

PHYSICS OF HEAVY QUARKS^{*,**}K. ZALEWSKI[†]Institute of Physics, Jagellonian University,
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Selected problems in heavy quark physics are discussed. The wealth of research problems in this field of physics is stressed.

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1. Introduction

Heavy quark physics is a broad and active field of particle physics. Within it, hundreds of theoretical papers are produced every year and the production rate keeps increasing. In this short presentation I shall concentrate on recently obtained insights and on open problems. The experimental data quoted without giving the source are either from the 1994 Tables of the Particle Data Group [1], or from the EPS Conference held in Brussels this summer.

According to the standard model there are six kinds of quarks. In order of increasing mass they are denoted u , d , s , c , b , t . The last three are considered heavy, because their masses are much larger than Λ_{QCD} , *i.e.* than about 0.5 GeV. The mass of the c -quark can be roughly estimated as half the mass of the lightest $c\bar{c}$ quarkonium, which gives $m_c \approx 1.5$ GeV. This in fact is not very heavy — only about three times Λ_{QCD} . The mass of the recently discovered t quark is $m_t = (180 \pm 12)$ GeV, which implies that the t -quark decays, usually into a W -boson and a b -quark, before it has time to hadronize. Consequently, the physics of the t -quarks is already well understood. The mass spectrum of the heavy quarks causes that most of the

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new ideas apply best to b -quarks. For t -quarks, the problems are fewer and they can be usually solved without making controversial assumptions. For c quarks, we are too far from the heavy quark limit, where all the quantities of order Λ_{QCD} can be neglected compared to the mass of the heavy quark. On the long run this may make the physics of the c -quarks more interesting than the physics of the b -quarks, but for the moment it is often just too difficult.

Let us begin by considering the problem: what is meant by the quark mass?

2. Quark masses

The standard definition of mass, $m = \sqrt{E^2 - \vec{p}^2}$ is not applicable to quarks, because the energy E and the momentum \vec{p} on its left-hand-side should be measured for free particles. Looking for a free quark not interacting with other coloured objects is like looking for one end of a string not attached to another end. There is no chance for success. The next choice, when trying to define the quark mass, is to make use of the mass parameter m_0 from the Lagrangian. This, however, has corrections. The fact that the quark is part of the time a quark-gluon system (the contribution of the gluon loop on the quark line) changes the mass by

$$\Sigma^{(1)} = \frac{m\alpha_s(\mu)}{\pi} \left[\frac{1}{\varepsilon} - \gamma + \log(4\pi) + \log \frac{\mu^2}{m^2} + \frac{4}{3} \right], \quad (1)$$

where $\gamma = 0.5772\dots$ is Euler's constant. From this formula one sees two difficulties; moreover, there are two others not directly visible.

- The limit $\varepsilon \rightarrow +0$ should be taken, thus the formula as it stands does not make sense.
- The scale parameter μ is arbitrary.
- The formula has been obtained using dimensional regularization. There are many other methods of regularizing (various cut off procedures, putting the theory on a lattice *etc.*), which yield different formulae.
- This correction is only the first term of an infinite series, in general convergence problems are expected.

The infinity is eliminated by replacing the mass $m_0 + \Sigma^{(1)}$ by the obviously equal number $(m_0 + \delta m) + (\Sigma^{(1)} - \delta m)$. The trick is to choose δm so that it cancels the infinity in $\Sigma^{(1)}$. Since nothing is known about m_0 , one can assume that $\Sigma^{(1)}$ does not introduce an infinity in the first term. This recipe leaves much freedom in the choice of δm . Choosing $\delta m = \frac{1}{\varepsilon}$ one gets

the so called minimal subtraction mass. Including in δm also $-\gamma + \log(4\pi)$, which is convenient, one obtains the very popular \overline{m} mass known as the MS-bar mass. Choosing $\delta m = \Sigma^{(1)}$ one obtains the pole mass m^P etc. Each of these masses depends on the scale μ . This arbitrary scale is usually chosen of the order of the mass of the quark being considered. For instance, the Particle Data Group [1] tabulates the quark masses $\overline{m}(\overline{m})$. The differences between the various masses are significant. For instance, using the formula for $\Sigma^{(1)}$ one finds for quark Q

$$\overline{m}_Q(\overline{m}_Q) = m_Q^P \left(1 - \frac{4\alpha_s(\overline{m}_Q)}{3\pi} \right). \quad (2)$$

Typical values of $\alpha_s(m_Q)$ for the heavy quarks are 0.35, 0.20, 0.10 for the c , b , t quarks respectively. This gives in the present (very crude) approximation the differences between the pole masses and the MS-bar masses 0.17 GeV, 0.34 GeV and 7 GeV. More careful calculations give for the c and b quarks 0.26 GeV and 0.51 GeV [2], while typical values for the t -quark are (8–9) GeV. An obvious question is: what is the mass found in Fermilab for the t quark? The description of the measurement provides an unambiguous operational definition of this mass, but to which of the theoretical mass parameters does it correspond? Somewhat surprisingly this problem is still controversial. The pole mass, however, seems to be the most popular interpretation.

For the other normalization schemes it is possible to perform analogous analyses, therefore, the existence of various renormalization schemes is not a serious difficulty.

The convergence problem, however, has been recently found to introduce an interesting complication. References can be traced starting from the recent review [3]. One finds (if one uses dimensional regularization) that the series used to define the pole mass is divergent. It can be used as an asymptotic series, but then it defines the pole mass only approximately, with an error of about 50 MeV. This is the reason why the MS-bar masses are now the popular ones for the c and b quarks. For the t quark the situation is different. With present experimental uncertainties an additional uncertainty of 50 MeV is irrelevant. On the other hand, the relation between the pole mass and the MS-bar mass has a much greater uncertainty. The calculations necessary to reduce this uncertainty are possible, but so hard that they have not yet been done and are unlikely to be performed in the nearest future. Therefore, if the measured mass is the pole mass, expressing it in terms of the MS-bar mass would be an unnecessary loss of precision.

3. Heavy particles

By heavy particles we mean here particles containing one or more heavy quarks or antiquarks. The best studied case is the nonrelativistic approximation for the quarkonia $\bar{Q}Q$. In particular for bottomonia, it is possible to get a very good fit to the masses (averages only for the P -states) below the threshold for strong decays, for the leptonic widths and for the dipole transition matrix elements. One can use the nonrelativistic Schrödinger equation with the simple spherically symmetrical potential

$$V(r) = a\sqrt{r} + \frac{b}{r} + c, \quad (3)$$

where a , b , c are constants [4]. How to make a relativistic theory is still controversial.

For heavy particles containing light quarks the situation is more difficult, because for them the nonrelativistic theory does not make much sense. A break through has been the idea to use expansions in the inverse of the heavy quark mass. For instance, for the mass of a particle with one heavy quark Q one finds

$$M_H = m_Q + \bar{A} + \frac{\langle \vec{p}^2 \rangle}{2m_Q} + \frac{\langle \vec{\sigma} \cdot \vec{B} \rangle}{2m_Q} + \frac{1}{m_Q^2} [\text{Darwin} + \text{Spin-orbit} + \text{IterII}]. \quad (4)$$

The leading term is just the mass of the heavy quark. The term of order m_Q^0 , denoted \bar{A} , is the energy of the light component in the colour-field of the heavy quark. The heavy quark is here considered as a static source of potential. Note the generality of this formulation. The light component may be an antiquark, as in valence models of $Q\bar{q}$ mesons, a pair of quarks, as in the valence models of Qqq baryons, or a more complicated combination of light quarks, light antiquarks and gluons, as in some more sophisticated models. The corrections of order $O(m_Q^{-1})$ correspond to the kinetic energy of the heavy quark and to the Pauli interaction of the magnetic moment of the heavy quark with the chromomagnetic field created by the light component. The magnetic term is responsible for the hyperfine mass splittings in the mass spectra. For instance the difference between the B meson and the B^* meson is that in the first the spins of the heavy meson and of the light component give the resultant spin of the particle equal zero, while in the second this spin equals one. One finds

$$\langle \vec{\sigma} \cdot \vec{B} \rangle = \frac{3}{4} (M_{B^*}^2 - M_B^2) \approx 0.37 \text{ GeV}^2. \quad (5)$$

Since this average should not depend on the mass of the heavy quark, one expects a similar value for the (D, D^*) system. In fact the experimental

number is 0.41 GeV^2 . This can be formulated differently: the experimental fact that the hyperfine splitting for $Q = b$ is about three times smaller than the hyperfine splitting for $Q = c$, is explained here as a consequence of the fact that the c -quark is about three times lighter than the b -quark. The kinetic energy term has no such direct connection to experimental data and, therefore, its value is controversial. It can be shown that $\langle \vec{p}^2 \rangle > \langle \vec{\sigma} \cdot \vec{B} \rangle$ ([5] and references given there) and typical estimates are between this lower limit and its double. For the terms of order m_Q^{-2} we have given only the names. The first two, the Darwin term and the spin-orbit interaction, are familiar from the Dirac theory of the hydrogen atom. The third term is the second perturbative iteration of the $O(m_Q^{-1})$ term.

One can apply this approach also to higher resonances. When the light component consists of a light antiquark in a P state, its angular momentum can be $\frac{1}{2}$ or $\frac{3}{2}$. The parity is plus. Combining that with the spin $\frac{1}{2}$ and positive parity of the heavy quark, one finds four excited states with spins and parities: 0^+ , 1^+ , 1^+ , 2^+ . Experimentally one finds two charmed mesons D_1^{**} and D_2^{**} with masses $(2423 \pm 3) \text{ MeV}$ and $(2458 \pm 2) \text{ MeV}$ respectively and one bottom meson B^{**} with mass $(5733 \pm 17) \text{ MeV}$. A D^{**} meson decays into a pion and a D or D^* meson. Using angular momentum and parity conservation, as well as the information that the pion is produced from the light component, one can see that the mesons with the angular momentum of the light component equal $\frac{1}{2}$, decay producing a pion in an S -state. Such mesons are broad and difficult to observe. The D^{**} mesons with the angular momentum of the light component equal $\frac{3}{2}$, on the other hand, produce pions in D -states and are narrow, because of the suppression of the decay probability by the angular momentum barrier. This explains, why only two D^{**} mesons have been observed. The hyperfine splitting between these mesons is about 30 MeV . Since this is an effect of order $O(m_Q^{-1})$, the corresponding splitting for the B^{**} mesons is expected to be about 10 MeV , and indeed cannot be seen at the present resolution of 17 MeV . This explains, why for the moment only one B^{**} meson has been seen. One also can predict that in order to distinguish the two B^{**} mesons, the resolution will have to be improved by about a factor of two.

4. Decays of heavy particles

Decays of heavy particles are an important source of information about the elements of the Cabibbo-Kobayashi-Masakawa (CKM) matrix. From the point of view of the standard model these matrix elements are coupling constants (not all independent from each other!) as fundamental as *e.g.* the electron charge. Where these constants are known, comparison of the

theoretical predictions with experiment yields interesting tests of the standard model.

Let us consider the semileptonic decay $\bar{B} \rightarrow D^* e \bar{\nu}$. In this decay the b -quark, with a probability amplitude proportional to the CKM matrix element V_{cb} , goes over into a c -quark. In the process it emits a virtual W^- intermediate boson, which decays into the e^- , $\bar{\nu}$ pair. The problem is to extract the modulus $|V_{cb}|$ from the experimental data.

In the heavy quark limit the heavy mesons \bar{B} and D^* are similar to hydrogen atoms. In each case the heavy quark sits in the middle, like the proton in hydrogen, and the light component surrounds it, like the electron cloud surrounds the proton in the hydrogen atom. The energy and momentum of the W -boson are very large on the scale of the momenta of the light components. An analogy would be a 1 MeV photon hitting the proton in hydrogen. In this situation the heavy "nucleus" behaves as if it were free. It gets ejected with large momentum (on the scale of the light stuff) from its original position. The b -quark absorbing (or equivalently emitting) the W -boson changes into a c -quark. Note that since the c -quark is very heavy, large momentum does not necessarily mean large velocity. This process, however, is not yet the process $\bar{B} \rightarrow D^*$. In order to get the probability amplitude for this decay it is necessary to multiply the probability amplitude for the ejection of the heavy quark by the probability amplitude that the light component of the original \bar{B} -meson will reorganize itself into the light component of the recoiling D^* -meson. This is given by the overlap of the two corresponding wave functions. Thus, omitting the less interesting (known) terms, the decay amplitude is

$$A = V_{cb} \bar{u}_{\vec{v}'} \gamma_\mu (1 - \gamma^5) u_{\vec{v}} F(\omega). \quad (6)$$

Here \vec{v} and \vec{v}' denote the initial and the final velocities of the heavy quark. In the heavy quark limit these velocities are equal to the velocities of the corresponding mesons. The argument $\omega = v^\mu v'_\mu$, which can be interpreted as the Lorentz factor of the D^* as seen in the rest frame of the \bar{B} , is a measure of the recoil velocity. The overlap factor, known as the Isgur-Wise function, is

$$F(\omega) = \int \psi_{\vec{v}'}^*(\vec{r}) \psi_{\vec{v}}(\vec{r}) d^3r. \quad (7)$$

Note that the overlapping wave functions of the light components differ only by the velocities of their centres. The change of the b -quark into a c -quark and the change of the relative spin orientation of the heavy and light quarks from antiparallel to parallel have in leading order no effect on the wave function of the light component. The remaining difficulty is how to extract from the data the factor $|V_{cb}|$ without using a specific model for the Isgur-Wise function.

Two solutions to this problem have been proposed. In the exclusive approach one notices that for $\vec{v} = \vec{v}'$ the two overlapping functions are identical and that consequently $F(1) = 1$ from the normalization of the wave function. In this approach one obtains from the data the product $|V_{cb}|F(\omega)$ and extrapolates it to zero recoil, where $F(\omega = 1) = 1$. In the inclusive approach, one gives up the constraint that the final charmed state must be a D^* meson. Then the Isgur-Wise function is replaced by the probability amplitude that the light component will reorganize itself into anything, which is, of course, equal one. Thus, one uses data for the inclusive process $\bar{B} \rightarrow X_c e \bar{\nu}$. Here X_c denotes any state containing the quark c . Since the b -quarks decay almost always into c -quarks, X_c can in practice be replaced by X meaning anything. We have presented here only the leading term analysis. In practice one includes various corrections, which are still somewhat controversial. Fortunately they change the calculated values of $|V_{cb}|$ by only a few percent. Incidentally, the analogous problem of extracting the CKM matrix element $|V_{ub}|$ from the data is much harder and is an active subject of research.

Let us mention two open problems connected with inclusive decays (*cf. e.g.* [6]). Theoretically one finds that the life times of the heavy particles containing single b -quarks are well estimated using the spectator model, *i.e.* neglecting the effect of the light components on the life times. This corresponds to equal life times for all such particles. It is possible to calculate corrections to this result and they turn out to be of a few percent. This agrees well with experiment for meson decays, but for Λ_b the experimental life time is only (0.72 ± 0.06) of the b -quark life time inferred from meson decays. The theoretical expectation for this ratio is below one, but almost surely above 0.9. The second problem is the measured fraction of the B mesons, which decay semileptonically. Theory can reproduce it, but at the condition that a large fraction of these decays leads to $\bar{c}c$ pairs. The average number of c and \bar{c} quarks per decay is experimentally (1.13 ± 0.05) , while the theoretical number necessary to get agreement with the semileptonic branching ratio is 1.3. This difference may seem small, but it should be kept in mind that one c -quark is present in almost every b -decay. Thus what counts is the surplus over this number. Here the experimental number is less than half the theoretical one.

Finally let us mention the so called rare decays, *i.e.* the decays, where the b -quark goes over into an s -quark and a photon, or lepton pair. Here the theory involves penguin diagrams, is quite complicated and is still being refined, but what is important is that it agrees well with experiment. This eliminates many ideas concerning "new physics" *i.e.* physics beyond the standard model.

5. Production of heavy particles

Heavy particle production is a broad and active subject. Here we shall only mention a few problems, which now are attracting particular interest.

The calculated cross-section for the process $p\bar{p} \rightarrow t\bar{t}X$ at the Tevatron is somewhat lower than measured. Since the experimental uncertainties are large, however, and since the discrepancy decreases as data improve, this does not seem to be a serious problem.

The ratio of the decay probability of Z^0 into $b\bar{b}$ to the decay probability of Z^0 into any hadrons should be about 0.2, because there are five kinds of quarks into which a Z^0 can decay and they all have masses negligible compared to the Z^0 mass. Experimentally

$$R_b = \frac{\Gamma(Z^0 \rightarrow b\bar{b})}{\Gamma(Z^0 \rightarrow \text{hadrons})} = 0.2205 \pm 0.0016 \quad (8)$$

in agreement with this crude estimate. Precise calculations, however, give $R_b = 0.2155$, *i.e.* a ratio smaller by about three standard deviations than the experimental one. This is considered as a possible problem for the standard model. It is interesting that supersymmetry can increase the predicted R_b so that it becomes lower than the experimental value by only about one standard deviation. If this is the correct explanation of the discrepancy, the lightest supersymmetric particles should have masses below 100 GeV and there is a good chance of discovering them in the upgraded LEP accelerator. This is, of course, a bold speculation, but it has recently triggered much discussion. Incidentally, the corresponding ratio $R_c = 0.154 \pm 0.07$, to be compared with the theoretical prediction 0.172. Here, however, the experiment is very difficult and a modification of the theory is not plausible, therefore this discrepancy is expected to disappear, when data improves.

Finally let us mention the production of charmonia at the Tevatron. According to theory those charmonia, which are not decay products of particles containing b -quarks, should be mostly produced in gluon-gluon interactions. Such interactions are much more likely to produce P -wave charmonia (χ -states) than S -wave charmonia (ψ -states). Therefore, the prediction was that the direct production of ψ -states will be small and that a large majority of such charmonia will come from decays of χ -states. Experimentally it seems that the direct production of ψ -charmonia is much stronger than expected, sometimes stronger by more than an order of magnitude. One way out of this difficulty is to assume that the $c\bar{c}$ systems in octet colour states are an important intermediate state.

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