preferentially in "sudden" nuclear and particle reactions. and a derivation of the second order Bose Einstein correlation function in the entire domain of its variables has been given for the practically important case of a chaotic superposition of squeezed states. Furthermore it has been shown that by measuring BEC in "sudden" reactions important new information about the dispersion of the hadronic medium before it emits can be obtained. I shall sketch below briefly these results.

Consider a blob of hadronic matter (for which Shuryak [9] proposed the name "pion liquid") created in particle collision which undergoes a sudden breakup into free pions. In other words, the pionic system, having its specific ground state and elementary pionic excitations (not coinciding exactly with the usual vacuum and free particles) converts rapidly into free pions. In this case the single and higher order inclusive cross section and the many-particle correlation functions will depend on the spectrum of excitations in the pionic system. The importance of the form of the spectrum of pionic excitations for multiparticle production was stressed in the same reference by Shuryak. I shall argue below that the above physical picture results in the production of quantum squeezed states.

Let us consider the transition from a pionic "liquid" to a free pion field in the spirit of local parton-hadron duality, *i.e.* we conjecture a close correspondence between particles (fields) in the two "phases". At the moment of this transition one can postulate the following relations between the generalized coordinate Q and the generalized momentum P of the field:

$$Q = \frac{1}{\sqrt{2E_b}}(b^+ + b) = \frac{1}{\sqrt{2E_a}}(a^+ + a)\,,$$
$$P = i\sqrt{\frac{E_b}{2}}(b^+ - b) = i\sqrt{\frac{E_a}{2}}(a^+ - a) \tag{1}$$

a^+, a are the free field creation and annihilation operators and b^+, b the corresponding operators in the "liquid". Eq. (1) holds for each mode p. Then we get immediately a connection between the a and b operators,

$$a = b\, cosh\, r \;+\; b^+\, sinh\, r\,,$$
$$a^+ = b\, sinh\, r \;+\; b^+\, cosh\, r \tag{2}$$

with

$$r = r(\vec{p}) = \frac{1}{2}\, log\,(E_a/E_b)\,. \tag{3}$$

The transformation (2) is just the squeezing transformation [4] with a momentum dependent squeezing parameter $r(\vec{p})$ given by Eq. (3) and the coherent eigenstate $|\beta\rangle_b$ of the b-operator is the squeezed state $|\alpha, r\rangle_a$ of the a-operator:

$$|\beta\rangle_b = |\alpha, r\rangle_a \,, \tag{4}$$

where α and β are related by the same transformation (2) as the a and b operators. This proves the above made statement.

In general the system may not be in a pure coherent or squeezed state and then a statistical averaging has to be performed both with respect to the coherent as well as for the squeezed states. Interestingly enough in the case of squeezed states this apparently routine task raises a new question of principle.

In practice it is easier to express the a, a^+-operators through the b, b^+-operators according to Eq. (2) and then perform the averaging over the coherent states $|\beta\rangle_b$. Considering charged identical pions (complex valued field) we shall use the Glauber–Sudarshan representation of the density matrix, and write the average value of an operator $\hat{O}$ as

$$\langle \hat{O}(a, a^+)\rangle = \prod_{\vec{p}} \int d^2\beta_k P\{\beta(\vec{p})\} \langle\beta|\hat{O}\left(a(b, b^+), a^+(b, b^+)\right)|\beta\rangle_b \,, \tag{5}$$

and assume a Gaussian form for the weight function $P\{\beta(\vec{p})\}$. Due to the linearity of the squeezing transformation (2) this form will hold also for the a, a^+-operators.

The particle source will be characterized by a primordial correlator determined by the number density $n(p)$ and by a function $f(\vec{x})$ describing its geometrical form, see Refs. [10, 11]. To make contact with the previous results of [10, 11] we note that the radius of the source R enters the function f and the correlation length L appears in $n(p)$. For simplicity we shall not consider the time dependence here and take the form function $f(\vec{x})$ to be dependent only on the space coordinates.

And now we arrive at a new surprise: the direct substitution of the transformation (2) into Eq. (5) leads to undefined (divergent) expressions of the form $\delta(0)$ when one tries to perform the normal ordering of b, b^+-operators (the last is necessary to use the coherent state representation of Eq. (5)). This situation can be avoided by introducing new creation and annihilation operators which are non-zero only inside the volume of the particle source,

$$\tilde{a}(\vec{x}) = a(\vec{x}) f(\vec{x}) \,, \quad \tilde{a}^+(\vec{x}) = a^+(\vec{x}) f(\vec{x}) \,, \tag{6}$$

or, for Fourier transformed quantities,

$$\tilde{a}(\vec{p}) = \int \frac{d^3k}{(2\pi)^3} a(\vec{k}) f(\vec{k} - \vec{p}) \quad , \quad \tilde{a}^+(\vec{p}) = \int \frac{d^3k}{(2\pi)^3} a^+(\vec{k}) f(\vec{p} - \vec{k}) \tag{7}$$

with standard commutation relations

$$\left[a(\vec{p}_1), a^+(\vec{p}_2)\right] = (2\pi)^3 \cdot \delta^3(\vec{p}_1 - \vec{p}_2)\,. \tag{8}$$

Then the equal momentum commutators of the modified operators are finite. For example:

$$[\tilde{a}(\vec{p}), \tilde{a}^+(\vec{p})] = \int \frac{d^3k}{(2\pi)^3} f(\vec{p}-\vec{k}) f(\vec{k}-\vec{p}) = \int d^3x |f|^2(\vec{x}) = V_{\text{eff}} \tag{9}$$

being equal to an effective volume V_{eff} of the particle source. While this finite size is quite natural in particle physics, it is not so in optics where the system is usually macroscopic. Furthermore it is remarkable that this problem of finite size appears only with squeezed states and only when correlations are considered. Thus in [12, 13, 6], where "thermal" squeezed states were introduced and applied to multiplicity distributions and their moments (these are the *integrals* of correlation functions) this did not happen.

With the smoothed operators $\tilde{a}(\vec{p}), \tilde{a}^+(\vec{p})$ substituted into Eq. (5) the form of the source is already taken into account and the remaining statistical averaging may be performed in the same way as for an infinite medium.

Now the evaluation of the averaged matrix elements is straightforward. Substituting Eqs. (7) and (2) into Eq. (5) and performing the Gaussian averaging over coherent states $|\beta\rangle_b$ we get the single-particle inclusive density in the form:

$$\begin{aligned}\rho_1(\vec{p}) &= \frac{(2\pi)^3}{\sigma}\frac{d\sigma^{in}}{d^3p} = \langle \tilde{a}^+(\vec{p})\tilde{a}(\vec{p})\rangle \\ &= \int \frac{d^3k}{(2\pi)^3}\left[n_b(\vec{k}) \cosh 2r(\vec{k}) + \sinh^2 r(\vec{k})\right] f(\vec{p}-\vec{k}) f(\vec{k}-\vec{p})\,,\end{aligned} \tag{10}$$

where the function f describes the effect of finite size of the particle source and $n_b(\vec{k})$ given by the equation

$$\langle \beta^*(\vec{k})\beta(\vec{k}')\rangle = (2\pi)^3\delta^3(\vec{k}-\vec{k}')n_b(\vec{k}) \tag{11}$$

represents the density of pionic "quasiparticles" (b-quanta) (in particular, for a thermal source the function $n_b(\vec{k})$ is the usual Planck distribution function).

The squeezed state effect is reflected in Eq. (10) in the factor $\cosh 2r$ multiplying the primary pionic density $n_b(\vec{k})$ and in the term $\sinh^2 r$ representing the ground state contribution. That is the final state pions are produced even if the pions in the pionic source are absent (zero temperature), just due to the decay of the squeezed vacuum state. According to

Eq. (10), the single particle density may be strongly enhanced in the presence of squeezed states if the squeezing parameter $r(\vec{p})$ is large enough.

We consider now the two-particle inclusive density

$$\rho_2(\vec{p}_1,\vec{p}_2) = \frac{(2\pi)^6}{\sigma} \cdot \frac{d\sigma}{d^3p_1 \cdot d^3p_2} = \langle \tilde{a}^+(\vec{p}_1)\tilde{a}^+(\vec{p}_2)\tilde{a}(\vec{p}_1)\tilde{a}(\vec{p}_2)\rangle \tag{12}$$

in the presence of squeezed states. With the finite size cut off the two-particle density is calculated in the same way as the single-particle density. Using Gaussian averaging one gets the simple expression

$$\rho_2(\vec{p}_1,\vec{p}_2) = \langle \tilde{a}^+(\vec{p}_1)\tilde{a}(\vec{p}_1)\rangle\langle \tilde{a}^+(\vec{p}_2)\tilde{a}(\vec{p}_2)\rangle + |\langle \tilde{a}^+(\vec{p}_1)\tilde{a}(\vec{p}_2)\rangle|^2 + |\langle \tilde{a}(\vec{p}_1)\tilde{a}(\vec{p}_2)\rangle|^2 \tag{13}$$

with

$$\langle \tilde{a}^+(\vec{p}_1)\tilde{a}(\vec{p}_2)\rangle = \int \frac{d^3k}{(2\pi)^3}\left[n_b(\vec{k}) cosh\, 2r(\vec{k}) + sinh^2 r(\vec{k})\right] f(\vec{p}_1-\vec{k})f(\vec{k}-\vec{p}_2)\,,$$

$$\langle \tilde{a}(\vec{p}_1)\tilde{a}(\vec{p}_2)\rangle = \int \frac{d^3k}{(2\pi)^3}\left[n_b(\vec{k}) + \tfrac{1}{2}\right] sinh 2r(\vec{k}) f(\vec{k}_1-\vec{p}_1)f(\vec{k}-\vec{p}_2)\,, \tag{14}$$

The first term in the right hand side of Eq. (13) is the product of single-particle densities $\rho_1(\vec{p}_1)\rho_1(\vec{p}_2)$, the second term is the exchange contribution characteristic for Bose–Einstein correlations modified by the squeezing factor r (for $r = 0$ it coincides with the usual BEC). The third term arises only in the presence of squeezed states (it vanishes for $r = 0$). This last contribution differs from the "surprising" terms in the two-particle correlation function discussed in Refs. [11, 3], which are absent in the case of charged identical pions under consideration and which have another dependence on momenta $\vec{p}_1, \vec{p}_2$, being maximal at $\vec{p}_1 + \vec{p}_2 = 0$, and not at $\vec{p}_1 - \vec{p}_2 = 0$ as is the case for all terms in Eq. (13).

As one can see from Eqs. (10), (13), (14) the second order correlation function

$$C_2(\vec{p}_1,\vec{p}_2) = \rho_2(\vec{p}_1,\vec{p}_2)/[\rho_1(\vec{p}_1)\rho_1(\vec{p}_2)]\,. \tag{15}$$

is enhanced due to the presence of the third term in the right hand side of Eq. (13) and in general the value of the ratio (15) is arbitrarily large. In particular, for $n_b(\vec{k}) = 0$ (that is for cold pionic matter when particle production is the result of the squeezed vacuum decay) and for small values of the squeezing parameter $r(\vec{k})$, one may have $C_2 >> 1$. For $r(\vec{k}) \sim 1$ and

$\vec{p}_1 \cong \vec{p}_2$ the ratio (15) is close to three. We call this effect "overbunching" to distinguish it from conventional Bose Einstein correlation where a bunching effect occurs, too, but where the intercept $C_2(p,p)$ does not exceed the value of two.

Possible applications of the rapid transition mechanism discussed above could include the explosion of a hadronic fireball after a phase transition from quark-gluon plasma [14] and annihilation. [2]

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[2] An overbunching effect similar to that discussed above has been seen in annihilation reactions [15] but was interpreted in [16] in terms of resonance production. The present approach in terms of squeezed states should be viewed as an alternative or a supplementary source of overbunching. It has the advantage that it leaves transparent the relation between BEC and the space-time characteristics of the source and it could perhaps also explain why overbunching may have been seen so far only in annihilation. Indeed our derivation of the squeezing effect assumes a rapid transition and it has been argued recently by Amado *et al.* [17] that annihilation is a fast process.

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