STUDIES ON THE RESCATTERING OF MESONS IN NUCLEAR MATTER*

CH. HARTNACK, CH. DAVID AND J. AICHELIN

SUBATECH
University of Nantes, CNRS, Ecole des Mines de Nantes
4 rue A. Kastler, F-44070 Nantes, France

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The effects of rescattering for pions and kaons in the nuclear medium were analysed. The strong influence of rescattering was found for both regarded mesons.

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1. Introduction

A major goal of studying heavy ion reactions is the unique opportunity to create hot and dense nuclear matter in the laboratory [1]. Unfortunately, this novel state of nuclear matter exists only for a very short time and expands afterwards. In order to gain information about nuclear matter under these extreme conditions, one must find probes which are most sensitive to the properties of this dense matter at the time of maximum density. The key mechanism for creating dense matter is the compression of nuclei due to inertial confinement [2]. Early hydrodynamical calculations [3] succeeded in describing the dynamics of the collisions of heavy nuclei. Mainly, there are three predictions of the hydrodynamical model which have now been confirmed by experiment [4–6] namely nuclear stopping of heavy systems at central collisions, transverse flow (in the reaction plane) — the so-called bounce-off effect, and the emission of high energetic particles perpendicular to the reaction plane — the so-called squeeze out.

The production of secondary particles has recently gained much attention since they are expected to yield direct information on the hot high density region. Pions had been proposed as direct messengers from the high density region [2, 8, 9] since they are produced during the time of maximum

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compression. However, pions have a large cross section for reabsorption by a nucleon forming a delta. This delta may decay, reemitting another pion: Thus, most of the observed pions have interacted several times with rather 'cold' nuclear matter and the signals from the high density region may have been washed out [10].

Kaons are assumed to conserve the density signature much better since they undergo much less interactions with the nuclear matter than pions. Especially the absorption of kaons is strongly suppressed due to strangeness conservation.

2. Quantum molecular dynamics

The QMD model [11] is a n body theory which simulates heavy ion reactions at intermediate energies on a event by event basis. It has been successfully used for the description of fragmentation processes The major aspects of the formulation of QMD shall now be discussed briefly. For a more detailed description we refer to Ref. [13].

In QMD each nucleon is represented by a coherent state of the form (we set $\hbar, c = 1$)

$$\phi_\alpha(x_1, t) = \left(\frac{2}{L\pi}\right)^{3/4} \mathrm{e}^{-(x_1 - x_\alpha(t) - p_\alpha(t)t/m)^2/L} \, \mathrm{e}^{i(x_1 - x_\alpha(t))p_\alpha(t)} \, \mathrm{e}^{-ip_\alpha^2(t)t/2m} .$$

Thus the wave function has two time dependent parameters x_α, p_α. The parameter L which is related to the extension of the wave packet in phase space, is fixed. The total n body wave function is assumed to be the direct product of coherent states

$$\phi = \phi_\alpha(x_1, x_\alpha, p_\alpha, t)\phi_\beta(x_2, x_\beta, p_\beta, t) \cdots .$$

Note that we do not use a Slater determinant (with $(A_p + A_t)!$ summation terms) and thus neglect antisymmetrization. First successful attempts with antisymmetrized states have been performed for smaller systems [14, 15]. A consistent derivation of the equations of motion for the testparticle under the influence of both, the real and the imaginary part of the G-matrix has not been achieved yet within this approach. Therefore we will add the imaginary part as a cross section.

The initial values of the parameters are chosen in a way that the ensemble of $A_T + A_P$ nucleons gives a proper density distribution as well as a proper momentum distribution of the projectile and target nuclei. The time evolution of the system is calculated by means of a generalized variational principle : we start out from the action

$$S = \int_{t_1}^{t_2} \mathcal{L}[\phi, \phi^*] dt$$

with the Lagrange functional $\mathcal{L}$

$$\mathcal{L} = \left(\phi \left| i\hbar \frac{d}{dt} - H \right| \phi \right) ,$$

where the total time derivative includes the derivation with respect to the parameters. The time evolution of the parameters is obtained by the requirement that the action is stationary under the allowed variation of the wave function.

If the true solution of the Schrödinger equation is contained in the restricted set of wave functions $\phi_\alpha(x_1, x_\alpha, p_\alpha)$ this variation of the action gives the exact solution of the Schrödinger equation. If the parameter space is too restricted we obtain that wave function in the restricted parameter space which comes closest to the solution of the Schrödinger equation.

For the coherent states and an Hamiltonian of the form $H = \sum_i T_i + \frac{1}{2} \sum_{ij} V_{ij}$ (T_i= kinetic energy, V_{ij} = potential energy) the Lagrangian and the variation can be easily calculated and we obtain:

$$\begin{aligned} \mathcal{L} &= \sum_\alpha \left[\dot{x}_\alpha p_\alpha - \frac{1}{2} \sum_\beta \langle V_{\alpha\beta} \rangle - \frac{3}{2Lm} \right] , \\ \dot{\bar{x}}_\alpha &= \frac{p_\alpha}{m} + \nabla_{p_\alpha} \sum_\beta \langle V_{\alpha\beta} \rangle , \\ \dot{p}_\alpha &= -\nabla_{\bar{x}_\alpha} \sum_\beta \langle V_{\alpha\beta} \rangle , \end{aligned}$$

with

$$\bar{x}_\alpha = x_\alpha + \frac{p_\alpha}{m} t \text{ and } \langle V_{\alpha\beta} \rangle = \int d^3x_1 d^3x_2 \langle \phi_\alpha \phi_\beta | V(x_1, x_2) | \phi_\alpha \phi_\beta \rangle .$$

These are the time evolution equations which are solved numerically. The interaction V_{ij} can be taken as the real part of the Brückner G-matrix supplemented by the Coulomb interaction. The equations of motion now show a similar structure like classical Hamiltonian equations.

$$\dot{p}_i = -\frac{\partial \langle H \rangle}{\partial q_i} , \qquad \dot{q}_i = \frac{\partial \langle H \rangle}{\partial p_i} . \tag{1}$$

The numerical solution can be treated in a similar way as it is done in classical molecular dynamics [17–20].

The Wigner distribution function f_i of the nucleons can be easily derived from the test wave functions (note that antisymmetrization is neglected).

$$f_i(\vec{r},\vec{p},t) = \frac{1}{\pi^2\hbar^2} e^{-(\vec{r}-\vec{r}_{i0}(t))^2 \frac{2}{L}} e^{-(\vec{p}-\vec{p}_{i0}(t))^2 \frac{L}{2\hbar^2}} . \tag{2}$$

The expectation value of the Hamiltonian can be written down as

$$\langle H\rangle = \langle T\rangle + \langle V\rangle = \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int f_i(\vec{r},\vec{p},t)\, V^{ij} f_j(\vec{r}\,',\vec{p}\,',t)\, d\vec{r}\, d\vec{r}\,' d\vec{p}\, d\vec{p}\,' . \tag{3}$$

The baryon-potential $V^{ij} = G^{ij} + V^{ij}_{\text{Yuk}} + V^{ij}_{\text{Coul}} + V^{ij}_{\text{mdi}}$ consists of

$$G^{ij} = t_1\delta(\vec{r}_i - \vec{r}_j) + t_2\delta(\vec{r}_i - \vec{r}_j)\rho^{\gamma-1}(\vec{r}_{i0}) , \tag{4}$$

$$V^{ij}_{\text{Yuk}} = t_3 \frac{\exp\{|\vec{r}_i - \vec{r}_j|/\mu\}}{|\vec{r}_i - \vec{r}_j|/\mu} , \tag{5}$$

$$V^{ij}_{\text{Coul}} = \frac{q_i q_j e^2}{|\vec{r}_i - \vec{r}_j|} \tag{6}$$

$$V^{ij}_{\text{mdi}} = t_4 \ln^2(1 + t_5(\vec{p}_i - \vec{p}_j)^2)\delta(\vec{r}_i - \vec{r}_j) . \tag{7}$$

The real part of the Brückner G-matrix is density dependent, which is reflected in the expression for G^{ij}.

The parameters μ and $t_1 \ldots t_5$ are adjusted to fit the real part of the G-matrix and to describe the properties of finite nuclei.

For the density dependence standard Skyrme type parametrizations are used. Two different equations of state are commonly used: A hard equation of state (H) with a compressibility of about $\kappa = 380$ MeV and a soft equation of state (S) with a compressibility of $\kappa = 200$ MeV [21, 22]. A fit of the momentum dependence to measurements [23, 24] of the real part of the nucleon optical potential [25, 12] yields:

$$\delta \cdot \ln^2 \left(\varepsilon \cdot (\Delta\vec{p})^2 + 1\right) \cdot \left(\frac{\rho}{\rho_0}\right) .$$

It should be noted that these measurements have been superseded by new data [26] and thus new parametrisation have been made [27].

As stated above the imaginary part of the G-matrix acts like a collision term. In our simulation we restrict us to binary collisions (two-body level) only and may therefore apply the Boltzmann collision ansatz. The collisions are performed in a point-particle sense in a similar way as in the cascade models [7, 8]:

Two particles collide if their minimum distance d in their CM frame fulfills the requirement:

$$d \le d_0 = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}}, \qquad \sigma_{\text{tot}} = \sigma(\sqrt{s},\ \text{type}).$$

where the cross section is assumed to be the free cross section of the regarded collision type ($N - N$, $N - \Delta$, ...).

A reduction of the effective cross section is obtained by the "Pauli-blocking". For each collision the phase space densities in the final states are checked in order to assure that the final distribution in phase space is in agreement with Pauli's principle. Since the phase space in QMD is not discretized into elementary cells and for getting smoother distribution functions the following procedure is performed. The phase space density f_i' at the final state is measured and interpreted as a blocking probability. Thus, the collision is only allowed with a probability of $(1 - f_1')(1 - f_2')$.

3. Rescattering of pions

Pions are secondary particles which interact very strongly with the nuclear medium. Therefore pions might be used to investigate the nucleon-delta and nucleon-pion interactions in the nuclear medium. Pions are produced (for the regarded energy domain of about 1 AGeV incident energy) by the reaction

$$NN \to N\Delta\,, \qquad \Delta \to N\pi\,.$$

Also the reverse channels occur in the reaction. A pion which is produced may be reabsorbed by another nucleon forming a delta. This delta may be absorbed in an inelastic collision

$$\pi N \to \Delta\,, \qquad \Delta N \to NN$$

which may be quoted as (true) pion absorption or it may decay again producing another pion.

$$\pi N \to \Delta \to \pi N$$

This effect may be called rescattering of a pion.

In order to study the importance of the effect of rescattering we selected the pions produced in a collision Au(1 AGeV)+Au according to the number of delta generations, *i.e.* the number of times the pion has been in a delta. The number of rescatterings is therefore just $N_\Delta - 1$. We see from Fig. 1 that only a smaller fraction of pions do not undergo any rescattering while most of the pions rescatter a least one time. It is also found that some pions rescatter quite often, up to about 10 times. Is there some special property

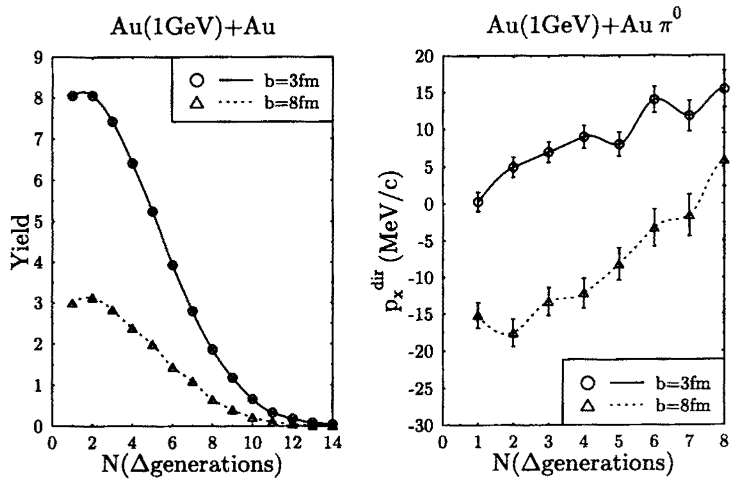

Fig. 1. Left: Yield of the number of delta generations. Right: $p_x^{\rm dir}$of pions as a function of the number of delta generations.

of pions which rescatter quite often? One possibility which was proposed in [28] is to regard the directed flow $p_x^{\rm dir}$which is defined by

$$p_x^{\rm dir} = \frac{1}{N_\pi} \sum_{i=1}^{N_\pi} p_x(i) {\rm sign}(Y(i) - Y(CM)).$$

It should be noted that nucleons at this energy show $p_x^{\rm dir} > 0$, which is called a 'positive flow'. Thę r.h.s of Fig. 1 shows that pions with different N_Δ show a different directed flow. Pions without rescattering show no or negative flow. Furthermore it can be seen that in general pions at large impact parameters show rather negative flow while pions in central collisions show rather positive flow.

What could be the cause of the pionic flow? One possibility may be the absorption and rescattering of pions in the spectator matter. This would cause (due to shadowing effects of the projectile and target remnants) a negative flow which should be strongest at large impact parameters. A further effect could be the flow of the sources, *i.e.* a collective motion of the matter producing the deltas in inelastic collisions. This effect which would cause a positive flow (due to the positive collective flow of the nucleons) would require a rather late production time since the nucleonic flow has to be build up before the deltas are produced. A further effect could be a potential interaction on the pions (pion optical potential) and on the deltas (which would be stronger for longer lifetimes of the delta and for pions with more rescattering).

Fig. 2 shows on the lhs. the transverse momentum transfer of pions as a function of the impact parameter for the Au (1 AGeV) + Au using a hard eos with momentum dependent interactions (potentials on the propagation of the deltas have been suppressed in this case in order to get a clearer signal on the forces on the pions). We find positive values for central collisions and negative values for peripheral collisions. This can be interpreted as an effect of rescattering (for the negative flow at large impact parameters) and of the potential (for the positive flow at small impact parameters). Furthermore we find some differences between positive and negative pions. It should be noted that the pions are propagated under the influence of Coulomb interactions. The positive pions are repelled by the positive charged baryons and therefore get a stronger negative flow than the negative pions which are attracted by the baryons. The inclusion of further interactions stemming from the real part of the pion-optical potential may therefore have strong additional influences on the results.

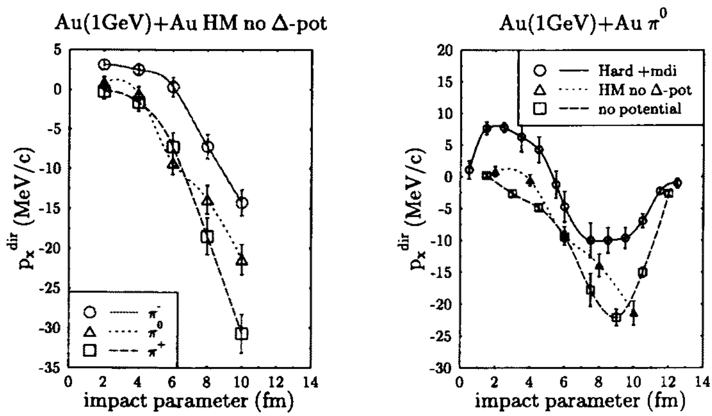

Fig. 2. Left: $p_x^{\mathrm{dir}}$of π^- π^0 and π^+ as a function of impact parameter for the system Au+Au at 1AGeV incident energy using a hard eos with mdi (and no delta-potentials). Right: $p_x^{\mathrm{dir}}$of π^0 for a hard eos with mdi, a calculation without nuclear forces on deltas, and a calculation without any potential

Since the deltas are baryons and feel nuclear potentials, an influence of the eos on the pion flow might be visible. The rhs of Fig. 2 therefore compares results obtained with a hard eos including mdi with a calculation without potentials. There are visible influences from the nuclear eos, *e.g.* the latter calculation never reaches positive pion flow. In the present simulations the nuclear eos is assumed to be valid for all kinds of baryons. However, there is no evidence that this should be the case. In order to estimate the effects we calculated the reaction assuming that there are no potential interactions between deltas and other baryons. The comparison of this simulation with the calculation with full potentials therefore gives some indication on the effects of nucleon-delta potentials, while the comparison with the cascade (which differ in the potentials on the other baryons) gives an indication on the flow of the sources (since the nucleonic flow differs between a calculation with and without potentials). The results from a calculation without delta-nucleon interactions (but nucleon-nucleon potentials) are quite similar to that of the calculation without potential. Therefore we can conclude that if a delta-nucleon interaction would be postulated, this would influence the pion flow.

It should be noted that similar considerations have been done concerning azimuthal anisotropies of pions [29] and that first experimental evidence of anisotropies have been reported [30, 31]

3.1. Flow in asymmetric systems

Let us come back to the point, where the question of pionic flow was addressed for the first time, namely in the asymmetric reaction Ne+Pb measured by the Diogène collaboration [32, 10]. It should be mentioned that here the term "flow" was used in a different meaning: while in symmetric systems the directed flow $p_x^{\rm dir}$ is used, the Diogène analysis regards the total average of p_X/m over the rapidity. Fig. 3 shows the dependence of

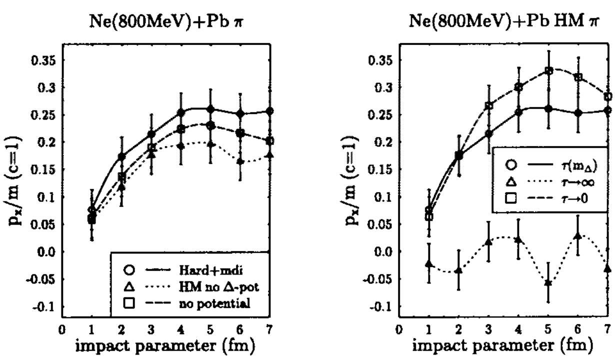

Fig. 3. Dependence of the average p_X/m as a function of the impact parameter, left for a hard eos with mdi, a calculation without delta-baryon interactions and a calculation without potentials, right hard +mdi, with a mass-dependent delta decay width and decay lifetimes going to zero and infinity.

this average on the impact parameter. We see differences between a calculation with a hard eos including mdi and a calculation without potentials. A calculation without delta-baryon interactions tends towards the results without potentials. The rhs shows the influence of the delta lifetime on p_X/m. We see stronger effects than that of the eos if we decrease the delta lifetime from a massdependent value to zero. If we increase the lifetime to infinity the "flow" p_X/m of the pions vanishes. Therefore we assume that modifications in the absorption/decay channels may also play an important role for understanding the pion kinematics, so that it is unclear whether the pion flow might be used to get information about nuclear-delta potentials.

4. Rescattering effects of kaons

Let us now consider the production of kaons. Subthreshold kaons are predominantly produced by the reaction

$$N/\Delta + N/\Delta \to NYK\,,$$

where for lower energies the $N\Delta$ contribution is the most dominant [33–36]. Kaons are found to be produced at rather high densities during the early time of the reaction. Therefore the study of kaon observables may be of high interest. Influences of effects of scalar and vector kaon nucleon potentials on the nucleon dynamics have recently reported [37]. However, effects of the kaon rescattering have rarely been studied in a realistic way in this energy region (1–2 AGeV). At threshold, the kaon production is normally treated only perturbatively by counting all high energy collisions and folding them with the probabilities of the production [12]. Rescattering can only be regarded by mean free path considerations. Calculations with realistic propagation suffer from a lack of statistics due to the low production probability of kaons.

We use in our calculations realistic propagation (in analogy to the pion propagation in QMD) with enhanced kaon production channels. This method allows to study the full dynamics of kaons with a reasonable statistic. We implemented the measured K–N cross sections for the elastic channels and the charge exchange channel with angular distribution fitted to the experimental data by use of a neural network. We present first calculations for the system Ni(1.93)+Ni which is measured by the FOPI collaboration.

We see in Fig. 4 a distribution of the number of rescatterings of kaons in semicentral events ($b \leq 4fm$) of Ni+Ni. A large number of kaons undergoes rescattering. This large fraction of rescattered kaons changes the observables strongly. As a demonstration we show a comparison of transverse spectra without rescattering (corresponding to the results of purely perturbative treatment) and with inclusion of collisions. We see that the

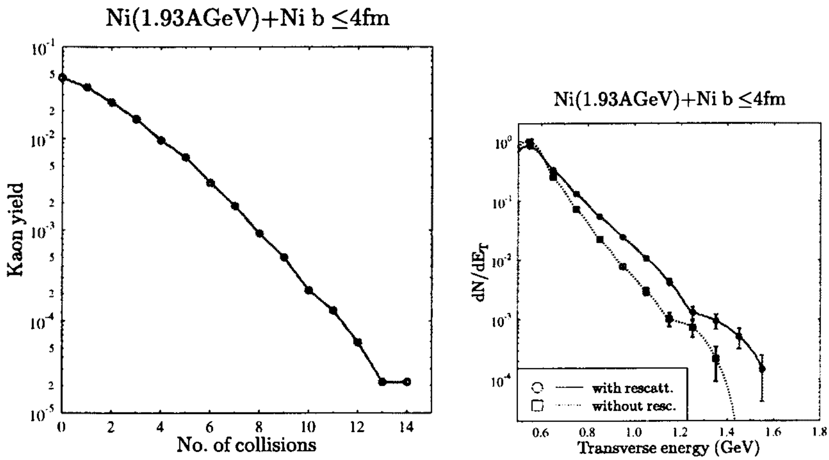

Fig. 4. Left: Distribution on the rescattering of kaons Right: Transverse energy spectra for kaons with and without rescattering

collisions broaden the energy distribution and higher transverse energies are reached.

Similar effects can also be found in the longitudinal direction. Fig. 5 presents rapidity distributions for K^+ and lambdas again without and with rescattering. Again the rescattering yields a broadening of the distributions. The kaons and lambdas can now be found at larger rapidities. This is for instance important for comparison to the present data of the FOPI collaboration, who measure the kaons especially at backward rapidities. Here strong effects of calculations with and without K-N collisions can be noted. The QMD results with collisions seem to be in a good agreement with recent FOPI-data [38].

First analysis has also be done on the flow of the kaons. Here nearly no flow is found for the kaons neither without nor with collisions. However it should be noted, that the present analysis has been done with a rather small system. It can be assumed that for heavier systems the effects of rescattering will still be stronger. A detailed investigation of large systems like Au + Au concerning rapidity distributions, energy spectra, flow and azimuthal anisotropies will be done in near future.

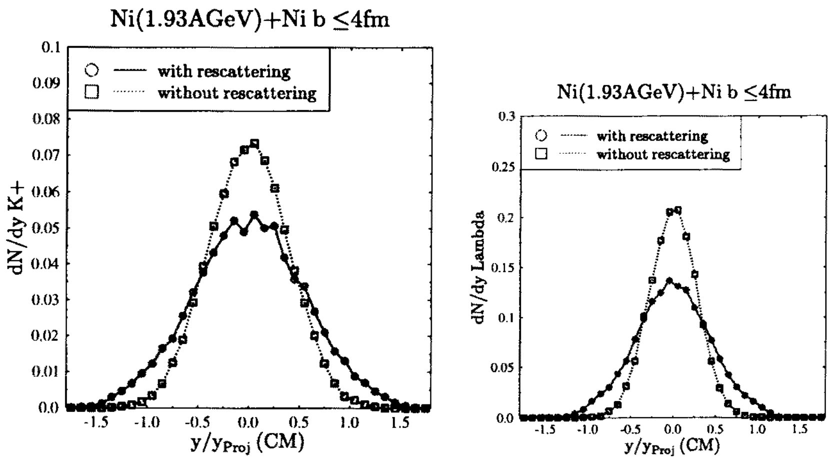

Fig. 5. Rapidity distribution for kaons and lambdas with and without rescattering

5. Conclusion

We have analyzed the effects of rescattering for pions and kaons. We find strong influences on the dynamics of the mesons. Pionic flow reflects to a large extend the interaction with the nuclear matter by potential as well as by collisions. Kaons also show strong effects of rescattering. We see changes in the transverse energy spectra as well as in the rapidity distributions. These changes are quite important for the description of particle multiplicities in present experiments with their constraints in acceptance.

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