EXTENDED-OBJECT FAMILIES IN STRING AND SUPERGRAVITY THEORIES*,**

K.S. Stelle

The Blackett Laboratory, Imperial College
Prince Consort Road, London SW7 2BZ, UK

(Received July 3, 1996)

Supergravity theories admit a large variety of extended-object solutions that are characterized by the saturation of a Bogomol'ny bound, with a consequent partial preservation of unbroken supersymmetry. We present a scheme for the classification of such solutions into families related by dimensional reduction and oxidation, each headed by a maximal non-isotropically-oxidizable, or "stainless" solution.

PACS numbers: 04.50.+h, 04.65.+e, 11.25.-w, 11.27.+d

The effective field theory for the massless modes of the bosonic string is described, up to order α', by the effective action

$$I_{\text{eff}} = \int d^Dx\sqrt{-g}e^{-2\phi}\Big[(D-26) - \tfrac{3}{2}\alpha'(R + 4\nabla^2\phi - 4(\nabla\phi)^2 - \tfrac{1}{12}H_{MNP}H^{MNP}\Big] + \mathcal{O}(\alpha')^2\,, \tag{1}$$

containing the following massless fields: the metric g_{MN}, the antisymmetric tensor gauge field B_{MN}, with field strength $H_{MNP} = \partial_M B_{NP} + \partial_N B_{PM} + \partial_P B_{MN}$, and the dilaton field ϕ. A similar set of fields occurs as a subset of the effective field theory for any string theory, and in a superstring theory corresponds to the Neveu–Schwarz, Neveu–Schwarz (NS-NS) sector of the theory. The effective action (1) provides a summary of the effective field equations of the theory. These equations are themselves directly derived by making a self-consistent coupling of the string to a background

* Presented at the II German–Polish Symposium "New Ideas in the Theory of Fundamental Interactions", Zakopane, Poland, September 1995.

** Research supported in part by the Commission of the European Communities under contracts SC1*–CT91–0674 and SC1*–CT92–0789.

"condensate" of its own massless modes, requiring self-consistency with the background through the cancellation of BRST anomalies [2], or by the vanishing of sigma-model beta functions (β^g, β^B, β^ϕ) [3]. The $(D-26)$ "cosmological term" indicates the critical dimension; in the effective action for superstrings, this changes to $(D-10)$.

One should note several major differences from General Relativity in the theory described by the effective action (1). First of all, the α' corrections continue on to infinite order, with finite, calculable coefficients. This gives rise to apparent ghost states, owing to the occurrence of higher-derivative terms in the effective action. The string theory is unitary, however, so these apparent ghost states must be purely artifactual. They are similar to analogous artifacts that would occur in the QED effective action for the massless photon after integrating out the massive electrons. As in that case, the apparent ghosts herald the onset of new physical effects not properly described by the effective action, once a certain energy scale is reached. In QED, this effect is electron-positron pair creation. In string theory, it is the excitation of massive-level string oscillations. One also needs to take account of the physical effects of the additional massless fields B_{MN} and ϕ in (1). The scalar dilaton field ϕ plays an especially important rôle, because its presence blurs the identification of the physically-relevant spacetime metric, owing to the possibility of conformal redefinitions $g_{MN} \to e^{\lambda\phi} g_{MN}$, giving rise to different "conformal frames". The effective action (1) is written in a conformal frame such that $I_{\text{eff}} = \int d^D \sqrt{-g} e^{-2\phi} \beta^\phi$. Another frame that is frequently used is the "Einstein frame," in which the $e^{-2\phi}$ factor in front of the Einstein–Hilbert Lagrangian $\sqrt{-g}R$ is scaled away.

In superstring theories, there appear additional massless-level antisymmetric tensor gauge fields. These fields couple to bilinears in the fermionic variables of these theories, and so belong to the Ramond, Ramond (RR) sector. For example, in the type IIA theory, the RR sector has a 3-form gauge-field potential A_{MNP} and a 1-form potential A_M in addition to the NS-NS fields. In the type IIB theory, there is a second 2-form potential, making up a doublet B^i_{MN}, together with a 4-form potential A^+_{MNPQ} (whose 5-form field strength is self-dual), in the superstring critical dimension $D = 10$.

In the following, we shall simplify our discussion and at the same time shall encompass the effects of all of the scalar and antisymmetric-tensor contributions to the effective theory by restricting attention to one scalar field, denoted ϕ, and one $(n-1)$-form gauge potential $B_{M_1 \cdots M_{n-1}}$, with an n-form field strength $H_{M_1 \cdots M_n}$. The D-dimensional Lagrangian for these fields will be taken to be

$$\mathcal{L} = \sqrt{-g}\Big[R - \tfrac{1}{2}(\partial\phi)^2 - \frac{1}{2n!} e^{-a\phi} H_{[n]}{}^2\Big], \tag{2}$$

where the parameter a governing the coupling of the scalar ϕ to the antisymmetric tensor kinetic terms will play a central rôle in our discussion. We have given a reminder of the order of the form $H_{[n]}$ in its subscript. The equations of motion following from (2) are

$$\begin{aligned}
\Box\phi &= -\frac{a}{2n!}\mathrm{e}^{-a\phi}H_{[n]}{}^2\,,\\
R_{MN} &= \tfrac{1}{2}\partial_M\phi\partial_N\phi + S_{MN}\,,\\
S_{MN} &= \frac{1}{2(n-1)!}\mathrm{e}^{-a\phi}[H^2_{MN} - \frac{n-1}{n(D-2)}H^2 g_{MN}]\,,\\
\nabla_{M_1}(\mathrm{e}^{-a\phi}H^{M_1\cdots M_n}) &= 0\,.
\end{aligned} \tag{3}$$

Kaluza–Klein dimensional reduction

Now we consider the reduction of the system described by (2), (3) from $D' = D+1$ to D-dimensional space-time. We shall consider only *consistent truncations* of the fields, *i.e.* restrictions of the fields such that solutions of the restricted theory are at the same time solutions of the *un*restricted theory. We shall let the $(D+1)$-dimensional quantities be indicated with careted indices: $x^{\hat{M}} = (x^M, z)$. The line element in $D+1$ dimensions is taken to be

$$d\hat{s}^2 = \mathrm{e}^{2\alpha\varphi}ds^2 + \mathrm{e}^{2\beta\varphi}(dz + A_M dx^M)^2\,. \tag{4}$$

The Kaluza–Klein ansatz for the metric is then (4) together with a restriction to *z-independent* fields $\varphi(x)$ and $A_M(x)$. The constants α and β will be chosen shortly. Insertion of the ansatz (4) into the Einstein–Hilbert action produces an action for a D-dimensional theory. Now fix $\beta = -(D-2)\alpha$, to maintain the Einstein-frame form of the D-dimensional action, and fix $\alpha^2 = [2(D-1)(D-2)]^{-1}$ to normalize the φ kinetic term.

We also need to specify an ansatz for the antisymmetric tensor gauge field $\hat{B}_{[n-1]}$ (where the superscript indicates the order of the form). Since only one of the $(n-1)$ antisymmetrized indices may take the value z, one has

$$\hat{B}_{[n-1]} = B_{[n-1]}(x) + B_{[n-2]}(x)\wedge dz\,. \tag{5}$$

For the field strength $\hat{H}_{[n]} = d\hat{B}_{[n-1]}$ it is convenient to define $G'_{[n]} = G_{[n]} - G_{[n-1]}\wedge A$, $G_{[n]} = dB_{[n-1]}$ and $G_{[n-1]} = dB_{[n-2]}$, giving

$$\hat{H}_{[n]} = G'_{[n]} + G_{[n-1]}\wedge(dz + A)\,. \tag{6}$$

Substituting these decompositions into (2), one obtains the dimensionally-reduced Lagrangian

$$\mathcal{L} = \sqrt{-g}\Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{4}e^{-2(D-1)\alpha\varphi}F^2 - \frac{1}{2n!}e^{-2(n-1)\alpha\varphi - \hat{a}\phi}{G'_{[n]}}^2 - \frac{1}{2(n-1)!}e^{2(D-n)\alpha\varphi - \hat{a}\phi}{G_{[n-1]}}^2\Big], \quad (7)$$

where $F = dA$ is the field strength for the Kaluza–Klein vector emerging from the $(D+1)$-dimensional metric $\hat{g}_{MN}$ and $\hat{a}$ is the ϕ coupling parameter for $\hat{H}_{[n]}$ in the $(D+1)$-dimensional theory.

Note that the prefactors of the terms ${G'_{[n]}}^2$ and ${G_{[n-1]}}^2$ are both of the form $e^{-a_{[n]}\tilde{\phi}_{[n]}}$, where the $\tilde{\phi}_{[n]}$ are SO(2)-rotated combinations of ϕ and φ. Restriction of the fields in the original $(D+1)$-dimensional Lagrangian (2) to obtain the dimensionally-reduced Lagrangian (7) is a consistent truncation of the theory. Further restriction to keep just one of the three n-form terms, F^2 (corresponding to $n = 2$), ${G'_{[n]}}^2$, or ${G_{[n-1]}}^2$, together with an appropriately-rotated scalar-field combination $\tilde{\phi}_{[n]}$ while setting the orthogonal scalar-field combination to zero is also a consistent truncation. This last truncation gives once more again a Lagrangian of our standard simple form (2). Since all of the restrictions made have been consistent truncations, solutions of the restricted theory will also be solutions of the original unrestricted theory. Consequently, studying solutions of (2) in the diverse possible spacetime dimensions D for supergravity theories will also give us sets of solutions of the original superstring effective field theories.

p-brane solutions

Now we concentrate on solutions to the standard system of field equations (3) following from (2). For the line-element in D dimensions, we make the metric ansatz

$$ds^2 = e^{2A}dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B}dy^m dy^n \delta_{mn}, \quad (8)$$

where x^μ ($\mu = 0, \ldots, d-1$) are coordinates on a translationally invariant d-dimensional subsurface embedded in the D-dimensional spacetime; these are to be interpreted as the "worldvolume" coordinates of the p-brane. The y^m ($m = 1, \ldots, D-d$) are coordinates of the "transverse" space to the p-brane. The functions $A(r)$ and $B(r)$ are taken to depend isotropically on the y^m coordinates, *i.e.* only through the $SO(D-d)$ rotationally-symmetric combination $r = \sqrt{y^m y^m}$.

For the $(n-1)$-form gauge potential $B_{M_1\cdots M_{n-1}}$, there are two types of ansatz for the solutions that we shall consider:

Elementary p-branes

In this case, the antisymmetric tensor $B_{M_1\cdots M_{n-1}}$ couples directly to the worldvolume of the p-brane, so $d = p+1 = n-1$, and we make the ansatz

$$B_{\mu_1\cdots\mu_{n-1}} = \varepsilon_{\mu_1\cdots\mu_{n-1}} e^{C(r)}, \tag{9}$$

where the as-yet undetermined function $C(r)$ again depends isotropically on the transverse y^m coordinates. Components of $B_{M_1\cdots M_{n-1}}$ with indices pointing into any of the transverse directions are set to zero. As a consequence of this and of isotropicity, the field strength $H_{[n]}$ takes the form

$$H_{m\mu_1\cdots\mu_{n-1}} = \varepsilon_{\mu_1\cdots\mu_{n-1}} \partial_m e^{C(r)} . \tag{10}$$

Solitonic p-branes

This case is dual to the elementary case above, and for it the worldvolume dimension is $d = D-n-1$. The antisymmetric tensor ansatz is most conveniently given directly in terms of the field strength,

$$H_{m_1\cdots m_n} = Q\varepsilon_{m_1\cdots m_n p} \frac{y^p}{r^{n+1}}, \tag{11}$$

where Q is a magnetic charge.

Given the above ansätze, one finds solutions by direct substitution into the equations of motion (3). First, define the worldvolume dual dimension by

$$\tilde{d} = D - d - 2 . \tag{12}$$

Then one finds solutions with

$$\begin{aligned} B &= -\frac{d}{\tilde{d}} A , \\ \phi &= \frac{a(D-2)}{\varepsilon \tilde{d}} A , \\ e^{-\tilde{k}A} &= 1 + \frac{k}{r^{\tilde{d}}} , \end{aligned} \tag{13}$$

where $\tilde{k} = d + (2\tilde{d})^{-1}a^2(D-2)$, $\varepsilon = +1(-1)$ for the elementary (solitonic) solution, and k is a constant related to the charge Q by $k = [2(D-2)]^{-1}[(D-2)^2a^2/\tilde{d}^2 + 2d(D-2)/\tilde{d}]^{1/2}\varepsilon Q$. In the elementary case (9), the function $C(r)$ is determined by

$$\frac{\partial}{\partial r}(e^C(r)) = Qe^{2\tilde{k}A} r^{-(\tilde{d}+1)} . \tag{14}$$

The line element determined by the solution (13) may be written in a nicely symmetrized form

$$ds^2 = \left(1 + \frac{k}{r^{\tilde{d}}}\right)^{-\frac{4\tilde{d}}{(D-2)\Delta}} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1 + \frac{k}{r^{\tilde{d}}}\right)^{\frac{4d}{(D-2)\Delta}} dy^m dy^m\,, \quad (15)$$

where the constant Δ is defined by

$$a^2 = \Delta - \frac{2d\tilde{d}}{D-2}\,. \quad (16)$$

The importance of the quantity Δ introduced here goes beyond the nice symmetrical form it gives to (15), for we shall see that Δ is invariant under dimensional reduction [1]. Within the present context of solutions derived from the ansätze (8)–(11), the values of Δ that have been found are $\Delta = 4, 2, {}^4\!/_3$. Generalisation of this ansatz to excite multiple scalar-antisymmetric combinations yields further Δ values [11, 12].

The elementary and solitonic solutions (13), (14) derived from the ansätze (8)–(11) have a special structure that permits Kaluza–Klein dimensional reduction to be carried out directly *on the solution*, and not only on the equations of motion. The special feature of these solutions permitting this is translational invariance along the worldvolume directions, as manifested in the coordinates used in (13), (14) by the lack of dependence of $A(r)$ and $C(r)$ upon the worldvolume coordinates x^μ. Thus, dimensional reduction may be effected directly upon such a solution by letting the reduction coordinate z be taken to be any one of the x^μ. Such a reduction automatically preserves isotropicity in the transverse coordinates and maps elementary $\rightarrow$ elementary and solitonic $\rightarrow$ solitonic solution types.

Oxidation, rustiness and stainlessness

The converse of Kaluza–Klein dimensional reduction has been called "dimensional oxidation." Whenever a theory in D spacetime dimensions may be obtained by dimensional reduction from a theory in $D+1$ dimensions, then any solution of the D-dimensional theory may be promoted, or "oxidized" to a solution of the $D+1$-dimensional theory. This procedure does not, however, guarantee that specific features of a solution, such as isotropicity in the transverse dimensions, will be preserved under oxidation. Thus, we introduce some more terminology (traditionally fanciful in the subject of supergravity): if an isotropic p-brane solution in D dimensions can successfully be oxidized into an isotropic $(D+1)$-dimensional solution in accordance with our ansätze (8)–(11), then we shall call such a solution "rusty." As is immediately apparent, isotropicity in the new $D+1-d$ transverse dimensions requires that the value of the worldsheet dimension d increase by one in this process (but note that $\tilde{d}$ remains constant). Thus, a rusty p-brane in D dimensions oxidizes into a $(p+1)$-brane in $(D+1)$ dimensions.

Clearly, all of the 'brane solutions that can be obtained by dimensional reduction from solutions in higher spacetime dimensions using translational symmetries along worldvolume directions are rusty. The process of isotropic dimensional oxidation and reduction of such solutions corresponds to the process of *double dimensional reduction* of p-brane worldsheet actions as originally discussed in [4]. We shall not concentrate here on the dynamics of the zero-mode fluctuations of our translationally-invariant p-brane solutions, but these should be described by worldvolume actions generalizing the Nambu–Goto action for the string. Several of the present families of solutions fit cleanly into known actions of this type [5]; others appear to require an extension of the known class of worldvolume actions.

A sequence of p-brane solutions related by isotropic dimensional oxidation must end somewhere. This can happen in two ways. One of these occurs at the top of a dimensional reduction/oxidation sequence of theories, where the top theory is not itself obtainable by reduction from any theory in a higher dimension. The other way involves a theory that can be oxidized to a theory in a higher dimension, but where the solution in question cannot be oxidized without loosing its isotropic transverse structure in the process, *i.e.* without going outside the form of our p-brane ansätze (8)–(11). In either of these cases where the solution cannot be isotropically oxidized, we shall call the solution "stainless." Classifying the stainless solutions will give us a classification of all p-brane solutions.

Examples

The $D=10$ effective action for any supergravity theory includes the fields present in the bosonic string effective action (1). After a conformal rescaling to put the action into Einstein frame, one has

$$I_{10} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2\cdot 3!}\mathrm{e}^{-\phi}H_{MNP}H^{MNP}\Big]\,. \qquad (17)$$

This is of our general form (2), with $a=1$, $n=3$. The two-form gauge field B_{MN} supports an elementary string ($p=1$) solution [6]

$$ds^2_{10} = \left(1+\frac{k_2}{r^6}\right)^{-3/4}\eta_{\mu\nu}dx^\mu dx^\nu + \left(1+\frac{k_2}{r^6}\right)^{1/4}\delta_{mn}dy^m dy^n\,,$$

$$B_{01} = -\mathrm{e}^{\phi_0/2}\left(1+\frac{k_2}{r^6}\right)^{-1} \qquad \mathrm{e}^{-2\phi} = \mathrm{e}^{-2\phi_0}\left(1+\frac{k_2}{r^6}\right)\,. \qquad (18)$$

This solution has $d = 2$, $\tilde{d} = 10 - 2 - 2 = 6$, $a = 1$, giving $1 = \Delta - 2 \cdot 2 \cdot 6/(10-2)$, so $\Delta = 4$.

The simplified action (17) is obtained from a consistent truncation of $D = 10$, $N = 2A$ supergravity, whose bosonic sector is:

$$I_{2A} = \frac{1}{2\kappa^2} \int d^{10}x\sqrt{-g}\Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2\cdot 3!}e^{-\phi}H_{[3]}^2 - \tfrac{1}{2\cdot 2!}e^{3\phi/2}F_{[2]}^2 - \tfrac{1}{2\cdot 4!}e^{\phi/2}H_{[4]}^{\prime\,2}\Big] - \tfrac{1}{2}H_{[4]} \wedge H_{[4]} \wedge B_{[2]}\,, \tag{19}$$

where $H'_{[4]} = dB_{[3]} + A_{[1]} \wedge H_{[3]}$. This action is in turn obtained from a consistent Kaluza–Klein dimensional reduction of the bosonic sector of $D = 11$ supergravity:

$$I_{11} = \frac{1}{2\kappa^2} \int d^{11}x\sqrt{-g}\Big[R - \tfrac{1}{48}H_{MNPQ}H^{MNPQ}\Big] + \tfrac{1}{(12)^4}H_{[4]} \wedge H_{[4]} \wedge B_{[3]}\,. \tag{20}$$

Since the action (17) giving rise to the elementary string solution (18) is obtained from a sequence of consistent truncations starting from the $D = 11$ action (20), the $D = 10$ string solution automatically oxidizes to a solution of the $D = 11$ theory. After a preliminary conformal rescaling to account for the canonical Einstein-frame normalization of the $D = 10$ action, one obtains the $D = 11$ solution

$$\begin{aligned} ds_{11}^2 &= \left(1 + \frac{k_2}{r^6}\right)^{-2/3} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1 + \frac{k_2}{r^6}\right)^{1/3} dy^m dy^n\,, \\ B_{\mu\nu\rho} &= -\varepsilon_{\mu\nu\rho}\left(1 + \frac{k_2}{r^6}\right)^{-1}\,. \end{aligned} \tag{21}$$

This is the isotropic elementary membrane solution ($p = 2$) of $D = 11$ supergravity [7]. Note that the $D = 10$ dilaton has disappeared, having been absorbed into the $D = 11$ metric as $g_{11\,11} = e^{4\phi/3}$. Since the $D = 11$ theory does not contain a dilaton, one should consider that the coupling parameter a takes the value zero, giving $0 = \Delta - 2 \cdot 3 \cdot 6/(11-2)$, so $\Delta = 4$, the same as for the string solution in $D = 10$.

By the above discussion, we have verified the rusty character of the ($p = 1$) elementary string in $D = 10$, since it is isotropically oxidizable to the ($p = 2$) elementary membrane in $D = 11$. The $D = 11$ membrane solution is itself stainless, since the $D = 11$ supergravity theory is the largest supergravity theory, and is not obtainable by dimensional reduction from any higher dimension. The $D = 11$ elementary membrane sits at the top of an oxidation/reduction pathway that reduces down to the $D = 10$ elementary string that we began this example with, and then to a $D = 9$

elementary particle solution, which turns out to be an extreme Reissner–Nordstrom black hole.

Now consider an example of a solution where oxidation can be performed, but where the isotropic character is lost. In $D = 9$, there are two independent 5-brane solitonic solutions, whose metrics are:

$$ds^2_{9\,\Delta=4} = \left(1+\frac{k}{r}\right)^{-1/7} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1+\frac{k}{r}\right) dy^m dy^n\,, \ \ \mu = 0,\cdots,5, \tag{22}$$

$$ds^2_{9\,\Delta=2} = \left(1+\frac{k}{r}\right)^{-2/7} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1+\frac{k}{r}\right)^{12/7} dy^m dy^n\,, \ \ m = 6,\cdots,8\,. \tag{23}$$

In both cases, there is a 1-form potential A_M, with field strength

$$H_{mn} = \frac{-2k}{\Delta^{1/2}}\varepsilon_{mnp}\frac{y^p}{r^3}\,. \tag{24}$$

Since the $D = 9$ theory can be obtained by a consistent Kaluza–Klein reduction from $D = 10$, both solutions (22), (23) can be oxidized, but with different results. The $\Delta = 4$ solution (22) can oxidize isotropically, since the $D = 10$ theory has a B_{MN} 2-form potential with a $\Delta = 4$ scalar coupling and so can support an isotropic solitonic 6-brane solution:

$$\begin{aligned} ds^2_{10\,\Delta=4} &= \left(1+\frac{k}{r}\right)^{-1/8} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1+\frac{k}{r}\right)^{7/8} dy^m dy\,, \\ H_{mnp} &= -k\varepsilon_{mnpq}\frac{y^q}{r^4}\,. \end{aligned} \tag{25}$$

Since it can isotropically oxidize, the $D = 9$, $\Delta = 4$ solution (22) is rusty.

By contrast, the $D = 9$, $\Delta = 2$ solution can only oxidize by having the $\Delta = 2$ 1-form A_M become part of the $D = 10$ metric, since there is no appropriate $\Delta = 2$ 2-form gauge field in $D = 10$. One then finds the $D = 10$ metric

$$\begin{aligned} ds^2_{10\,\text{warped}} &= \left(1+\frac{k}{r}\right)^{-1/7} (dx^\mu dx^\nu \eta_{\mu\nu} + (dz + A_m dx^m)^2) \\ &\quad + \left(1+\frac{k}{r}\right)^{7/4} dy^m dy^m\,. \end{aligned} \tag{26}$$

In this metric, z has become a coordinate on a non-trivial $U(1)$ fibre bundle: the metric has become "warped." Thus, we have an example of the second kind of stainless p-brane in the $D = 9$, $\Delta = 2$ solution (23): although it can be oxidized to a higher-dimensional spacetime, this oxidation does not preserve the isotropic character of our p-brane metric ansatz (8).

Supersymmetry

All of the solutions so-far discussed have been purely bosonic; although they are solutions to supersymmetric theories, fermion fields have been set to zero in these backgrounds. As with the simplest flat-space solution, however, such solutions may nonetheless preserve several supersymmetries unbroken. Since we are dealing with supergravity theories, the full supersymmetries of the action are local; what might remain unbroken in a given background is generally only a rigid supersymmetry. Nonetheless, since in gravitational theories one is frequently dealing with solutions that asymptotically tend to flat space, the standard of comparison for the unbroken supersymmetries is not the original full local supersymmetry of the action, but the asymptotic supersymmetry of flat space. Thus, one may speak of "half" of the supersymmetry being preserved by a solution, meaning that the solution leaves unbroken half as many supersymmetries as the usual asymptotic flat spacetime. The preservation of half of the flat-space supersymmetry turns out to be a hallmark of the class of solutions considered here.

To see how some supersymmetry may remain unbroken in a purely bosonic background, consider once more the membrane solution of $D = 11$ supergravity, in which the spin-$^3/_2$ gravitino field ψ_M is set to zero. Under the full local $D = 11$ supersymmetry transformations, but restricted to a vanishing gravitino background, the gravitino transformation is

$$\begin{aligned}\delta\psi_M|_{\psi=0} &= \tilde{D}_M\varepsilon \\ &= \Big(\partial_M + \tfrac{1}{4}\omega_M^{AB}\Gamma_{AB} - \tfrac{1}{288}(\Gamma^{PQRS}{}_M + 8\Gamma^{PQR}\delta^S{}_M)H_{PQRS}\Big)\,\varepsilon\,. \end{aligned} \tag{27}$$

Now make a $3+8$ split of the Γ matrices:

$$\begin{aligned}&\Gamma_A = (\gamma_\mu\otimes\Gamma_9, \mathbb{1}\otimes\Sigma_m), \qquad \mu = 0,1,2, \quad m = 3,\ldots,10\,,\\ &\Gamma_9 = \Sigma_3\Sigma_4\cdots\Sigma_{10}\,, \qquad \Gamma_9^2 = 1\,.\end{aligned} \tag{28}$$

Using this split, one may retain covariance under the unbroken SO(2,1)$\otimes$ SO(8) symmetry of the membrane solution (21). In searching for unbroken supersymmetries, we shall take the SO(2,1)$\otimes$SO(8)-covariant ansatz for the supersymmetry parameter

$$\varepsilon(x^\mu, y^m) = \varepsilon\otimes\eta(r)\,, \tag{29}$$

where ε is a constant spinor under SO(2,1) and $\eta(r)$ is an as-yet undetermined SO(8) spinorial function of the isotropic transverse coordinate $r = \sqrt{y^m y^m}$.

Using now the fact that the membrane solution (21) obeys our elementary-form ansatz (8), (9) with $A = C/3$, $B = -C/6 + \text{const.}$, one finds that one can make a supersymmetry transformation that maintains $\delta\psi_m = 0$ with the parameter ansatz (29) provided $\eta(r)$ satisfies

$$\eta(r) = e^{-C(r)/6}\eta_0 \,, \tag{30}$$

$$(1 - \Gamma_9)\eta = 0 \,, \tag{31}$$

where η_0 is a constant SO(8) spinor, and the condition (31) requires η to be chiral with respect to SO(8). The chirality condition and the functional-dependence condition (30) reduce the number of independent components (ε, η_0) in the supersymmetry parameter (29) to half the number of rigid supersymmetries of $D = 11$ flat space, *i.e.* to 16 real constant spinor components. Thus, we say that the solution (21) preserves half the supersymmetry [7]. Half-supersymmetry preservation also characterizes all the other p-branes that we are considering here.

The preservation of half of the supersymmetry is closely related to another feature of the class of solutions that we have discussed. Although these solutions describe "windows" of infinite extent, and hence have an infinite amount of field energy, their field energy per spatial unit volume remains finite [6]. For example, for the $D = 11$ membrane, one has a finite ADM mass/unit area expressed as an integral over the transverse coordinates,

$$\mathcal{M} = \int d^8y\, \theta_{00} \,, \tag{32}$$

where θ_{00} is a stress-tensor component. From the local $D = 11$ supersymmetry algebra, it follows (subject to certain assumptions about nonsingularity) that the mass/unit area satisfies the Bogomol'ny inequality [7]

$$\kappa^2\mathcal{M} \geq |P| \,, \qquad P = \tfrac{1}{2}\int_{S^7} ({}^*H + \tfrac{1}{2}B \wedge H) \,, \tag{33}$$

where P is the conserved Page charge [8] of the $D = 11$ theory. Preservation of half the supersymmetry implies that the inequality (33) is saturated,

$$\kappa^2\mathcal{M} = |P| \,. \tag{34}$$

The saturation of the Bogomol'ny inequality guarantees the stability of the solution.

Given the preservation of half of the supersymmetry, one can organize the fluctuations about p-brane solutions into multiplets. Especially important among these fluctuations are the Goldstone zero-modes, for which there is no restoring potential, so they behave like massless wave-like excitations superimposed on the flat background p-brane "window." Each broken rigid symmetry gives rise to a Goldstone mode. Thus, for the membrane solution in $D = 11$, one has $11 - 3 = 8$ bosonic Goldstone modes coming from the broken translational symmetries (corresponding to the transverse location of the membrane) and $^{32}/_{2} = 16$ fermionic Goldstone modes, coming from the broken supersymmetries. This set of 8 bosonic and 16 fermionic fields is just what is needed to fill out a multiplet of the unbroken supersymmetries, which may be considered to be an $N = 8, d = 3$ worldvolume supermultiplet. Recall that supersymmetry requires a balance of bosonic and fermionic degrees of freedom, but that fermionic wave equations are of first-order in derivatives while bosonic wave equations are of second order, so that twice as many fermionic fields are needed to describe the zero modes as for the bosonic fields.

The stainless brane scan

To summarize the classification of p-brane solutions to supergravity theories, one may plot just the stainless solutions, as we have discussed. Each of these gives rise to a dimensional-reduction family of descendant p-branes in lower dimensions, following a diagonal trajectory on the $(D, d = p + 1)$ plane. The status of these solutions as possibly *exact* string-theory solutions varies according to the different cases [9]. There is accumulating evidence, however, that the saturation of Bogomol'ny bounds for these solutions gives a strong possibility that such solutions will persist in the full theory, perhaps with some renormalizations.

In the following diagram of the stainless p-branes are included only purely elementary or purely solitonic solutions; dyonic solutions are also known to exist, but are not shown. To simplify the diagram, the various dual formulations of supergravity have also been factored out, with theories being considered in their forms with antisymmetric tensor field strengths satisfying $n \leq D/2$. Next to each stainless solution is indicated its Δ value. The p-brane solutions discussed here preserve $^1/_2$ of the supersymmetry of the smallest supergravity theory in which the given solution can exist, or an amount $(^1/_2, ^1/_4, ^1/_8)$ of the maximal possible supersymmetry (corresponding to dimensional reductions of $D = 11$ supersymmetry) for the cases $\Delta = (4, 2, ^4/_3)$. The $D = 10$ 7-brane was recently obtained in [10]. Additional recent solutions preserving lower amounts of supersymmetry with new values $\Delta = (1, ^4/_5, ^2/_3,)$ involving generalizations of our ansätze

(8)–(11) have also been found in [11, 12]. Another class of recently-found solutions for supergravity theories with dilaton potentials, but without $n \geq 1$ antisymmetric tensor field strengths, has been discussed in [13–15]; the relation of these solutions (not shown in the diagram) to the p-branes and dimensional-reduction classification discussed here remains to be clarified. Clearly, an interesting kind of p-brane "chemistry" seems to be emerging.

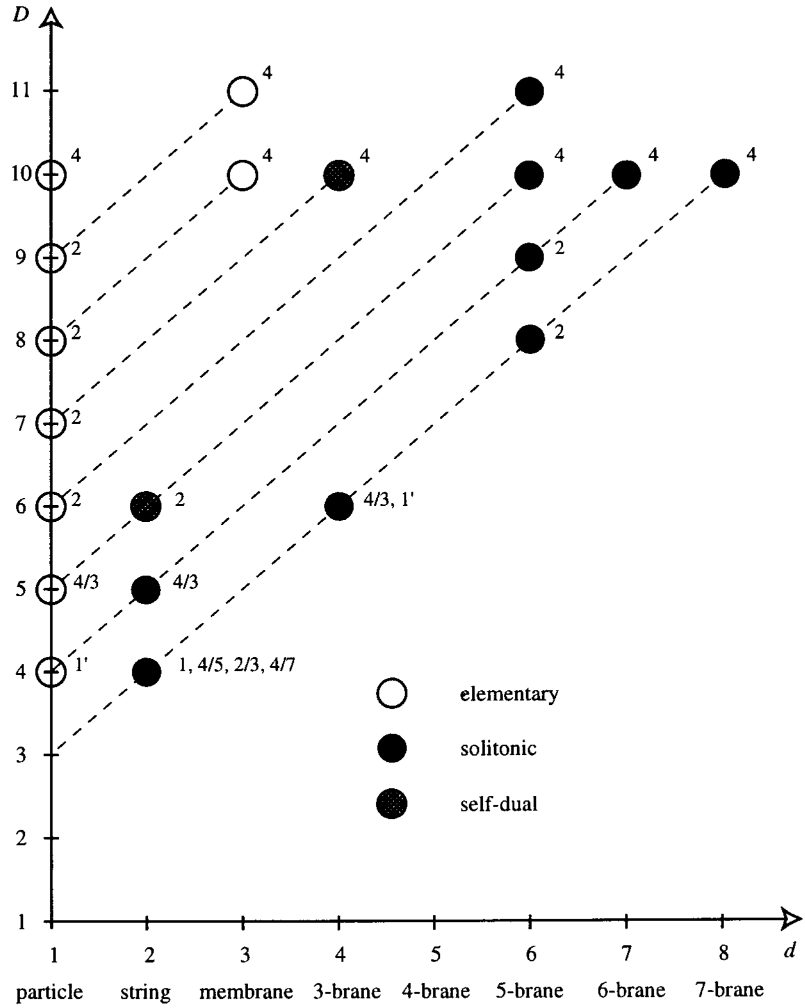

Fig. 1. The stainless supersymmetric p-branes

REFERENCES

[1] H. Lü, C.N. Pope, E. Sezgin, K.S. Stelle, Stainless Super p-branes, preprint hep-th/9508042, *Nucl. Phys.* **B** (in press).
[2] T. Banks, D. Nemeschansky, A. Sen, *Nucl. Phys.* **B277**, 67 (1986).
[3] C.G. Callan, D. Friedan, E.J. Martinec, M.J. Perry, *Nucl. Phys.* **B262**, 593 (1985).
[4] M.J. Duff, P.S. Howe, T. Inami, K.S. Stelle, *Phys. Lett.* **B191**, 70 (1987).
[5] A. Achucarro, J. Evans, P.K. Townsend, D. Wiltshire, *Phys. Lett.* **B198**, 441 (1987).
[6] A. Dabholkar, G.W. Gibbons, J.A. Harvey, F. Ruiz-Ruiz, *Nucl. Phys.* **B340**, 33 (1990).
[7] M.J. Duff, K.S. Stelle, *Phys. Lett.* **B253**, 113 (1991).
[8] D.N. Page, *Phys. Rev.* **D28**, 2976 (1983).
[9] A.A. Tseytlin, *Class. Quantum Grav.* **12**, 2365 (1995).
[10] G.W. Gibbons, M.B. Green, M.J. Perry, Instantons and Seven-Branes in Type IIB Superstring Theory, preprint hep-th/9511080.
[11] M.J. Duff, H. Lü, C.N. Pope, E. Sezgin, Supermembranes with Fewer Supersymmetries, preprint hep-th/9511162.
[12] Supergravities, preprint hep-th/9512012.
[13] J. Polchinskski, E. Witten, Evidence for Heterotic–Type I String Duality, preprint hep-th/9510169.
[14] E. Bergshoeff, M.B. Green, G. Papadopoulos, P.K. Townsend, The IIA supereightbrane, preprint hep-th/9511079.
[15] H. Lü, C.N. Pope, E. Sezgin, K.S. Stelle, Dilatonic p-brane Solitons, preprint hep-th/9511203.