ON THE RELATION BETWEEN QUADRATIC AND LINEAR CURVATURE LAGRANGIANS IN POINCARÉ GAUGE GRAVITY*

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(Received July 3, 1996)

We discuss the choice of the Lagrangian in the Poincaré gauge theory of gravity. Drawing analogies to earlier de Sitter gauge models, we point out the possibility of deriving the Einstein-Cartan Lagrangian *without* cosmological term from a *modified* quadratic curvature invariant of *topological* type.

PACS numbers: 04.50. +h, 04.20. Cv

1. Introduction

One of the main achievements of the gauge approach to gravity (see [1–5] and references therein) lies in a better understanding of the deep relations between the symmetry groups of spacetime and the nature of the sources of the gravitational field. At the same time a satisfactory *kinematical* picture of gauge gravity emerges which specifies metric, coframe, and connection as the fundamental gravitational field variables. In contrast, the *dynamical* aspect of the gravitational gauge theory is far less developed. In general, the choice of a dynamical scheme, *i.e.* of the gravitational field Lagrangian, ranges from the simplest Einstein-Cartan model with the Hilbert type Lagrangian (linear in curvature) to the 15-parameter theory with Lagrangian quadratic in torsion and curvature, or even to non-polynomial models.

Some progress was achieved in gauge theories based on the de Sitter group [6–11] which is, in a sense, the closest semi-simple "relative" of the

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* Presented at the II German–Polish Symposium "New Ideas in the Theory of Fundamental Interactions", Zakopane, Poland, September 1995.

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(non-semi-simple) Poincaré group. The main idea behind the derivation of the gravitational field Lagrangian was to consider it as emerging, via a certain spontaneous breakdown symmetry mechanism, from a unique invariant, the Chern–Pontryagin or the Euler invariant, *e.g.*, which have the meaning of topological charges. Recently this approach has been reanalyzed in [12].

In this paper we report on an attempt to exploit the analogy with the de Sitter gauge approach. In a Riemann-Cartan spacetime, we construct a gravitational Lagrangian by starting from a topological invariant quadratic in curvature, deform it suitably, and arrive, apart from an exact form, at an Einstein–Cartan Lagrangian (linear in curvature). Whereas in the traditional approach of (4.6) the emergence of a cosmological term cannot be prevented, our new method, see our main result (4.9), yields a pure Einstein-Cartan Lagrangian *without* cosmological term.

2. Two four-dimensional topological invariants

The spacetime which we consider obeys a Riemann-Cartan geometry with *orthonormal* coframe ϑ^α, a metric $g = o_{\alpha\beta}\,\vartheta^\alpha \otimes \vartheta^\beta$, and a Lorentz connection $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha} = \Gamma_i{}^{\alpha\beta} dx^i$. Here the anholonomic frame indices are denoted by $\alpha, \beta, \ldots = \hat{0}, \hat{1}, \hat{2}, \hat{3}$, the holonomic coordinate indices by $i, j, \ldots = 0, 1, 2, 3$, and $o_{\alpha\beta} = diag\,(-1, +1, +1, +1)$ is the local Minkowski metric (with the help of which we raise and lower Greek indices).

As it is well known, in four dimensions there are two topological invariants which are constructed from the (in general, Riemann-Cartan) curvature two-form $R_\alpha{}^\beta = d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta$. These are the *Euler* invariant defined by the four-form

$$E := R_{\alpha\beta} \wedge R^{\star\alpha\beta} = \tfrac{1}{2}\eta^{\alpha\beta\mu\nu} R_{\alpha\beta} \wedge R_{\mu\nu}\,, \tag{2.1}$$

and the *Chern–Pontryagin* invariant described by the four-form

$$P := -R_\alpha{}^\beta \wedge R_\beta{}^\alpha = R_{\alpha\beta} \wedge R^{\alpha\beta}\,. \tag{2.2}$$

Both forms (2.1) and (2.2) are functionals of the Lorentz connection $\Gamma^{\alpha\beta}$ and of the local metric $o_{\alpha\beta}$. The $\eta^{\alpha\beta\mu\nu}$ is the Levi–Civita tensor, and no any other geometrical variables are involved. The right star * denotes the so-called Lie dual with respect to the Lie algebra indices.

The Gauss–Bonnet theorem states that an integral of (2.1), with a proper normalization constant, over a compact manifold without a boundary describes its Euler characteristics (the alternating sum of the Betti numbers which count the simplexes in an arbitrary triangulation of the manifold). As for the integral of (2.2), also introducing proper normalization, this represents the familiar "instanton" number specialized to the gravitational gauge case.

3. Ordinary and twisted deformations of the curvature

Due to the peculiar properties of the Lie algebra of the de Sitter group, a new object appears within the framework of de Sitter gauge gravity part of the generalized SO(1,4) or SO(2,3) curvature, a two-form

$$\Omega_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{\ell^2}\,\vartheta_{\alpha\beta}\,, \qquad \text{with} \qquad \vartheta_{\alpha\beta} := \vartheta_\alpha \wedge \vartheta_\beta\,. \tag{3.1}$$

We may call it a deformation of the original curvature form by a specific contribution constructed from the translational gauge potentials, namely the coframe one-forms ϑ^α. The constant ℓ with the dimension of length provides the correct dimension. If we recall that the curvature of a Riemann-Cartan spacetime can be decomposed into six irreducible pieces ${}^{(N)}R_{\alpha\beta}$, with $N = 1,\ldots,6$, see [4], then we find that $\vartheta_{\alpha\beta}$ is proportional to the sixth pieces ${}^{(6)}R_{\alpha\beta}$, the curvature scalar, that is, in (3.1) we subtracted a certain constant *scalar* curvature piece from the total curvature.

Similarly to (3.1), by means of the Hodge star, we can define another deformation

$$\mathcal{R}_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{\ell^2}\,\eta_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{\ell^2}\,{}^*\!\left(\vartheta_\alpha \wedge \vartheta_\beta\right)\,. \tag{3.2}$$

We may call this a twisted translational deformation, since $\eta_{\alpha\beta}$ has the opposite parity behavior compared to $R_{\alpha\beta}$. In fact, the term $\eta_{\alpha\beta}$ is proportional to a constant pseudoscalar piece ${}^{(3)}R_{\alpha\beta}$ of the curvature or, in components, to $R_{[\gamma\delta\alpha\beta]}$. In other words, in (3.2) a constant *pseudoscalar* curvature piece is subtracted out. Note that ${}^{(3)}R_{\alpha\beta}$ vanishes together with the torsion since, by means of the first Bianchi identity,

$$DT^\alpha = R_\beta{}^\alpha \wedge \vartheta^\beta \qquad \text{or} \qquad DT^\alpha \wedge \vartheta_\alpha = R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta = {}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta\,. \tag{3.3}$$

Hence, in a Riemannian spacetime, ${}^{(3)}R_{\alpha\beta}$ vanishes identically.

Using also the irreducible decomposition of the torsion into three pieces ${}^{(M)}T^\alpha$, with $M = 1,2,3$, the last equation can be rewritten as

$$\begin{aligned} {}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta &= d\left(\vartheta_\alpha \wedge T^\alpha\right) - T_\alpha \wedge T^\alpha \\ &= d\left(\vartheta_\alpha \wedge {}^{(3)}T^\alpha\right) - {}^{(1)}T_\alpha \wedge {}^{(1)}T^\alpha - 2\,{}^{(2)}T_\alpha \wedge {}^{(3)}T^\alpha\,. \end{aligned} \tag{3.4}$$

For a proof of this equation see [4] Eq. (B.2.19).

Before we consider some Lagrangians in the next section, we develop some algebra for the quadratic expressions of $\vartheta_{\alpha\beta}$ and $\eta_{\alpha\beta}$. We find:

$$\vartheta_{\alpha\beta}\wedge\vartheta^{\alpha\beta} = -\left(\vartheta_\alpha\wedge\vartheta^\alpha\right)\wedge\left(\vartheta_\beta\wedge\vartheta^\beta\right) = 0\,. \tag{3.5}$$

Moreover, for any two-form Φ, we have $^{**}\Phi = -\Phi$. Consequently, we find

$$\vartheta_{\alpha\beta}\wedge\vartheta^{\alpha\beta} = -\left({}^{**}\vartheta_{\alpha\beta}\right)\wedge\vartheta^{\alpha\beta} = -\,{}^*\vartheta_{\alpha\beta}\wedge{}^*\vartheta^{\alpha\beta} = -\eta_{\alpha\beta}\wedge\eta^{\alpha\beta} \tag{3.6}$$

or

$$\eta_{\alpha\beta}\wedge\eta^{\alpha\beta} = 0\,. \tag{3.7}$$

The mixed term can be expanded as follows:

$$\vartheta_{\alpha\beta}\wedge\eta^{\alpha\beta} = \tfrac{1}{2}\,\eta^{\alpha\beta\gamma\delta}\vartheta_{\alpha\beta}\wedge\vartheta_{\gamma\delta} = 12\,\eta\,. \tag{3.8}$$

Here η is, as usually, the volume four-form. If we transform the Hodge star $*$ into the Lie star $\star$, we have

$$\eta^{\alpha\beta} = \vartheta^{\star\alpha\beta} = {}^*\vartheta^{\alpha\beta}\,. \tag{3.9}$$

Eventually, we take the Lie star of the last equation:

$$\eta^{\star\alpha\beta} = -\vartheta^{\alpha\beta}\,. \tag{3.10}$$

4. Deformed topological invariants and the Einstein–Cartan Lagrangian

Let us calculate the Euler and Pontryagin four-forms with the curvature replaced by the deformed curvature. We denote the Lagrangian of the Einstein–Cartan theory by

$$L_{\mathrm{EC}} := -\frac{1}{2\ell^2}\,\eta_{\alpha\beta}\wedge R^{\alpha\beta}\,. \tag{4.1}$$

Later we will meet similar Lagrangians with $\eta_{\alpha\beta}$ substituted by $\vartheta_{\alpha\beta}$. We do the corresponding algebra first:

$$\begin{aligned}\vartheta_{\alpha\beta}\wedge R^{\alpha\beta} &= \tfrac{1}{2}R^{\gamma\delta\alpha\beta}\,\vartheta_{\gamma\delta}\wedge\vartheta_{\alpha\beta}\\ &= \tfrac{1}{2}R_{[\alpha\beta\gamma\delta]}\,\vartheta^\alpha\wedge\vartheta^\beta\wedge\vartheta^\gamma\wedge\vartheta^\delta = {}^{(3)}R_{\alpha\beta}\wedge\vartheta^\alpha\wedge\vartheta^\beta\,, \end{aligned}\tag{4.2}$$

$$\eta_{\alpha\beta}\wedge R^{\alpha\beta} = -2\ell^2 L_{\mathrm{EC}}\,, \tag{4.3}$$

$$\vartheta_{\alpha\beta}\wedge R^{\star\alpha\beta} = \vartheta^{\star\alpha\beta}\wedge R_{\alpha\beta} = \eta_{\alpha\beta}\wedge R^{\alpha\beta} = -2\ell^2 L_{\mathrm{EC}}\,, \tag{4.4}$$

$$\eta_{\alpha\beta}\wedge R^{\star\alpha\beta} = \eta^{\star\alpha\beta}\wedge R_{\alpha\beta} = -\vartheta^{\alpha\beta}\wedge R_{\alpha\beta} = -{}^{(3)}R_{\alpha\beta}\wedge\vartheta^\alpha\wedge\vartheta^\beta\,. \tag{4.5}$$

In the formulas (4.2) and (4.5) it is of course possible to substitute the first Bianchi identity (3.4) in order to splitt off a boundary term, if desireable.

For the deformations (3.1) and (3.2) one finds, respectively, the following generalized Euler forms:

$$V_{\mathrm{Eu}} = \Omega_{\alpha\beta} \wedge \Omega^{\star\alpha\beta} = E + 4L_{\mathrm{EC}} + \frac{12}{\ell^4}\,\eta\,. \tag{4.6}$$

$$V'_{\mathrm{Eu}} = \mathcal{R}_{\alpha\beta} \wedge \mathcal{R}^{\star\alpha\beta} = E + \frac{2}{\ell^2}\,{}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta - \frac{12}{\ell^4}\,\eta\,, \tag{4.7}$$

$$V''_{\mathrm{Eu}} = \Omega_{\alpha\beta} \wedge \mathcal{R}^{\star\alpha\beta} = E + 2L_{\mathrm{EC}} + \frac{1}{\ell^2}\,{}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta\,. \tag{4.8}$$

It is interesting to note that the translational Chern–Simons term $\vartheta_\alpha \wedge T^\alpha$ [13], via (3.4), appears as boundary term in (4.7) and (4.8). The other mixed term, $\mathcal{R}_{\alpha\beta} \wedge \Omega^{\star\alpha\beta}$, is the same as that in (4.8), since the Lie star can be moved to $\mathcal{R}_{\alpha\beta}$.

Three more generalized topological Lagrangians are defined according to the Chern-Pontryagin pattern as follows:

$$V_{\mathrm{Po}} = \mathcal{R}_{\alpha\beta} \wedge \mathcal{R}^{\alpha\beta} = P + 4L_{\mathrm{EC}}\,, \tag{4.9}$$

$$V'_{\mathrm{Po}} = \Omega_{\alpha\beta} \wedge \Omega^{\alpha\beta} = P - \frac{2}{\ell^2}\,{}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta\,, \tag{4.10}$$

$$V''_{\mathrm{Po}} = \Omega_{\alpha\beta} \wedge \mathcal{R}^{\alpha\beta} = P + 2L_{\mathrm{EC}} - \frac{1}{\ell^2}\,{}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta + \frac{12}{\ell^4}\eta\,. \tag{4.11}$$

As we can see, both deformed curvatures, (3.1) and (3.2), generate the Einstein–Cartan Lagrangian (4.1) from the topological type invariants, since the variational derivatives of E and P are identically zero. In the case of $\Omega_{\alpha\beta}$ one should use the Euler type form (4.6), while for $\mathcal{R}_{\alpha\beta}$ the Chern–Pontryagin type invariant (4.9) suggests itself. Actually, the case (4.6) was studied in the work of MacDowell and Mansouri [6] (see also [7–12]. The problem of this de Sitter gauge approach was a very large cosmological constant $\sim 1/\ell^4$ which is generated simultaneously with the Einstein-Cartan Lagrangian. To the best of our knowledge, the possibility (4.9) of using the twisted deformation of the curvature was not reported in the literature, even if Mielke [14] had somewhat related thoughts, see his Eq. (9.8). A nice improvement of the usual de Sitter result is then the *absence of the cosmological term* in (4.9). It can certainly happen that the cosmological constant would reappear due to other physical mechanisms (through the quantum vacuum corrections, *e.g.*), but the huge initial value $1/\ell^4$ is avoided.

The inspection of the Lagrangians (4.7) and (4.10) shows that they are trivial from the dynamical point of view. Since, up to a boundary term, ${}^{(3)}R_{\alpha\beta} \wedge \vartheta^\alpha \wedge \vartheta^\beta \sim T_\alpha \wedge T^\alpha$, see (3.4), the vacuum field equations leave the curvature undetermined while the torsion turns out to be zero. In the non-vacuum case, like in the Einstein–Cartan theory, the torsion is related to the spin current. More exactly, it is proportional to the Hodge dual of the spin. A similar thing happens in the curvature sector where the left hand side of the gravitational field equation is then represented not by the Einstein form but rather by its dual. This theory evidently has no Newtonian limit and thus appears to be physically irrelevant.

The cases (4.8) and (4.11) also induce an Einstein-Cartan Lagrangian. However, in both cases additionally a definite parity violating term [15, 16] emerges, see also [17] Sec. 5.3, a possibility which one has to keep in mind, but presently a need for such terms is not obvious. The Lagrangian (4.11) represents the MacDowell-Mansouri Lagrangian with an additional parity violating admixture, whereas (4.8) is attached to our new Lagrangian (4.9) in an analogous way.

It would be tempting to include the twisted deformation of the curvature (3.2) into a generalized gravitational gauge theory analogously to the way (3.1) appears in the de Sitter model. However this seems to be impossible, at least at the present level of understanding this problem. A simple argument runs as follows: The de Sitter group describes the symmetry of a four-dimensional model spacetime the curvature of which is defined by putting (3.1) equal zero, *i.e.* the model is a de Sitter spacetime with $R^{\alpha\beta} = \frac{1}{\ell^2}\vartheta^\alpha \wedge \vartheta^\beta$. Calculating the exterior covariant derivative of this equation, one finds that, *in four dimensions*, torsion is equal to zero, and one is left with the usual Riemannian spacetime of constant curvature. Unlike this, the condition $R^{\alpha\beta} = \frac{1}{\ell^2}\eta^{\alpha\beta}$ specifies a spacetime of constant pseudoscalar curvature, but does not seem to define any sound spacetime geometry. Again taking the covariant exterior derivative, one discovers that torsion is absent, and the remaining equation, which involves the purely Riemannian curvature, turns out to be inconsistent. Hence it looks as if no fundamental spacetime existed with a symmetry property which would make it possible to include the twisted deformation of the curvature in some sector of a generalized gauge group.

5. Scalar field

In general case, the third and sixth irreducible parts of the Riemann–Cartan curvature read [4]

$$^{(3)}R_{\alpha\beta} = -\tfrac{1}{12}X\eta_{\alpha\beta}, \qquad ^{(6)}R_{\alpha\beta} = -\tfrac{1}{12}R\,\vartheta_\alpha \wedge \vartheta_\beta\,, \tag{5.1}$$

where the curvature pseudoscalar and scalar are defined by

$$X := {}^*(\vartheta_\alpha \wedge \vartheta_\beta \wedge R^{\alpha\beta}) \qquad \text{or} \qquad R := e_\alpha \rfloor e_\beta \rfloor R^{\alpha\beta}\,, \tag{5.2}$$

respectively. This suggests a natural generalization of the deformations (3.1) and (3.2) by introducing a scalar field Φ according to

$$\Omega^{\Phi}_{\alpha\beta} := R_{\alpha\beta} - \Phi^2 \vartheta_{\alpha\beta} \tag{5.3}$$

and

$$\mathcal{R}^{\Phi}_{\alpha\beta} := R_{\alpha\beta} - \Phi^2 \eta_{\alpha\beta} \,. \tag{5.4}$$

There is no need to introduce a constant factor ℓ^{-2} since the canonical dimension of a scalar field is already ℓ^{-1}.

The analysis of the arising gravitational Lagrangians is straightforward, since it is only necessary to replace in the formulas (4.6)–(4.11) everywhere $1/\ell$ by Φ. In the *absence of matter* one notices immediately that all these models possess the Weyl conformal symmetry

$$\vartheta^\alpha \longrightarrow e^\lambda \vartheta^\alpha \,, \qquad \text{hence} \qquad g = o_{\alpha\beta}\, \vartheta^\alpha \otimes \vartheta^\beta \longrightarrow e^{2\lambda} g \,, \tag{5.5}$$

$$\Phi \longrightarrow e^{-\lambda}\Phi \,. \tag{5.6}$$

With the suitable choice of the conformal parameter function λ, it is always possible to eliminate the scalar field completely by picking the gauge $\Phi = 1/\ell$. Thus the Lagrangians (4.6)–(4.11) provide a Poincaré gauge gravity analog of the Riemannian gravity sector in the model of Pawłowski and Rączka [18, 19]. Introducing the matter sector in such a way so that the conformal symmetry (5.5), (5.6) is preserved, it is possible to arrive at the Riemann–Cartan generalization of the Pawłowski and Rączka model. If, however, the coupling with matter is introduced without respecting (5.5), (5.6), we discover the non-Riemannian generalizations of the scalar–tensor theroies of gravity.

6. Discussion

One may notice that both deformations (3.1) and (3.2) are particular cases of the general deformation

$$R^{\alpha\beta} - \frac{A}{\ell^2}\,\vartheta^\alpha \wedge \vartheta^\beta - \frac{B}{\ell^2}\,\eta^{\alpha\beta} \,, \tag{6.1}$$

with arbitrary constants A and B. Then a straightforward calculation demonstrates that both topological forms, of the Euler type as well as of the Chern–Pontryagin type, generate the same gravitational Lagrangian which includes the true topological invariant, the Einstein–Cartan term modified by the square of torsion ("twisted" Einstein–Cartan) term, plus the cosmological constant $\sim 1/\ell^4$. One can verify that the Lagrangian

$A\, L_{\rm EC} + B\, T^{\alpha} \wedge T_{\alpha}$ has the same physical contents as the usual Einstein–Cartan model, provided $A \neq \pm iB$. This confirms earlier observations made within the framework of the self-dual two-form approaches to gravity theory. In accordance with the discussion above, in the most general case A and B may be (nonconstant) scalar fields, thus yielding either conformal invariant or scalar-tensor versions of the "twisted" Einstein–Cartan model.

The general deformation (6.1) is again closely related to the de Sitter symmetry group, and hence one could probably obtain this from a kind of a twisted de Sitter gauge theory. However, the gravitational Lagrangian should inevitably contain a large classical cosmological constant with all the known difficulties of physical interpretation of such a model.

We are grateful to Frank Gronwald for helpful discussions. One of the authors (FWH) would like to thank Ryszard Rączka for the invitation to the Polish–German Theoretical Physics Meeting in Zgorzelisko (near Zakopane) in September 1995 where a part of this paper was drafted. The work of YNO was supported by the grant He 528/17-1 of the Deutsche Forschungsgemeinschaft, Bonn.

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