HOT AND DENSE PION GAS WITH FINITE CHEMICAL POTENTIAL*

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A dense and hot pion system with a dynamically fixed particle number and an arbitrary charge is investigated with respect to ultra-relativistic heavy-ion collisions. The Lagrangian for describing such a system is derived from Weinberg's chiral Lagrangian for the pion-pion interaction. Pion polarization operators are calculated within the Hartree approximation. The pion spectrum in the isospin symmetrical gas is presented. The effective in-medium pion gap depends sensitively on the density and the temperature, and it is found to exceed both the free pion mass and the corresponding value for the pion gas in chemical equilibrium. The possibility of a Bose-Einstein condensation is also discussed.

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1. Meson-enriched systems can be created in ultra-relativistic heavy-ion collisions (URHIC) due to the kinematic separation of the light meson component (as assumed in Bjorken's model) or in the processes of hadronization and decay of the quark-gluon plasma possibly formed in the mid-rapidity region. The heavy meson resonances decay thereby to pions, and at subsequent time one deals with an expanding pion enriched gas with some admixture of other light mesons [1].

Relying on the number of produced pions in URHIC and the typical radius of the pion system evaluated from the $\pi\pi$ correlation data [1], the

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pion gas density can be estimated as $\rho \sim (1-6)\rho_0$, where $\rho_0 = 0.17\ \mathrm{fm}^{-3}$ is the nuclear saturation density. At such densities the pion gas is obviously highly imperfect and the proper account of the strong pion-pion interaction is required.

In Ref. [2] the Weinberg Lagrangian [3] has been employed for the description of the pion gas. To its advantage, allowing for analytical calculations with the self-consistent treatment of the intermediate in–medium pion states, Weinberg's Lagrangian reflects properly the genuine chiral nature of the pionic excitations. The corresponding $\pi\pi$ interaction satisfies the low-energy theorems following from the current algebra and reproduces well the experimental pion scattering lengths at rather low pion energies.

In Ref. [2] the chemically and thermally equilibrated pion gas has been considered. However, according to estimates of Ref. [4], at temperatures $T \lesssim (0.8-1.2)m_\pi$ the rate of the pion absorption (W_{abs}) is substantially suppressed in contrast to the rate of the elastic pion rescattering (W_{res}). Then one arrives at the scenario wherein the pion system cools down, on the one hand, rather rapidly, so that the typical cooling time τ_{cool} is much less than $\tau_{abs} = W_{abs}^{-1}$, whereas, on the other hand, τ_{cool} time still essentially exceeds the rescattering time $\tau_{\mathrm{res}} = W_{\mathrm{res}}^{-1}$. As a result, while expanding, the pion gas has enough time to be driven out of the chemical equilibrium, remaining in the thermal quasi-equilibrium. This inspires us to study the properties of the strongly interacting pion gas with a *dynamically* fixed total number of particles, i.e., with a finite pion chemical potential.

2. The Lagrangian, proposed by Weinberg [3] for the description of the $\pi\pi$ interaction in the first-order of the coupling constant, has the form

$$L = \int d^3x \left[\tfrac{1}{2}(\partial\vec{\varphi})^2 - \tfrac{1}{2}m_\pi^2\vec{\varphi}^2 + \tfrac{1}{2}m_\pi^2\lambda\vec{\varphi}^4 - \lambda(\partial\vec{\varphi})^2\vec{\varphi}^2\right], \qquad (1)$$

where $\lambda = 1/(2f)^2$ is the pion self-interaction constant, and $f = 93$ MeV stands for the pion decay constant. The isospin vector $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$ is associated with the fields of the positive (π^+), negative (π^-) and neutral (π^0) pions by relations $\pi^\pm = \frac{1}{\sqrt{2}}(\varphi_1 \pm i\varphi_2)$, and $\pi^0 = \varphi_3$. The positive and the negative pions, being introduced as particles and anti-particles, are described by the one complex field, $(\pi^-)^\dagger = \pi^+$, where $(...)^\dagger$ means the Hermitian conjugation. The neutral pions as the self-conjugated particles are represented by the real field $(\pi^0)^\dagger = \pi^0$.

In a system with fixed and, in principle, different numbers of pions of each species the particle–anti-particle symmetry is lost. Therefore, we turn in the Lagrangian (1) to the new fields φ_-, φ_+, φ_0, corresponding to the pions with the positive frequencies, $\pi^{\pm 0} = \varphi_{\pm 0} + \varphi_{\mp 0}^\dagger$. Now we have $\varphi_- \neq \varphi_+^\dagger$, and φ_0 is described by a complex field. Replacing the new fields

in the Lagrangian (1), we are able to separate the terms corresponding to the conserved particle number.

In terms of new fields the Lagrangian (1) renders $L = \int d^3x(\mathcal{L}_{\text{fixed}} + \mathcal{L}')$ with

$$\begin{aligned} \mathcal{L}_{\text{fixed}} &= \mathcal{L}_+ + \mathcal{L}_- + \mathcal{L}_0 + \mathcal{L}_{\text{res}}, \\ \mathcal{L}' &= \mathcal{L}_{2\leftrightarrow 2} + \mathcal{L}_{3\leftrightarrow 1}, \end{aligned} \tag{2}$$

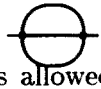
where $\mathcal{L}_{\pm 0} = |\partial\varphi_{\pm 0}|^2 - m_\pi^2|\varphi_{\pm 0}|^2$ is the free pion Lagrangian density and the pion elastic rescattering is described by

$$\begin{aligned} \mathcal{L}_{\text{res}} &= 2\lambda m_\pi^2|\varphi_\pm|^2\,(|\varphi_\pm|^2 + 2|\varphi_\mp|^2 + |\varphi_0|^2) \\ &+ \lambda m_\pi^2|\varphi_0|^2\,(2|\varphi_+|^2 + 2|\varphi_-|^2 + 3|\varphi_0|^2) \\ &- 4\lambda(\partial_\nu\varphi_+ \cdot \varphi_+^\dagger)(\partial^\nu\varphi_- \cdot \varphi_-^\dagger) - 4\lambda(\varphi_+\partial_\nu\varphi_+^\dagger)(\varphi_-\partial^\nu\varphi_-^\dagger) - \lambda(\partial\varphi_0 \cdot \varphi_0^\dagger)^2 \\ &- \lambda(\varphi_0\partial\varphi_0^\dagger)^2 - 4\lambda\,(|\partial\varphi_-|^2 + |\partial\varphi_+|^2 + |\partial\varphi_0|^2)\,(|\varphi_-|^2 + |\varphi_+|^2 + |\varphi_0|^2). \end{aligned}$$

The Lagrangian density $\mathcal{L}_{\text{fixed}}$ describes the processes which conserve the total number of particles and separately the number of pions of each sort. These are the interaction of pions of a certain sort and the reactions of the different pion species with each other, e.g., $\pi^+\pi^- \leftrightarrow \pi^+\pi^-$ and $\pi^\pm\pi^0 \leftrightarrow \pi^\pm\pi^0$. The part $\mathcal{L}_{2\leftrightarrow 2}$ of the Lagrangian density corresponds to the processes $\pi^0\pi^0 \leftrightarrow \pi^+\pi^-$ which, while keeping the total number of pions fixed, change the relative fractions of the pion species. Such processes impose the restriction on the chemical potentials of pions $2\mu_0 = \mu_+ + \mu_-$ and determine the final isospin composition of the pion system.

The pion self-interaction contained in the terms (2) gives the contribution to the pion polarization operator already in the first order in the coupling constant λ. Such a contribution can be depicted by the tadpole graph  , where the fat pion line corresponds to the full in-medium pion propagator.

The last part of the Lagrangian density, $\mathcal{L}_{3\leftrightarrow 1}$, contains the terms with non-equal numbers of the creation and annihilation operators. It corresponds to the processes $\pi \leftrightarrow \pi\pi\pi$ with a change of the pion number. These processes drive the system towards chemical equilibrium.

The terms $\mathcal{L}_{2\leftrightarrow 2}$ and $\mathcal{L}_{3\leftrightarrow 1}$ contribute to the pion polarization operator only in the second order in the coupling constant and are given by the sandwich diagram of the type  with different possibilities for the pion species in the internal lines allowed by the charge conservation. The direction of the internal lines takes also into account the distinct processes

$\pi\pi \leftrightarrow \pi\pi$ (two lines are directed to the one side and one line to the other side) and $\pi \leftrightarrow \pi\pi\pi$ (all lines go from the left to the right).

We evaluated the contribution of the sandwich diagram to the real part of the pion polarization operator and showed that it is substantially suppressed in comparison with the contribution of the tadpole graph at temperatures $T \lesssim (1 - 1.5)m_\pi$. The symbol m_π denotes here the vacuum pion mass. The higher-order vertex corrections are estimated as small at temperatures of interest. In the calculation of the real part of the pion polarization operator this allows us to stick furthermore to the Hartree approximation, accounting only for the tadpole diagram (with the in-medium pion propagator).

The imaginary part of the sandwich diagram determines the probabilities of the rescattering and absorbative processes $W_{\mathrm{abs,res}}$. We have evaluated these quantities and obtained for the isospin symmetrical pion gas that $\tau_{\mathrm{abs}}/\tau_{\mathrm{res}} \simeq 17$ for $T = 200$ MeV and at the pion density $\rho = 3\rho_0$ and $\tau_{\mathrm{abs}}/\tau_{\mathrm{res}} \simeq 7$ for $T = 150$ MeV and $\rho = \rho_0$. These evaluations support earlier estimates [4] and our former suggestion on the lack of the chemical equilibrium in the pion gas in URHIC. Thus, for the description of the pion gas with a fixed number of particles we shall furthermore use the Lagrangian (2), dropping the term $\mathcal{L}'$ and introducing instead the non-vanishing pionic chemical potentials connected by condition $2\mu_0 = \mu_+ + \mu_-$.

3. Varying the Lagrangian (2) with respect to $\varphi_+^\dagger$, $\varphi_-^\dagger$, $\varphi_0^\dagger$, we obtain three coupled non-linear equations of motion for the corresponding fields. We solve this system within the Hartree approximation. By this approach the behaviour of a certain pion is determined by the averaged interaction with surrounding particles which form a thermal bath. The properties of particles in the bath are, in turn, determined by the same equation of motion as that for the considered pion.

Formally, we represent the fields $\varphi_{\pm 0}$ as a superposition of some picked-out field $\tilde{\varphi}_{\pm 0}$ and environmental fields $\xi_{\pm 0}$, $\varphi_{\pm 0} \to \tilde{\varphi}_{\pm 0} + \xi_{\pm 0}$. Then, in equations of motion we keep only these terms that are linear in the fields $\tilde{\varphi}_{\pm 0}$ and quadratic in the fields $\xi_{\pm 0}$ (other terms vanish by averaging). After averaging over the fields $\xi_{\pm 0}$ with the Gibbs' weight factor, for the retarded pion Green's function in the momentum representation we obtain

$$\left[G_j^R(\omega, k)\right]^{-1} = \omega^2 - k^2 - m_\pi^2 - \Pi_j^R(\omega, k) + i\,0, \qquad j = +, -, 0, \tag{3}$$

with the polarization operator $\Pi_j(\omega, k)$ of a certain pion species $j = \pm, 0$ given by

$$\Pi_\pm^R(\omega, k) = -\,4\lambda\, m_\pi^2 \left[2d_0^+ + 2d_0^- + d_0^0\right] + 4\lambda\, m_\pi^2 \left[d_2^+ + d_2^- + d_2^0\right] \tag{4}$$

$$+ 4\lambda \left[d_0^+ + d_0^- + d_0^0\right] (\omega^2 - k^2) \pm 4\lambda \rho_{ch} \omega,$$

$$\Pi_0^R(\omega, k) = - 2\lambda m_\pi^2 \left[2d_0^+ + 2d_0^- + 6d_0^0\right] + 4\lambda m_\pi^2 \left[d_2^+ + d_2^- + d_2^0\right]$$

$$+ 4\lambda \left[d_0^+ + d_0^- + d_0^0\right] (\omega^2 - k^2). \qquad (5)$$

Taking into account that in the Hartree approximation the polarization operators have vanishing imaginary parts, and, therefore, the pion spectra $\omega_j(k)$ are well determined by the dispersion equation $\left[G_j^R(\omega, k)\right]^{-1} = 0$, we can write the d functions as

$$d_0^j = \int \frac{d^3k}{(2\pi)^3} \Gamma_j(k) n_j(\omega_j(k)), \quad m_\pi^2 d_2^j = \int \frac{d^3k}{(2\pi)^3} (\omega_j^2(k) - k^2) \Gamma_j(k) n_j(\omega_j(k)).$$

Here $n_j(\omega)$ are the Bose distributions with the chemical potentials μ_j, connected by the above balance equation and determined by the total pion density $\rho_{tot} = \sum_j \rho_j$ and the charge density $\rho_{ch} = \rho_+ - \rho_-$, where

$$\rho_j = \int \frac{d^3k}{(2\pi)^3} 2\omega_j(k) \Gamma_j(k) n_j(\omega_j(k)),$$

and $\Gamma_j(k) = \left(2\omega - \frac{\partial \Pi_j^R}{\partial \omega}\right)\Big|_{\omega_j(k)}^{-1}$.

4. We employ the derived Eqs. (4,5) to the isospin symmetrical pion gas which probably arises in the mid-rapidity region of URHIC. Then, the pions of all three sorts are described by the polarization operator

$$\Pi^R(\omega, k) = -20\lambda m_\pi^2 d_0 + 12\lambda m_\pi^2 d_2 + 12\lambda d_0 (\omega^2 - k^2). \qquad (6)$$

For $\mu = 0$ this expression coincides with that obtained in Ref. [2] for the pion gas in the chemical equilibrium. In our case, however, the functions d_0 and d_2 essentially depend on the value of the chemical potential $\mu = \mu_+ = \mu_- = \mu_0 \neq 0$. Making use of the relation between d_0 and d_2 we obtain the dispersion relation

$$\omega^2(k) = m_\pi^{*2} + k^2, \qquad m_\pi^{*2} = m_\pi^2 \frac{1 - 20\lambda d_0}{1 - 24\lambda d_0}, \qquad (7)$$

where the quantity m_π^* is the effective pion mass.

Fig. 1 displays the effective pion masses (7) (solid lines) and the chemical potentials (dashed lines) as functions of the temperature for the pion densities $\rho = \rho_0$ and $3\rho_0$. Dotted lines indicate the chemical potential of the ideal pion gas. The dash-dotted line in Fig. 1 depicts the effective pion mass for $\mu = 0$ which recovers the result of Ref. [2]. We observe a principal

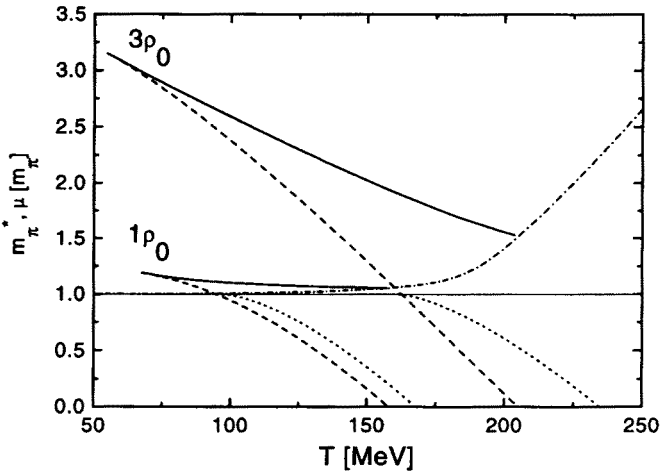


Fig. 1. The pion mass and the chemical potential of an isospin symmetrical pion gas vs. the temperature. See text for explanations.

difference in the behaviour of the effective pion mass $m^*_\pi(\mu \neq 0)$ from that for $\mu = 0$. For a fixed number of pions the value $m^*_\pi(\mu \neq 0)$ decreases with increasing temperature, whereas the value $m^*_\pi(\mu = 0)$ remains approximately constant staying nearby m_π for $T \leq 175$ MeV and grows rapidly for larger temperatures.

Fig. 1 also enables us to discuss the possibility of a Bose–Einstein condensation (*c.f.* [5]). The crossing points of the solid and dashed lines determine the critical temperatures $T_c^{\rm ind}$ of an 'induced' Bose condensation caused by the singularity in the pionic momentum distribution at $\mu = m^*_\pi$. The critical point of the Bose condensation T_c^{id} in the ideal gas corresponds to temperatures at which the chemical potential μ^{id} (dotted lines in Fig. 1) reaches the value of the vacuum pion mass. When the chemical potential of the interacting pion gas reaches this value, then there exists the principal possibility of a first-order phase transition from the interacting gas phase to a condensate one. At such temperatures the Bose condensate becomes energetically favorable.

The density dependences of $T_c^{\rm ind} = T(\mu = m^*_\pi)$, $T(\mu = m_\pi)$ and $T_c^{id} = T(\mu^{id} = m_\pi)$ are shown in Fig. 2 (dashed, solid and the dash-dotted lines, respectively) together with the solution of the equation $\mu(T, \rho) = 0$ depicted by the dotted line. The latter curve shows the density in the pion gas being in chemical equilibrium at given temperature.

We observe a remarkable behaviour of the value $T_c^{\rm ind}$. In contrast to the ideal gas case, for interacting pions the value $T_c^{\rm ind}$ decreases with a growing

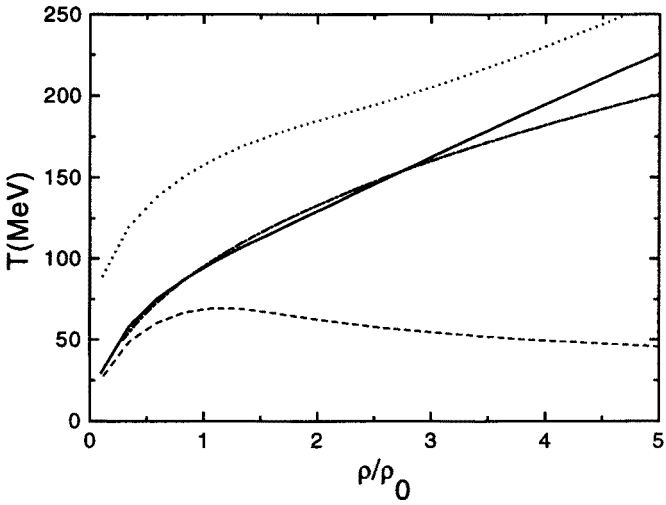


Fig. 2. The dependence of T_c^{ind} (dashed line), $T(\mu = m_\pi)$ (solid line), T_c^{id} (dashed-dotted line) and $T(\mu = 0)$ (dotted line) on the density.

density for $\rho \geq \rho_0$. The values of T_c^{ind} are smaller than the characteristic values of the experimental inverse slope factors measured in URHIC. Therefore one could argue that the induced Bose condensation is unlikely to occur in the quasi-equilibrium pion gas state. As one observes from Fig. 2, the values of the critical temperatures for a first-order phase transition $T(\mu = m_\pi)$ are large enough to be reached in URHIC. Thus there is a principal possibility of the occurrence of a first-order phase transition at least in some heavy-ion collisions.

5. Above we utilized the Weinberg Lagrangian for the pion-pion interaction. Although it allows for describing reasonably well the pion-pion scattering data in the energy region 280 MeV $< \sqrt{s} <$ 500 MeV, at higher energies the account of the ρ meson degrees of freedom is required. We have found that the $\pi\rho$ interaction, introduced with preserving the chiral symmetry [3], causes a contribution to the pion polarization operator, which is rather small at temperatures $T \ll m_\rho$, where m_ρ is the ρ meson mass. Thus our findings do not substantially change when including the ρ meson contribution.

6. In summary, in the framework of the Lagrange formalism we have considered the pionic modification in a pion gas with a fixed particle number using Weinberg's interaction. We have calculated self-consistently the pion spectrum within the Hartree approximation. The in-medium pion gap is

found to be essentially larger than the vacuum pion mass, and it dependents sensitively on the density, temperature and isotopical composition. The present analysis might serve as a first step towards detailed modeling of a strongly interacting meson gas composed of various light mesons like π, ρ, ω, K, η etc. Of particular interest is thereby the $\rho \leftrightarrow \pi\pi$ reaction, which is important for the dilepton signals measured by CERES.

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