GEOMETRICAL QUANTIZATION HADRONIC RESONANCES*

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The Balmer-like mass formula and quantization conditions derived from the first principles have been applied to systematic analysis of the gross structure of all known hadronic resonances. These equations are very useful at least for prediction and estimation of the invariant masses of unknown resonances. The correspondence principle between old classical and new quantum theories plays an outstanding role in the interpretation of the results and allows us to go even into fine details. We have demonstrated that the dimensional analysis, the principles of similitude and automodelity and the methods of analogy can put some bridge between the various branches of physics.

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A comparatively small number of examples of the so called "crude" systems is known in physics whose the "gross" or "global" properties can be described by the simplest approaches while the accuracy of description deteriorates in the case of detailed elaboration. For the first time, such a behaviour of systems in classical mechanics was established by Poincare. The reduction of accuracy with increasing number of model parameters has in detail been studied in the mathematical theory of the polynomial approximation of tabulated functions. It is well-known that the beating starts at the interjunction points with increasing of polynomial order and the problem of correct interpolation of experimental data becomes in general not feasible. A classical example of correct deal with crude systems in physics is the theory of diffraction. The aim of this paper is the application of diffraction theory methods (together with the dimension analysis, the principles of similitude and automodelity, the methods of analogy) to study the gross structure of resonances in micro- and macrophysics.

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We have developed (see review papers and references therein [1]) the following general physical conception of resonances: the periodic motion and refraction of waves in the restricted region of space are responsible for the creation of resonances in any resonating system. This conception is considered here for quantum mechanical systems, whose wave nature plays a decisive role in our approach. Within the R-matrix formalism we put at the boundary of this region a condition of radiation of physical particles which can be observed at large (asymptotic) distances and requires proper matching of the corresponding "external" wave functions with the "inner" part of the wave function of the considered system.

TABLE I

The invariant masses (MeV) of $\pi^-\pi^+$ or $\pi^-\pi^-$ systems

n/j	$m(\pi^-\pi^+)_{\text{theor}}$	$m(\pi^-\pi^+)_{\text{exp}}$	$m(\pi^-\pi^-)_{\text{exp}}$
1/4	287		
1/2	310		313 ± 3
3/4	345	332 ± 3	340
1	388	388 ± 2	388 ± 5
5/4	438	435	441
3/2	492	$K_S^0(497.672 \pm 0.031)$	
7/4	549	$\eta(547.45 \pm 0.19)$	
2	608		
9/4	668	652 ± 2	640 ± 5
5/2	730	$\rho(769.9 \pm 0.8)$	
11/4	793	$\omega(781.9 \pm 0.12)$	
3	856		
13/4	921		
7/2	985	$f_0(980 \pm 10)$	
15/4	1050		
4	1115		
17/4	1181		
9/2	1246	$f_2(1275 \pm 5)$	
19/4	1312	$f_0(1000 - 1500)$	
5	1378		
21/4	1444	$\rho(1465 \pm 25)$	
11/2	1511		
23/4	1577		
6	1644		
25/4	1710	$f_J(1710)$	

The new quantization condition for asymptotic momenta of decay products of a resonance has been obtained in the framework of this conception. It results in the Balmer-like mass formula used in our study; its accuracy is surprisingly high and unusual for this branch of physics. Following the outlined conception we carried out the systematical investigation of the gross structure of spectra and mass distributions of all known hadronic resonances starting from light mesons and ending with bottomonium resonances. As a final result we have shown below (Table I) the invariant masses for the $(\pi^-\pi^+)$ resonances calculated using a model [1]. Note, we have no free parameters. One can see a good description of existing experimental data. We hope that this is not accidental.

In conclusion, we would like to say that the Balmer-like mass formula have been applied to systematic analysis of the gross structure of all known hadronic resonances. It means that these equations could be useful at least for prediction and estimation of the invariant masses of unknown resonances. We can say that the correspondence principle between old classical and new quantum theories plays an outstanding role in the interpretation of the results and this "correspondence" allows us to go even into fine details. We have demonstrated that the dimensional analysis, the principles of similitude and automodelity and the methods of analogy can put some bridge between the various branches of physics.

Therefore, we can conclude that the classical and quantum mechanical principles **are sufficient** for explanation of the gross properties of hadron resonances. That means that new quantum numbers, new particles or other exotics **are not necessary**.

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