THERMODYNAMICS OF AN INTERACTING πN-SYSTEM*

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The thermodynamic potential of an interacting system of pions and nucleons is calculated including the P_{33} πN resonance $\Delta(1232)$. The resonance width is taken into account in a consistent manner.

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1. Introduction and conclusions

In relativistic heavy-ion collisions a zone of hot and dense hadronic matter is created. At beam energies of the order of 1 AGeV or higher, hadron resonances play an important role in the dynamics of such collisions. Some of these resonances (*e.g.* Δ and ρ) have large widths, comparable to or larger than the typical temperature of the interaction zone. Since the corresponding life-times are smaller than the life-time of the hot and dense zone, one expects several generations of these resonances. Consequently, it is interesting to study the thermodynamics of many-body systems with short-lived resonances, taking the width of the resonances into account.

At beam energies around 1 AGeV, the most abundant particles in the hot and dense zone are the nucleon, the pion and the P_{33} πN resonance $\Delta(1232)$. The thermodynamics of such systems has been studied by many authors. In such calculations, the width of the Δ-isobar is either neglected or treated in an *ad hoc* manner. In the following we present results of a general approach to thermodynamics, where the widths of resonances are treated consistently, applied to a system of interacting nucleons, pions and Δ-isobars [1]. The thermodynamic weight function of the resonance can

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be written as the sum of two distinct parts: The first part depends on the spectral function and counts the Δ-isobars, whereas the second takes into account correlated πN pairs as they occur in πN loops of the Δ selfenergy. The weight function can also be expressed as twice the energy derivative of the phase shift and may be interpreted as a time scale of the πN interaction as a function of energy.

In order to study the consequences of a consistent treatment of the resonance width in heavy-ion collisions, we consider a simple fireball model. The system freezes out when the mean free paths of the constituents exceed the fireball radius. In this model the pions observed in a heavy-ion collision are of three different origins: there are thermal pions, which are present at freeze-out, and there are pions, created after freeze-out in decays of Δ-isobars and of correlated πN pairs. Clearly all contributions must be taken into account when comparing to experimental results. The correlated πN pairs are found to give an important, hitherto neglected, contribution to the pion spectrum at low energies. At the present stage, the freeze-out parameters deduced from comparison to data are still preliminary. We note that higher resonances, in particular $N^*(1440)$, as well as collective flow may change the numerical values obtained so far. However, we expect the qualitative features of the present analysis to remain valid.

2. Baryon density

Our starting point for computing the thermodynamical potential $\Omega\,(T,\mu,V)$ is the self-consistent Green-function method of Kadanoff and Baym [2]. The advantage of this method is that the resulting thermodynamic potential is free of double-counting problems and consistent with conservation laws. However, a fully self-consistent treatment is extremely involved even in the simplest non-trivial approximation. Therefore, we relax the self-consistency for the nucleon and the pion and approximate their propagators by the free ones. The expression for the thermodynamical potential then reads as

$$\Omega = \tfrac{1}{2}\mathrm{Tr}\{\ln[-(\mathcal{D}_\pi^0)^{-1}]\} - \mathrm{Tr}\{\ln[-(\mathcal{G}_N^0)^{-1}]\} - \mathrm{Tr}\{\ln[-\mathcal{G}_\Delta^{-1}]\} - \mathrm{Tr}\{\Sigma_\Delta \mathcal{G}_\Delta\} + \Phi\,, \tag{1}$$

where Φ denotes a functional from where all selfenergies are derived. In our case only the Δ selfenergy is left. Choosing Φ as shown in Fig. 1(a), the Δ selfenergy consists of a πN loop and the last two contributions in Eq. (1) cancel. In a low-density limit medium effects like 'Pauli-blocking' or 'Bose-enhancement' can be neglected. Consequently, in this limit, the Δ selfenergy is equal to its vacuum form. In that case, the baryon density is found as

$$n_{\rm B}(T,\mu) = -\frac{1}{V}\left.\frac{\partial\,\Omega(T,\mu,V)}{\partial\mu}\right|_{T,V} = n_N(T,\mu) + \tilde{n}_\Delta(T,\mu)\,, \tag{2}$$

where n_N denotes the contribution of the thermal nucleons and $\tilde{n}_\Delta$ the contribution due to the πN interaction. The former is given by the Fermi-Dirac distribution with a degeneracy factor four integrated over momentum while the latter is of the form

$$\tilde{n}_\Delta(T,\mu) := 16\int\frac{d^3p}{(2\pi)^3}\int_{m_N+m_\pi}^{\infty}\frac{dm_\Delta}{2\pi}B_\Delta(m_\Delta) \times\left[\exp[(\sqrt{\vec{p}^{\,2}+m_\Delta^2}-\mu)/T]+1\right]^{-1}. \tag{3}$$

The weight function B_Δ in Eq. (3) is given by

$$B_\Delta(m_\Delta) = A_\Delta(m_\Delta) + 2{\rm Im}\left\{\frac{\partial\Sigma_\Delta}{\partial m_\Delta}(m_\Delta)\,G_\Delta(m_\Delta)\right\} = 2\frac{\partial}{\partial m_\Delta}\delta_{33}(m_\Delta)\,, \tag{4}$$

where the quantity A_Δ denotes the spectral function of the Δ-isobar, Σ_Δ the Δ selfenergy, G_Δ the full Green function of the Δ-isobar and δ_{33} the πN scattering phase shifts in the P_{33} channel. It is now obvious that the interaction contribution to the baryon density $\tilde{n}_\Delta$ is made up of two different parts. The first one, involving the Δ spectral function, is the contribution of the Δ-isobars to the baryon density. The second part, which has not been considered in most previous calculations, is the contribution of the interacting nucleons as they occur in the πN loops of the Δ selfenergy. Fig.1(b) visualizes the three contributions to the baryon density in terms of diagrams, corresponding to the contributions of nucleons, Δ-isobars and correlated πN pairs, respectively. A popular *ad hoc* prescription has been to use the spectral function A_Δ instead of B_Δ, thus neglecting the second term in Eq. (4) or in other words the third diagram of Fig. 1(b). In order to get a thermodynamically consistent description of the baryon density, not only the contributions of the nucleons and Δ-isobars but also that of the correlated πN pairs has to be included.

A complementary interpretation of the function B_Δ is obtained by considering the second equality in Eq. (4). This form of B_Δ is the well known Beth-Uhlenbeck result [3] and coincides with that obtained in the S-matrix approach to statistical physics [4]. The energy derivative of the phase shift defines a time scale, which in elastic scattering is interpreted as the time delay or the interaction time. Hence, an appealing interpretation emerges,

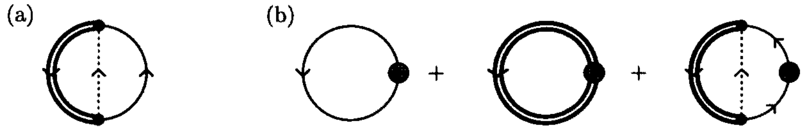


Fig. 1. (a) Functional Φ; (b) Different contributions to the baryon density. Here a double line denotes $\mathcal{G}_\Delta$, a solid line $\mathcal{G}_N^0$ and a dashed line $\mathcal{D}_\pi^0$. The gray circle indicates the baryon number operator. See the text for details.

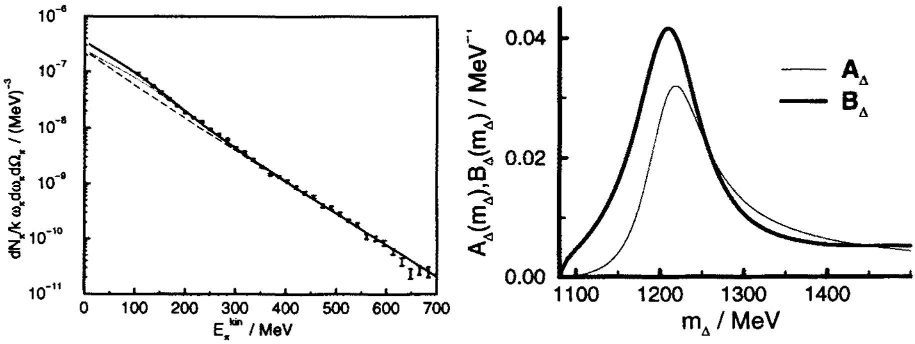
namely that the contribution of a resonance to thermodynamic potentials is proportional to the time spent in the resonant state. Furthermore, Eq. (4) implies that — neglecting medium effects — the thermodynamical potential and the baryon density can be computed in a model independent way, directly from the empirical phase shifts. As shown in the inset of Fig. 2 the second term in B_Δ shifts the strength towards lower masses.

The following results are computed using a model describing the πN scattering process in the P_{33} channel by considering the direct Δ Born-diagram and the crossed nucleon Born-diagram. The parameters, the bare mass of the Δ-isobar, the $\pi N \Delta$ coupling constant and a cut-off parameter at the $\pi N \Delta$ vertex, are fixed by fitting the corresponding phase shifts.

3. Pion spectrum

Given the population of resonant states, Eq. (3), which after freeze-out decay into a nucleon and a pion, it is straightforward to compute the resulting pion spectrum. The shape of the spectrum, including the contribution of thermal pions, is then fitted to the experimental data. This fixes the freeze-out temperature and density.

In Fig. 2 we show such a fit to the π^+ spectrum of the reaction Au+Au at 1 AGeV measured at GSI [5]. The spectrum is shown, as a function of the kinetic energy of the pions at midrapidity, in the so called Boltzmann representation, where a Boltzmann distribution would appear as a straight line with slope $-T^{-1}$. In the model, the high energy regime is dominated by the thermal pions. The influence of pions resulting from decays is noticeable only for pion kinetic energies below 350 MeV. We find that the contribution of the correlated πN pairs is decisive for the agreement at the lowest energies. The resulting values for the freeze-out temperature and density are $T = 76$ MeV and $n_\mathrm{B} = 0.2\ \rho_0$, respectively. Here ρ_0 denotes normal nuclear matter density. The fraction of Δ-isobars at freeze-out is $n_\Delta/n_\mathrm{B} = 0.07$ and that of Δ's and πN pairs $\tilde{n}_\Delta/n_\mathrm{B} = 0.13$. Assuming an



(a) (b)

Fig. 2. (a) Fit to π^+ spectrum. The dashed curve shows the contribution of thermal pions, the dotted curve one that of thermal pions plus pions resulting from Δ-isobar decays. Finally, the solid line corresponds to the full spectrum including thermal pions, pions from Δ-isobar decays as well as pions from decays of correlated πN pairs. Part (b) shows a comparison of the weight function B_Δ with respect to the spectral function A_Δ. See the text for details.

adiabatic expansion we can compute the corresponding fractions at preceding stages of the reaction. We find $n_\Delta/n_B = 0.28$ and $\tilde{n}_\Delta/n_B = 0.43$ at a density $n_B = 2\rho_0$.

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