PIONS IN THE NUCLEAR MEDIUM*

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We discuss various aspects of pion physics in the nuclear medium. We first study s-wave pion-nucleus interaction in connection with chiral symmetry restoration and quark condensate in the nuclear medium. We then adress the question of p-wave pion-nucleus interaction and collective pionic modes in nuclei and draw the consequences for in medium $\pi\pi$ correlations especially in the scalar-isoscalar channel. We finally discuss the modification of the rho meson mass spectrum at finite density and/or temperature in connection with relativistic heavy ion collisions.

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1. Introduction

The physics of pions in the nuclear medium has already a long story. This domain contains two complementary aspects. The first one concerns the use of pions as a probe of the nucleus and the second one is devoted to the key role of the pion in the nuclear many-body problem. Most of the considerable knowledge which has been accumulated can be found in [1]. In this paper we will show that all these results are of prime importance for the understanding of new problematics of present day hadronuclear physics. In that respect we will discuss in Section 2 the connection between chiral symmetry restoration and the s-wave pion-nucleus interaction. Section 3 is devoted to the physics of collective pion-delta modes, sometimes called pionic branch or pisobars. We discuss the consequences of these collective phenomena on two-pion propagation and correlations in nuclear matter (Section 4). Finally we examine in Section 5 the role of interacting $\pi N \Delta$ configurations in highly excited matter produced in relativistic heavy ion collisions; special emphasis will be put on the rho meson whose mass spectrum may reveal significant modification of the vacuum structure at finite density and/or temperature.

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2. Chiral symmetry restoration and s-wave pion-nucleus interaction

Asymptotic freedom and color confinement are usually considered as the most prominent properties of our theory of strong interaction, Quantum Chromodynamics (QCD). However QCD also possesses an almost exact symmetry, the $SU(2)_R \otimes SU(2)_L$ chiral symmetry which is certainly the most important key for the understanding of many phenomena in low energy hadron physics. This symmetry originates from the fact that the QCD Lagrangian is almost invariant under the separate flavor $SU(2)$ transformations of right handed $q_R = (u_R, d_R)$ and left handed $q_L = (u_L, d_L)$ light quark fields u and d

$$q_R \to e^{i\vec{\tau} \cdot \vec{\alpha}_R/2} q_R \qquad q_L \to e^{i\vec{\tau} \cdot \vec{\alpha}_L/2} q_L \,. \qquad (1)$$

The small explicit violation of chiral symmetry is given by the mass term of the QCD Lagrangian which is, neglecting isospin violation,

$$\mathcal{L}_{\chi SB} = -m_q \left(\bar{u}u + \bar{d}d \right), \qquad (2)$$

where the averaged light quark mass $m_q = (m_u + m_d)/2 \leq 10$ MeV, the scale of explicit chiral symmetry breaking, has to be compared with typical hadron masses of order 1 GeV, indicating that the symmetry is excellent. The normal Wigner realization would imply a doubling of the flavor $SU(2)$ symmetry built on left-handed and right-handed quarks. Each hadron should have a chiral partner with opposite parity and about the same mass. This is obviously not the case since, for instance, the first hypothetical chiral partner of the nucleon is the S_{11} resonance whose mass is larger than 1.5 GeV. However a symmetry present at the level of the Lagrangian is not necessarily realized in the spectrum. This is precisely what happens for chiral symmetry which is realized in the Goldstone manner or, equivalently stated, is spontaneously broken. One order parameter of this breaking in the QCD vacuum is provided by the quark condensate

$$< \bar{q}q >_{\text{vac}} = \tfrac{1}{2} < \bar{u}_R u_L + \bar{u}_L u_R + \bar{d}_R d_L + \bar{d}_L d_R > \simeq -(220\,\text{MeV})^3 \qquad (3)$$

which is an observable which mixes left-handed and right-handed quarks. The associated Goldstone boson is the pion whose small mass comes from the small explicit chiral symmetry breaking. There is a very important relation relating the explicit and spontaneous breaking at the quark scale and at the hadronic scale namely the Gell-Mann-Oakes-Renner relation [2] valid to lowest order in the explicit symmetry breaking parameters (m_q, m_π^2)

$$-2m_q < \bar{q}q >_{\text{vac}} = m_\pi^2 f_\pi^2 \,, \tag{4}$$

where the pion decay constant f_π plays the role of the order parameter for the spontaneous breaking at the hadronic level. The dynamical origin of this spontaneous breaking, highly non pertubative in nature, is not yet fully understood. However, we have very good reasons to believe (based on lattice simulation or effective hadron theories) that the quark condensate decreases, indicating partial restoration of the symmetry, with increasing baryonic density and/or temperature. Hence, before the full restoration at some critical density ρ_c and critical temperature $T_c \simeq 150$–200 MeV we expect that the mesons, which are the first excitations of the QCD vacuum, will be appreciably modified.

Already in nuclear matter at normal density ρ_0 there is a sizeable restoration of chiral symmetry whose origin can be understood in a very simple picture. A nucleon embedded in nuclear matter can be seen as a bubble in the QCD vacuum in which chiral symmetry is restored. Therefore on the average, the quark condensate will decrease. This qualitative statement can be made quantitative with the first order result [3], derived with help of the GOR relation

$$R_{\text{first order}} = \frac{< \bar{q}q(\rho) >}{< \bar{q}q >_{\text{vac}}} = 1 - \frac{\rho \Sigma_N}{f_\pi^2 m_\pi^2} \,. \tag{5}$$

Hence, to leading order in density the amount of chiral symmetry restoration, given by the ratio R, is governed by the pion-nucleon sigma commutator $\Sigma_N = < N|[Q_i^5,[Q_i^5,H]]|N > \simeq 45$ MeV, which is a measure of the explicit symmetry breaking on the nucleon. Putting the numbers together we find a 30% restoration at normal nuclear matter density or in the interior of heavy nuclei. This result, valid for a non-interacting medium, can be promoted to an exact result [4] provided the πN sigma commutator is replaced by the full π-Nucleus sigma commutator (per nucleon) $\tilde{\Sigma}_N(\rho)$

$$R = \frac{< \bar{q}q(\rho) >}{< \bar{q}q >_{\text{vac}}} = 1 - \frac{\rho \tilde{\Sigma}_N(\rho)}{f_\pi^2 m_\pi^2} \tag{6}$$

$$\tilde{\Sigma}_N(\rho) = \frac{1}{A} < A|[Q_i^5,[Q_i^5,H]]|A > \,. \tag{7}$$

An alternative form of the exact result (6) can be obtained by introducing the interpolating PCAC pion field related to the divergence of the axial current according to $\varphi_i = -\partial_\mu \mathcal{A}_i^\mu / f_\pi m_\pi^2$. There is a venerable low energy theorem [5] which states that the double sigma commutator on any target is proportionnal to the total isoscalar scattering amplitude between the target

and PCAC soft pion *i.e.* a pion with a vanishing energy momentum. As a consequence, it can be shown that Eq. (6) takes the equivalent form [6]

$$R = \frac{m_\pi^2}{m_\pi^2 + \Pi(0)} = 1 - \frac{\Pi(0)}{m_\pi^2 + \Pi(0)}, \qquad (8)$$

where $\Pi(0)$ is the full PCAC pion self-energy at four momentum $q = 0$ and corresponds physically to the soft pion-nucleus optical potential. The advantage of this formulation, which is at least to me more intuitive, is that all the mechanisms governing chiral symmetry restoration can be described in terms of contributions to the full soft pion-nucleus amplitude per unit volume $T = \Pi(0)/[1 + \Pi(0)/m_\pi^2]$. Keeping only in T the Born term $\Pi(0)$ limited to first order in density, namely $T^0_{Born} = \Pi^0(0) = \rho \Sigma_N / f_\pi^2$, we recover back the first order result (5). However, the full T matrix deviates from the optical potential $\Pi(0)$ by coherent rescattering (i.e the denominator in T). The fact that the first order optical potential Π^0 is repulsive may lead to the conclusion that coherent rescattering hinders chiral symmetry restoration. In other words, as proposed by Ericson [7], there is a possible reaction of the medium against chiral symmetry restoration.

In two recent papers [4, 8] we have studied how much this conclusion might be altered by higher order contributions to the irreducible pion self energy $\Pi(0)$ such as pion exchange effects or the influence of short range correlations combined with higher order exchange effects.

The effect of pion exchange can be obtained quantitatively by the direct evaluation of the ground state expectation value $< H_{\chi SB} >= (1/2) < A| \int dr \, m_\pi^2 \vec{\varphi} \cdot \vec{\varphi}(r)|A >$ on a correlated ground state wave function or, in the PCAC picture, by calculating the scattering amplitudes, such as the one depicted in fig 1a, within the standard non linear chiral Lagrangian. We found [4] a slight acceleration of chiral symmetry restoration given by $\delta R \simeq -0.04 \, (\rho/\rho_0)^2$, a 4% effect at normal nuclear matter density, closely linked to the pion excess.

The incoherent rescattering represents the rescattering of the soft pion in presence of short range correlations represented by a double wavy line on Fig. 1b. The intermediate pion of Fig. 1b acquires a momentum $q_c \simeq 1/R_c \simeq m_\omega \simeq 700$ MeV where R_c is the size of the pair correlation hole. Consequently, the corresponding contribution to the pion self energy possesses a piece going as q_c^2 in manifest conflict with chiral symmetry. We have shown [8], using a linear sigma model (Fig. 1c) or non linear sigma model (Fig. 1d), that higher order corrections exactly cancel this q_c^2 piece, as it should be. The net effect of short range correlations is thus weak. Their effect becomes negligibly small if the scalar-isoscalar radius R_S, much larger than the correlation hole radius R_c, is taken into account.

In the linear sigma model, the remaining effect of higher order contributions such as depicted on Fig. 1c is extremely large. For instance Birse and Mac Govern [9] found a contribution

$$\delta \Pi = \frac{5}{2} m_\pi^2 \left(\frac{\rho \Sigma_N}{f_\pi^2 m_\pi^2} \right)^2 \qquad \rightarrow \qquad \delta R = -\frac{5}{2} \left(\frac{\rho \Sigma_N}{f_\pi^2 m_\pi^2} \right)^2 \simeq -0.25 \left(\frac{\rho}{\rho_0} \right)^2 \tag{9}$$

but such a result is ruled out by standard π-nucleus phenomenology yielding for instance a unrealistically large $(b_0)_{eff}$. To look, in a more realistic way, at the inluence of higher order terms we start from the non linear Lagrangian

$$\begin{aligned} \mathcal{L} = \quad & i\bar\psi\gamma^\mu\partial_\mu\psi - g\bar\psi(\sigma + i\vec\tau \cdot \vec\pi\gamma_5)\psi + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec\pi \cdot \partial^\mu\vec\pi) \\ & + f_\pi^2 m_\pi^2\,\sigma - \frac{\Sigma_N}{f_\pi^2}\,\sigma\,\bar\psi(\sigma + i\vec\tau \cdot \vec\pi\gamma_5)\psi \end{aligned} \tag{10}$$

with $\sigma = (f_\pi^2 - \vec\pi^2)^{1/2}$. We have explicitly included in Eq. (10) a chiral symmetry breaking piece proportionnal to Σ_N to obtain the correct pion-nucleon sigma term at the tree level while maintaining the QCD algebraic identity $H_{\chi SB} = [Q_i^5, [Q_i^5, H]]$. For convenience we introduce a new nucleon field and the PCAC pion field according to

$$N = \left(\frac{\sigma + i\vec\tau \cdot \vec\pi\gamma_5}{f_\pi} \right)^{1/2} \psi \qquad \vec\varphi = \vec\pi \left(1 - \frac{\Sigma_N\,\bar N N}{f_\pi^2 m_\pi^2} \right) . \tag{11}$$

This generates a new form for the chiral symmetry breaking piece [8]

$$\mathcal{L}_{\chi SB} = f_\pi^2 m_\pi^2 \left(1 - \frac{\Sigma_N\,\bar N N}{f_\pi^2 m_\pi^2} \right) - \frac{1}{2} m_\pi^2 \frac{\varphi^2}{1 - \Sigma_N\,\bar N N/f_\pi^2 m_\pi^2} + O(\varphi^4) . \tag{12}$$

From the above equation we can directly read off the result for the soft pion optical potential

$$T_{\text{soft}}^{\text{Born}} = \Pi(0) = \frac{\rho \Sigma_N/f_\pi^2}{1 - \rho \Sigma_N/f_\pi^2 m_\pi^2} . \tag{13}$$

We see that the first order optical potential is renormalized by a set of higher order contact terms such as displayed on Fig. 1d. The distorsion of the soft pion wave through coherent rescattering modifies the Born amplitude in such a way that the free nucleon amplitude is recovered

$$T = \frac{\Pi(0)}{1 + \Pi(0)/m_\pi^2} = \frac{\rho \Sigma_N}{f_\pi^2} . \tag{14}$$
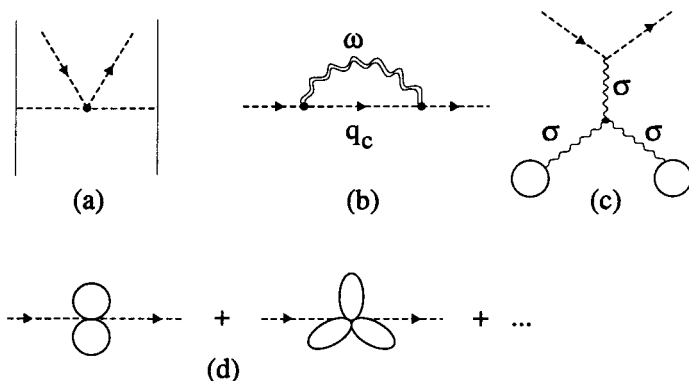
Fig. 1. Examples of medium correction to the pion-nucleus sigma term governing chiral symmetry restoration. The external dashed lines represent soft pions. (1a) pion exchange. (1b) Incoherent rescattering in presence of short range correlations. (1c) Higher order exchange in the linear sigma model. (1d) Higher order contact terms in the non linear sigma model.

One can also notice that this result could have been directly obtained by taking the expectation value of $-\mathcal{L}_{\chi SB}$. Hence coherent rescattering is exactly cancelled by the contact terms, contrary to what happens in the linear sigma model where it is overcompensated.

From the above discussion one can draw the following conclusions :

— The role of short range correlations in chiral symmetry restoration is always very small.

— The reaction of the medium against chiral symmetry restoration, associated to coherent rescattering, seems to be always compensated by higher order exchange contributions

— The main deviation of the nuclear sigma term with respect to the bare nucleon sigma term originates from pion exchange associated to pion excess in nuclei. Nevertheless, some higher order terms in the density may very well be present in the effective chiral Lagrangian governing low energy hadron and nuclear physics. These terms still largely unknown should be determined by experiments and may alter this last conclusion. However, an experimental information on the nuclear sigma term can be obtained through a Fubini-Furlan type analysis whose only input are scattering data. As shown in [4] the deviation of the nuclear sigma term with respect to the bare pion-nucleon sigma term lies in the range 3–9 MeV (depending on the assumptions on a dispersive correction) and is compatible with the calculated pion exchange contribution.

3. Collective pionic modes and p-wave pion-nucleus interaction

The propagation of a high energy pion (say $\omega > m_\pi$) is modified mainly by its p-wave self-interaction in the nuclear medium. The pion propagator has the form :

$$D_\pi(\boldsymbol{k},\omega) = [\omega^2 - \omega_k^2 - S(\boldsymbol{k},\omega)]^{-1}.$$ (15)

The p-wave self-energy is dominated by the virtual Δh excitations and it is extremely important to incorporate the effect of the repulsive short-range correlations through the introduction of the g' parameter. In a more refined description (see below) the coupling of the pion to ph and $2p2h$ excitations should also be included. It is possible to understand the main aspects of pion propagation and pionic collective modes within a very simple two-levels model [10]. Keeping only the coupling to Δh states in the static approximation ($\epsilon_{\Delta k} = \sqrt{k^2 + M_\Delta^2} - M_N$) and neglecting the delta width, the pion self-energy has the form :

$$S(\boldsymbol{k},\omega) = k^2\, \tilde{\Pi}^0(\boldsymbol{k},\omega) = k^2\, \Pi^0(\boldsymbol{k},\omega)\, / \left(1 - g'_{\Delta\Delta}\Pi^0(\boldsymbol{k},\omega)\right).$$ (16)

Here $g'_{\Delta\Delta} \simeq 0.5$ accounts for the short range screening of the polarisation bubble Π^0 which has the usual form :

$$\Pi^0(\boldsymbol{k},\omega) = \frac{4}{9}\left(\frac{f^*_{\pi N\Delta}}{m_\pi}\Gamma(k)\right)^2 \rho \left(\frac{1}{\omega - \epsilon_{\Delta k} + i\eta} - \frac{1}{\omega + \epsilon_{\Delta k}}\right).$$ (17)

It is a simple matter to show that the full pion propagator takes the following form

$$D_\pi(\boldsymbol{k},\omega;\rho) = \frac{Z_1(k,\omega;\rho)}{\omega^2 - \Omega_1^2(k,\omega;\rho) + i\eta} + \frac{Z_2(k,\omega;\rho)}{\omega^2 - \Omega_2^2(k,\omega;\rho) + i\eta}.$$ (18)

Due to its coupling to the Δ it is clear that the π (and the Δ as well) is no longer an eigenstate of the system but becomes a mixture of two $\pi\Delta$ modes with eigenenergy Ω_1, Ω_2 ($\Omega_1 < \Omega_2$) and strength Z_1, Z_2 ($Z_1 + Z_2 = 1$) shown on figure 2. At low momentum the lower branch sometimes called the pionic branch is dominantly pionic whereas the upper branch is mainly made of Δh states; the two branches exchange their structure at momentum transfers of the order of $2 - 3m_\pi$. The very important point for the following discussion (next section) is the smallness of the group velocity $d\Omega_1/dk$ associated to the pionic branch at low momentum transfer and at sufficiently high density. In spite of its simplicity this model gives the position and the strength of these branches close to the results of much more detailed calculation [11].

The lower (pionic) branch is mainly space-like and is in principle accessible by looking at the longitudinal spin-isospin response with a well chosen
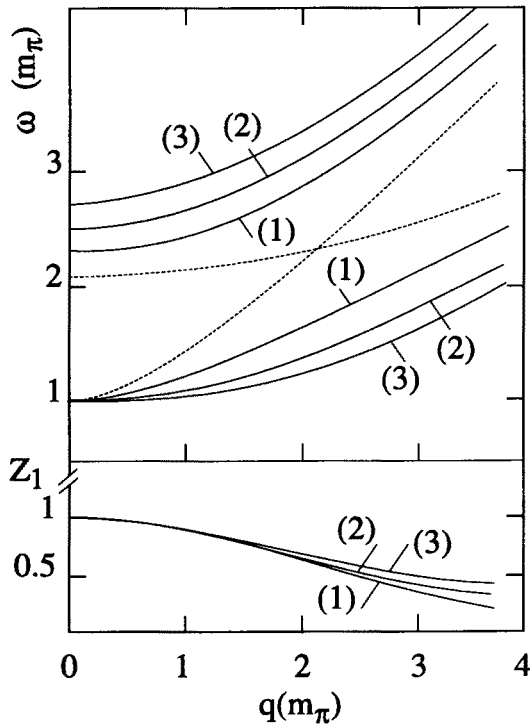
Fig. 2. Dispersion curves for Ω_1, Ω_2 and Z_1 weight factor for various values of ρ/ρ_0. The free pion and delta branches are also shown.
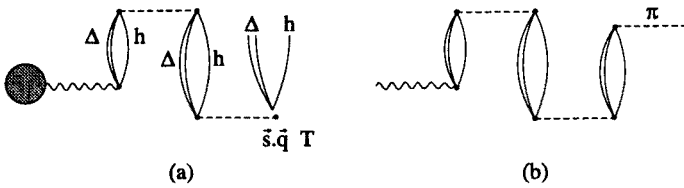

Fig. 3. (3a) Propagation of the longitudinal spin-isospin excitation in the nuclear medium. (3b) Coherent pion.

probe. The propagation of the excitation in the medium through the pion builds up the collective response of the nucleus as shown in Fig. 3a. Consequently, the delta-hole bump will be depopulated in favor of the pionic branch. Hence, a signature of these collective effects should be the down-
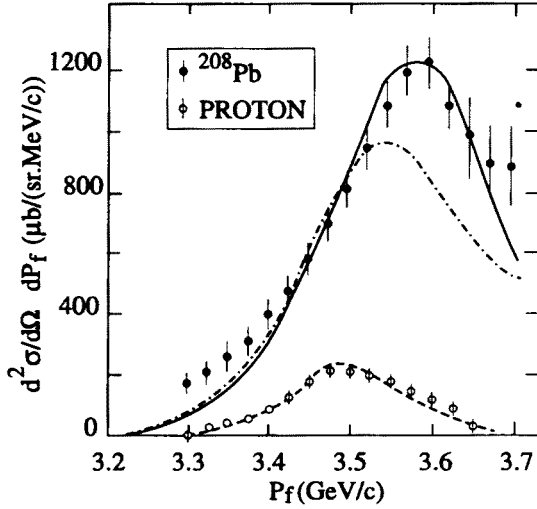
Fig. 4. Comparison of theory to experiment for Δ excitation in ^{208}Pb by the $(^3He, T)$ reaction at 2 GeV and zero angle. The dot-dashed curve is the first order response and the full curve includes collective $\pi\Delta$ effects. The dashed curve corresponds to a calculation on a proton target.

wards shift of the delta peak with respect to its position in the free nucleonic response or in the transverse response measured with real and virtual photons. Despite their peripheral character, we believe that the $(^3He, T)$ experiment performed at SATURNE at 2 GeV incident energy [12] gives a clear indication in favor of the existence of these collective modes. The $(^3He, T)$ data on various nuclei ranging from carbon to lead show a systematic downwards shift of about 70 MeV with respect to the proton data (see Fig. 4). Sophisticated calculations [13] without free parameters (in the sense that all the needed ingredients have been extracted from phenomenology) have shown that the collective effects are absolutely needed to explain the data (Fig. 4); similar conclusions have been reached in [14]. The decay channels of these collective modes have also been studied at SATURNE [15] and are still compatible with the above collective mechanism. The most direct evidence of the real existence of these collective modes is probably the experimental observation of coherent pion production, the target nucleus remaining in its ground state, shown of Fig

3b. A new dedicated experiment is presently performed to study in detail coherent pion production; details on this last topic can be found in the papers of B. Ramstein and L. Farhi in these proceedings.

4. Pion-pion correlation and two-pion resonances in nuclear matter

Since the pion propagation properties are significantly modified in nuclear matter, one can also expect two-pion systems such as two-pion resonances to be also appreciably modified. For this purpose let us consider the two-pion propagator in nuclear matter. For a two-pion system in its center of mass frame with relative momentum k and total energy E, it reads :

$$G_{2\pi}(k, E) = \int \frac{i dk^0}{2\pi} D_\pi(k, k_0) D_\pi(-k, E - k_0) . \qquad (19)$$

In the two level model, its explicit expression is :

$$G_{2\pi}(k, E) = \sum_{i,j=1}^{2} \frac{\Omega_i(k) + \Omega_j(k)}{2\Omega_i(k)\, \Omega_j(k)} \frac{1}{E^2 - (\Omega_i(k) + \Omega_j(k))^2 + i\eta} . \qquad (20)$$

Obviously in the zero density limit one recovers the bare two pion propagator namely $(E^2 - 4\omega_k^2 + i\eta)^{-1}$. To construct the full effective interaction (T matrix) one starts from a certain model for the $\pi\pi$ potential built with bare meson exchanges whose parameters are fitted on phase shifts and scattering lengths. The T matrix (invariant scattering amplitude) is obtained as a solution of the Lippman-Schwinger equation

$$T(q, q'; E) = V(q, q'; E) + \int \frac{dk}{(2\pi)^3} V(q, k; E)\, G_{2\pi}(k, E)\, T(k, q'; E) \quad (21)$$

projected on the various channels such as $I = J = 0$ (σ channel) or $I = J = 1$ (ρ channel). A very important remark can be made : at low energy the two pion strength distribution (i.e. Im T) receives a contribution of the type :

$$\int \frac{dk}{(2\pi)^3} \frac{Z_1(k)}{\Omega_1(k)} \delta(E^2 - 4\,\Omega_1^2(k)) \qquad (22)$$

which gives after phase space integration a factor $[d\Omega_1(k)/dk]^{-1}$. Since the group velocity $d\Omega_1(k)/dk$ may become very small at low momentum with increasing density, an accumulation of strength near the $E = 2m_\pi$ threshold can be produced. This very simple argument can be strictly applied only in the sigma channel since gauge corrections introduce other medium effects in the rho channel; this last point will be discussed in the next section.

The in medium pion-pion T matrix has been studied first in two phenomenological models. In the simplest one, described in [10] the $\pi\pi$ interaction is described by a simple scalar exchange roughly simulating the

coupling to the $K\bar{K}$ channel. The other model is the much more sophisticated Jülich model [16] which explicitly incorporates the coupling to the $K\bar{K}$ channel. Although very different, these two models give very similar results for the $I = J = 0$ channel. Both predict a very sharp structure near two-pion threshold with even the occurence of a two-pion $I = J = 0$ bound state for a density slightly below normal nuclear matter density. It is important to check wether this effect survives when the width of the in medium quasi-pion is incorporated. This can be done by making the substitution

$$\Omega_j(k) \rightarrow \Omega_j(k) + i\frac{\mathrm{Im}\,\tilde{\Pi}^0(\boldsymbol{k}, \Omega_j(k))}{2\,\Omega_j(k)}, \tag{23}$$

where $\mathrm{Im}\,\tilde{\Pi}^0$, calculated along the line j, takes into account the delta width, corrected from Pauli blocking [17], together with extra $2p2h$ contributions not reducible to a delta width piece [18]. The imaginary part of the two-pion propagator is obtained through a spectral representation :

$$\mathrm{Im}\,G_{2\pi} = -\frac{1}{\pi}\sum_0^E d\omega\,\mathrm{Im}D_\pi(\boldsymbol{k}, E)\,\mathrm{Im}D_\pi(\boldsymbol{k}, E - \omega) \tag{24}$$

and the real part is calculated using dispersion relation. The results, with widths included, is depicted on Fig. 5 for the first model. It appears that the sructure near two-pion treshold is still there.

When the coupling of the pion to the *ph* continuum is explicitly taken into account, a strong accumulation of stength below threshold is observed. In both models an instability with respect to pion pair condensation is reached at a density of $1.3\rho_0$ [18]. Obviously such a phenomenum would have dramatic consequences for the nuclear equation of state. It is likely an artefact of the approach which is in conflict with some chiral symmetry constraints; let us now discuss this point. It is possible to relate the $I = J = 0$ $\pi\pi$ scattering amplitude at the soft point, defined through the Mandelstam variables $s = m_\pi^2$, $t = 0$, $u = 0$, to the $\pi\pi$ sigma commutator. The low energy theorem, valid up to m_π^4 correction, reads [18]

$$T(s = m_\pi^2, t = 0, u = 0)$$
$$= \frac{\Sigma_{\pi\pi}}{f_\pi^2} = \frac{1}{f_\pi^2} < \pi^i(\boldsymbol{k} = 0)|[Q^5, [Q^5, H]]|\pi^i(\boldsymbol{k} = 0) > = \frac{m_\pi^2}{f_\pi^2}. \tag{25}$$

This result should be contrasted with the Weinberg result [19] at the physical threshold which gives

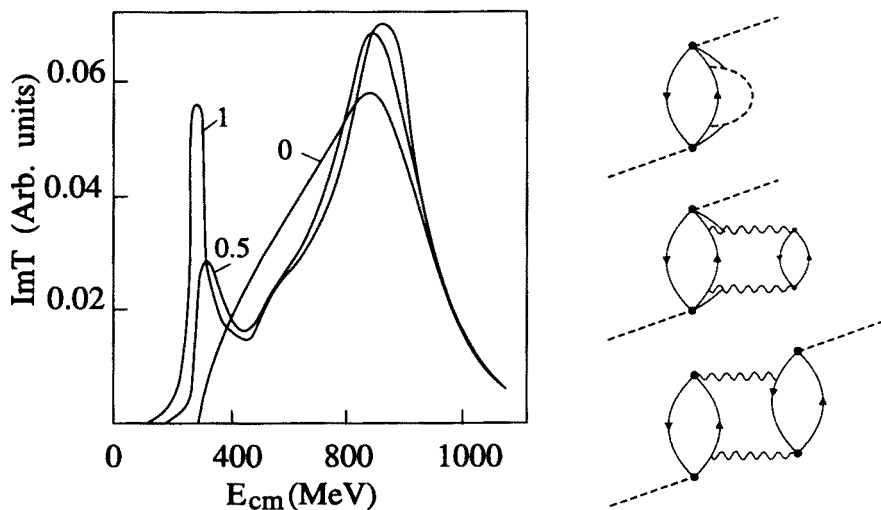$$T(s = 4m_\pi^2, t = 0, u = 0) = -7\frac{m_\pi^2}{f_\pi^2} \tag{26}$$

Fig. 5. $\pi\pi$ strength distribution in the sigma ($I = J = 0$) channel for various values of ρ/ρ_0. The various sources of width of the quasi-pions are shown on the right side.

and indicates a sign change when going off-shell. Although the soft-pion amplitude is not directly relevant to the $\pi\pi$ scattering amplitude it is possible to reach kinematical conditions which are very close to the point where the low energy theorem (25) applies. Hence, one may suspect that the soft-pion constraint will generate repulsion below threshold which is completely absent in the two above phenomenological models. For this reason chirally consistent models have been considered such as linear sigma model, gauged non linear sigma model or an improved version of the Jülich model [18]. An other well known problem is the way of preserving symmetries in the unitarization procedure. One needs a kinematical prescription for the off-shell continuation of the $\pi\pi$ potential in the scattering equation. Depending on the prescription, the chiral properties of the type of (26) present at the tree level may or may not be conserved in the iteration. The problem is especially critical within the Blankenbecler- Sugar choice (the intermediate pion are on the energy shell but not on the mass shell). One possible solution is to employ a substracted dispersion relation to extract the real part of the $\pi\pi$ propagator. For instance, in the free case this is equivalent to make the replacement :

$$G_{2\pi}(\boldsymbol{k}, E) = \frac{1}{\omega_k} \frac{1}{E^2 - 4\,\omega_k^2 + i\eta} \quad \rightarrow \quad \frac{E^2}{4\,\omega_k^2} \frac{1}{\omega_k} \frac{1}{E^2 - 4\,\omega_k^2 + i\eta}. \qquad (27)$$

The presence of the explicit E^2 factor ensures that the contribution of the iteration terms are of higher order in m_π^2, thus maintaining the chiral properties present at the Born term level. This procedure is nevertheless not totally satisfactory and a very good discussion of this important but rather technical problem can be found in [20]. However, once these chiral symmetry constraints are incorporated the pion pair condensation becomes strongly disfavored although some accumulation of strength below or near 2π threshold remains present.

Needless to say that a sizeable reshaping of the in medium $\pi\pi$ interaction is of utmost interest for the saturation mechanism since correlated pion exchange is a leading piece of nucleon-nucleon attraction. It is thus crucial to have some experimental information about this kind of in medium two-pion correlation. A possible evidence for these effects is provided by the $\pi - 2\pi$ data of the CHAOS collaboration at TRIUMF [21]. As shown on Fig. 6a, the measured $\pi^+\pi^-$ invariant mass spectrum shows a strongly A dependent structure near threshold. This effect, absent in the $\pi^+\pi^+$ channel which is a pure $I = 2$ isospin, indicates this is not a threshold (cusp) effect. A plausible explanation would be the in medium rescattering (Fig. 6b) of the pions in the scalar-isoscalar channel, generating a sharp structure as explained above. We are now in the process of incorporating this medium rescattering [22] on top of the detailed model of [23] which itself (full line of Fig. 6) cannot explain the data.
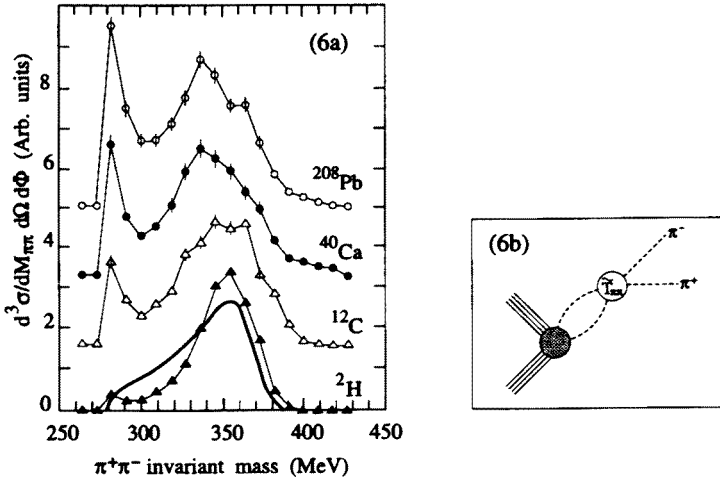

Fig. 6. (6a): Invariant mass distribution for the π^+, $\pi^+\pi^-$ reaction on various nuclei. The full curve is a prediction of the model of Ref. [23]; (6b): Rescattering of the two pions through the in medium T matrix

5. The rho meson in hot and dense hadronic matter

The modification of vector meson masses at high temperature and/or density is a vividly debated subject. In particular, many theoretical works have been devoted to the rho meson whose mass spectrum in dense matter can be studied, through dileptons production, in relativistic heavy ion collisions. For instance in the phenomenologically well established vector dominance picture (VDM), the dilepton production rate is directly proportionnal to the imaginary part of the rho meson propagator

$$\frac{dN_{l^+l^-}}{d^4x\, d^4q} = -\frac{\alpha^2}{3\pi^2 M^2} \frac{3m_\rho^4}{\pi g^2} \frac{1}{e^{q_0/T}-1} \frac{\mathrm{Im}\,\Sigma(q_0,T)}{[q_0^2 - m_\rho^2 - \Sigma(q_0,T)]^2}, \tag{28}$$

where g is the VDM coupling constant and m_ρ the free space rho meson mass. Here in Eq. (28) we have assumed, for simplicitely, back to back kinematics for which the invariant mass M of the pair coincides with the energy q_0 of the rho at rest. T is the temperature of the fireball and the physics is contained in the in medium rho self-energy $\Sigma(q_0,T)$. We will see below how this formula can be adapted to a realistic non-equilibrium situation.

Using current algebra and PCAC [24] it is possible to show that at finite temperature the mass of the rho meson remains unchanged to order T^2. Corrections to higher order in temperature are not controlled by chiral symmetry alone. Indeed to order T^4 explicit chiral models show opposite behaviour. For instance the linear sigma model implemented with vector mesons [25] predicts a decrease of the rho meson mass to order T^4. On the other hand a non linear version of the sigma model [26] finds an increase of this mass to order T^4. At critical temperature both agree qualitatively in that the masses of the ρ and the a_1 become degenerate at a value around 1 GeV.

It is well known that in the limit of vanishing quark masses the QCD action is scale invariant. However this approximate scale invariance is explicitely broken by quantum fluctuations in the renormalization procedure. The divergence of the associated scale current, equal to the trace of the energy-momentum tensor, is given by the so called trace anomaly [27]

$$\partial_\mu D^\mu = T_\mu^\mu = m_q\, \bar{q}q - \frac{\beta(g)}{2g} G_{\mu\nu}^a G_a^{\mu\nu}, \tag{29}$$

where $g = (\alpha_S/4\pi)^{1/2}$ is the QCD coupling constant and $\beta(g)$ the usual Gell-Mann-Low function governing its evolution. In addition there is a large spontaneous breaking of scale invariance by the gluon condensate

$< G \cdot G > = < (\alpha_s/\pi) G^a_{\mu\nu} G^{\mu\nu}_a >_{\text{vac}} \simeq (360\,\text{MeV})^4$. It has been proposed that this scale symmetry should be formally present in the Lagrangian of effective theories [28]. This can be achieved by introducing, through simple dimensional analysis, a scalar dilaton field χ in the standard chiral Lagrangian, according to

$$\mathcal{L} = \frac{f_\pi^2}{4} \left(\frac{\chi}{\chi_0}\right)^2 \text{tr}_f\, \partial_\mu U \partial^\mu U^\dagger - c \left(\frac{\chi}{\chi_0}\right)^3 tr_f \left(m_q (U + U^\dagger)\right)$$
$$-\tfrac{1}{2}\partial^\mu\chi\partial_\mu\chi - V(\chi) + \dots, \qquad (30)$$

where $U = \exp\left(i\vec{\tau}\cdot\vec{\phi}/2\right)$ and $\vec{\phi}$ is the pion field. The potential $V(\chi)$ models the quantum effects responsible for the scale anomaly [29] and drives the vacuum expectation value of the χ field to a non zero value given by the gluon condensate $\chi_0^4 \sim< G \cdot G >$. This approach may be questionable since, at the principle level, a low energy effective theory should be built, using renormalization group techniques, from the elimination of short wavelength quark-gluon degrees of freedom in favor of effective hadronic degrees of freedom. In that respect it is not easy to imagine how such an effective theory can keep a memory of the basic scale invariance since, by definition, its construction breaks scale invariance. However it leads to interesting scaling laws at finite temperature and/or baryonic density [30].

$$\frac{\chi^*}{\chi_0} = \left(\frac{< G \cdot G >^*}{< G \cdot G >}\right)^{\frac{1}{4}} = \left(\frac{< \bar{q}q >^*}{< \bar{q}q >}\right)^{\frac{1}{3}} = \frac{f_\pi^*}{f_\pi} = \frac{m_V^*}{m_V} = \dots. \qquad (31)$$

Incorporation of vector mesons, with help of the KSFR relation, in such a scheme shows that their masses decrease with increasing ρ and/or T like fractional powers of the condensates.

Recent dilepton production data from the CERES collaboration at CERN/SPS [31] show a spectacular reshaping of the mass spectrum in $S + Au$ collisions at 200 GeV per nucleon. The rho itself is depleted and there is an enhancement in the 500 MeV invariant mass region (see Fig. 8). Calculations based on the quark-gluon scenario turn out to be unable to reproduce the observed enhancement. A recent transport model calculation [32] has shown that the data can be reproduced provided the free space rho meson mass is replaced by the in medium dropped rho meson mass (Fig. 8a). In other words one is tempted to say that these data constitute an experimental evidence of partial chiral symmetry restoration. Nevertheless this statement should be at least moderated since more conventionnal many body effects [33] may explain the major part of the effect.

In the invariant mass region of interest the basic process is the $\pi^+\pi^-$ annihilation into a rho wich subsequently converts into a virtual photon. The
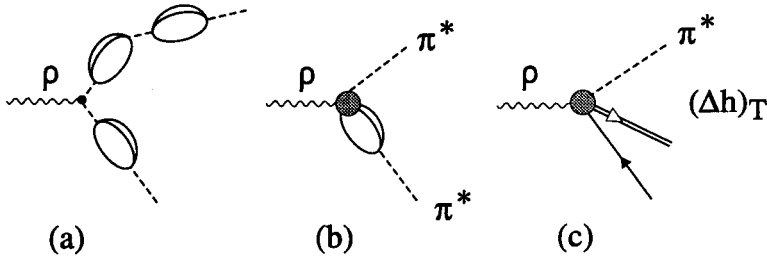
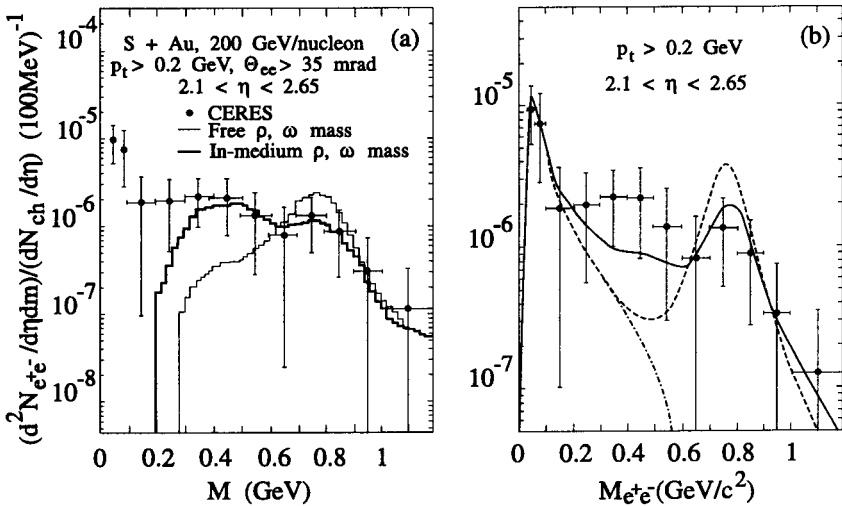Fig. 7. Medium corrections to the rho meson propagation; see explanation in the text.

Fig. 8. Effect of the in medium modified ρ meson mass (8a) and of many body mechanisms associated with π^* and Δ (8b) compared to CERES data. The dotted curves correspond to the bare rho calculation.

first medium correction is the replacement of the bare pions by collective quasi pion modes π^* (*i.e.* the pionic branch) dressed mainly by delta-hole excitations (Fig. 7a). Gauge invariance in the medium requires to incorporate other processes such as, among others, those represented on Fig. 7b and 7c. The main effect of the vertex correction diagram (Fig. 7b) is to kill the structure at $2m_\pi$ coming from the dressing of pions (Fig. 7a). The diagram of Fig. 7c represents the coupling of the rho to $\pi^*(\Delta h)_T$ states. The suffix T means that these Delta-hole states are the spin transverse ones, *i.e.* those excited in photon reactions and not directly coupled to the pionic branch. They lead to a structure in the invariant mass re-

gion $M \geq m_\pi + \omega_\Delta \sim 500$ MeV. Similar conclusions have been reached in Ref. [34, 35]. This effect accelerated by the consistent modification (of RPA type) of the in-medium form factor (the denominator of the propagator in Eq. 28) may provide an alternative explanation of the experimental enhancement.

We have quantitavely checked this mechanism [36] by performing the full calculation of dilepton production within the scheme of [33]. We have extracted the time evolution of temperature and baryonic abundance from transport results [32] and integrated (28) over time, taking into account acceptance corrections [37]. For instance, at the initial time the temperature is 170 MeV and the nucleon and delta densities are $1.0\rho_0$ each. Some theoretical improvements (inclusion of NN^{-1}, $N\Delta^{-1}$, $\Delta\Delta^{-1}$ and thermal effects in the corresponding bubbles) have been added to the original calculation. The result is displayed on Fig. 8b where we see that these many-body mechanisms may explain a very important part of the effect. Nevertheless, there is still some room for those mechanisms associated with a fundamental modification of the QCD vacuum. Thus, a consistent theoretical framework, including both types of effect, has to be elaborated in view of the next generation of dilepton experiments with the HADES detector at SIS.

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