

## CHIRAL SYMMETRY IN THE $K\bar{K}$ AND $KN$ SYSTEMS\*

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In this talk we present the results of recent developments in the application of Chiral perturbation theory to the  $K\bar{K}$  system and to the  $K^+N \rightarrow KN\pi$  reaction close to threshold. In the first case we study the decay channels of the  $a_0$  and  $f_0$  mesons assumed to be made largely from  $K\bar{K}$ . In the second case comparison is made with the present data. A qualitative agreement with experiment is found in both cases.

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### 1. Introduction

The nature of the  $f_0(980)$ ,  $I^G(J^{PC}) = 0^+(0^{++})$  and  $a_0(980)$ ,  $I^G(J^{PC}) = 1^-(0^{++})$  mesons is a matter of intense debate. They have been studied as  $q\bar{q}$  pairs [1, 2],  $q\bar{q}q\bar{q}$  systems [3], as results of a collaboration of poles and resonances in a  $K\bar{K}$  or  $\pi\pi$  amplitude [4], or as bound states of  $K\bar{K}$  [5]. In Ref. [6], these states are treated in a coupled channel formalism with  $K\bar{K}$  and  $\pi\pi$  components and in Ref. [7], the components are  $K\bar{K}$  and  $s\bar{s}$ . In both cases the mesons  $f_0$  and  $a_0$  appear bound, but the  $\pi\pi$  or  $s\bar{s}$  components in either case, although small, play an important role in the stability of the systems. Furthermore, the  $K\bar{K}$  is largely dominant in both cases.

In the work reported here, we take advantage of these findings and, using chiral perturbation theory, we evaluate the decay widths of these states into  $K\bar{K}$ ,  $\pi\pi$ ,  $\pi\eta$  and  $\gamma\gamma$ .

On the other hand, we also take advantage of the chiral Lagrangians involving mesons and nucleons and evaluate the cross sections of the different

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$K^+N \rightarrow KN\pi$  channels near threshold. We show here that the study of this reaction can serve better as a test of chiral perturbation theory than the analogous one  $\pi N \rightarrow \pi\pi N$ , since one does not have resonances in the  $K^+N$  system, while one has many  $\pi N$  resonances which contribute to the  $\pi N \rightarrow \pi\pi N$  reaction [8–12].

## 2. Chiral Lagrangians with mesons and baryons

We use the standard Chiral perturbation theory Lagrangians which contains the most general low-energy interactions of the pseudoscalar meson and the baryon octets of SU(3), consistent with the chiral symmetry properties of QCD [13–15]. At lowest order in derivatives and quarks masses it is given by

$$\mathcal{L}_{\text{ChPT}} = \mathcal{L}_2 + \mathcal{L}_1^{(B)} + \dots, \quad (1)$$

where  $\mathcal{L}_2, \mathcal{L}_1^{(B)}$  are the Lagrangians in the meson and baryon sectors. We have

$$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu U^\dagger \partial^\mu U + M(U + U^\dagger) \rangle, \quad (2)$$

where the symbol  $\langle \rangle$  denotes the flavour trace of the SU(3) matrices and  $f$  is the pion decay constant,  $f = 93$  MeV. The matrix  $U(\phi)$  is given by

$$U(\phi) = u(\phi)^2 = \exp \left( i\sqrt{2} \frac{\Phi}{f} \right) \quad (3)$$

$$\begin{aligned} \Phi(x) &\equiv \frac{\vec{\lambda} \cdot \vec{\phi}}{\sqrt{2}} = \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \end{aligned} \quad (4)$$

On the other hand  $M$  is proportional to the quark mass matrix and can be written in terms of the meson masses, in the limit of  $m_u = m_d$  as

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}. \quad (5)$$

One then expands  $\mathcal{L}_2$  in powers of the meson fields. In second order one obtains the standard free meson Lagrangians and to fourth order we find interaction terms which contribute to the  $M_1 M_2 \rightarrow M_3 M_4$  reactions. From there we can evaluate the amplitudes  $K^+ K^- \rightarrow K^+ K^-$ ,  $K^0 \bar{K}^0$ ,  $\pi^+ \pi^-$ ,  $\pi^0 \pi^0$ ,  $\pi^0 \eta$  and  $K^0 \bar{K}^0$  going to the same states.

### 3. Connection to the $f_0, a_0$ meson states

The  $f_0$  has a mass  $980 \pm 10 \text{ MeV}$  and quantum numbers  $T^G(J^{PC}) = 0^+(0^{++})$ . These are the same quantum numbers as a  $K\bar{K}$  state at threshold in  $T = 0$  isospin state and  $L = 0$ . On the other hand the  $a_0$  has  $M = 982 \pm 1.4 \text{ MeV}$  and quantum number  $1^-(0^{++})$ . These are the same quantum numbers of the  $K\bar{K}$  at threshold in  $T = 1$ ,  $L = 0$ . Furthermore,  $M_{K^+K^-} = 987 \text{ MeV}$ . All these things make it very appealing to assume that  $f_0, a_0$  are bound states of  $K\bar{K}$  with these quantum numbers. In such a case, we would have

$$\begin{aligned} f_0 \rightarrow |K\bar{K}, T=0\rangle &= \frac{1}{2}(K^+K^- + K^-K^+ + K^0\bar{K}^0 + \bar{K}^0K^0), \\ a_0 \rightarrow |K\bar{K}, T=1\rangle &= \frac{1}{2}(K^+K^- + K^-K^+ - K^0\bar{K}^0 - \bar{K}^0K^0). \end{aligned} \quad (6)$$

With these states, and assuming the kaons at threshold, we find

$$\begin{aligned} \langle K\bar{K}, T=0 | t | K\bar{K}, T=0 \rangle &= -\frac{6m_K^2}{f^2}, \\ \langle K\bar{K}, T=1 | t | K\bar{K}, T=1 \rangle &= -2\frac{m_K^2}{f^2}, \\ \langle K\bar{K}, T=0 | t | \pi\pi, T=0 \rangle &= \frac{6}{\sqrt{3}} \frac{m_K^2}{f^2}, \\ \langle K\bar{K}, T=1 | t | \pi\eta, T=0 \rangle &= -\frac{1}{\sqrt{6}} \frac{1}{3f^2} \{28m_K^2 - m_\pi^2 - 3m_\eta^2\}, \\ \langle K\bar{K}, T=0 | t | \pi\eta, T=0 \rangle &= 0, \\ \langle K\bar{K}, T=1 | t | \pi\pi, T=1 \rangle &= 0. \end{aligned} \quad (7)$$

where we have taken the  $\pi\pi$  and  $\pi\eta$  states symmetrized with respect to the exchange of the two mesons. The last two equations (7) come because of isospin conservation in the first case and because of the symmetry of the bosons in isospin formalism together with conservation of isospin in the last case. In addition to the decay of the  $a_0$  and  $f_0$  mesons into  $K\bar{K}, \pi\pi$  or  $\pi\eta$  we also study the decay into the  $\gamma\gamma$  system. The standard  $K^+K^- \rightarrow \gamma\gamma$  coupling for kaons at rest, and using the Coulomb gauge, contains only the diamagnetic term

$$t(K^+K^- \rightarrow \gamma\gamma) = 2e^2 \epsilon_\lambda^\dagger(k') \epsilon_{\lambda'}^\dagger(p'). \quad (8)$$

The  $K^0\bar{K}^0$  system does not couple directly to  $\gamma\gamma$ . It does it through loops of chiral perturbation theory [16] and we find

$$t(K^0\bar{K}^0 \rightarrow \gamma\gamma) = 2e^2 B \epsilon_\lambda^\dagger(k') \epsilon_{\lambda'}^\dagger(p') \quad (9)$$

with  $B = 0.07$  coming from both pionic and kaonic loops.

The partial decay widths of  $f_0$  and  $a_0$  are given by

$$\Gamma_i = \frac{|\varphi_\alpha^2(0)|^2}{4m_K^2} S_i \frac{1}{4\pi} \frac{k'}{M_\alpha} |t_i|^2 \quad (10)$$

with  $K'$  the momentum of one particle in the final state and  $S_i$  the symmetry factor,  $\frac{1}{2}$ , in all cases since the final states considered are all symmetrized. In Eq. (10),  $\varphi_\alpha(0)$  is the relative wave function of the  $K\bar{K}$  system at the origin.

For the case of the decay into  $K\bar{K}$  the formula of eq. (10) must be modified. Here, even if the  $K\bar{K}$  system is bound, the decay into  $K\bar{K}$  is allowed because of the width of the resonance. Thanks to this, a certain fraction of the mass distribution of the  $f_0$  or  $a_0$  states appears for masses bigger than  $2M_K$  and hence the decay is allowed.

We find

$$\Gamma_{K\bar{K}} = \frac{1}{2} |\varphi_\alpha(0)|^2 \frac{1}{8\pi^2} \int_{m_K}^{\infty} d\omega' \frac{k'}{\omega'} |t_i|^2 \frac{\Gamma_\alpha}{(M_\alpha - 2\omega')^2 + (\frac{\Gamma_\alpha}{2})^2}. \quad (11)$$

#### 4. Results for the partial widths

We take one piece of information from experiment, since  $\varphi_\alpha(0)$  is not known for us. Then, the rest of the magnitudes come as a prediction.

For instance, in the case of the  $f_0$  meson we take  $R_{f_0} = \Gamma_{K\bar{K}} / \Gamma_{\pi\pi}$  from experiment, and within the experimental uncertainties we find the results of Table I.

TABLE I

$\Gamma_{\text{tot}}$ [MeV]			
$M_{f_0}$	970	980	990 [MeV]
$R_{f_0}$			
0.24	40.1		
0.28	64.0	49.3	30.2
0.32	59.3		

to be compared with experiment which gives  $\Gamma_{\text{tot}} \simeq 50 \pm 15$  MeV [17]. On the other hand we find the value  $0.65 \cdot 10^{-5}$  for the ratio  $\Gamma_{\gamma\gamma} / \Gamma_{\text{tot}}$  compared to

the experimental value  $(1.19 \pm 0.33)10^{-5}$ . This value is somewhat lower than experiment, however, the absolute value which we find for  $\Gamma_{\gamma\gamma} = (0.32 \pm 0.10)$  MeV is compatible with most of the values quoted in Ref. [17]. In any case, as discussed in the talk of Metsch in this session [18], even a discrepancy of about a factor two must be considered a success since other models fail in orders of magnitude.

With respect to the  $a_0$  we take from experiment the value of  $\Gamma_{a_0}$  and find the results of Table II,

TABLE II

$$a_0 : A = \Gamma_{\eta\pi}\Gamma_{\gamma\gamma}/\Gamma_{a_0}; M_{a_0} = 982.7 \text{ MeV}$$

[MeV]

$\Gamma_{a_0}$	$\Gamma_{\pi\eta}$	$\Gamma_{K\bar{K}}$	$10^4\Gamma_{\gamma\gamma}$	$10^4A$
35	33.7	1.3	3.5	3.3
50	47.2	2.8	4.8	4.6
65	60.7	4.2	6.2	5.8

Experimentally we find [17]  $A = (2.4 \pm 0.8)10^{-4}$  MeV, which would be compatible with the results, using the lower value of  $\Gamma_{a_0}$ . One piece of information, however, disagrees with present data, the ratio  $\Gamma_{K\bar{K}}/\Gamma_{\pi\eta}$  for which there are two values given in [17],  $0.7 \pm 0.3$  and  $0.25 \pm 0.008$  contradictory among themselves but both of them bigger than the predictions from Table 2. Better data would be needed to clarify the present experimental situation.

### 5. The $K^+N \rightarrow KN\pi$ reaction

Here we need the term  $\mathcal{L}_1^{(B)}$  of Eq. (1) which is given by

$$\mathcal{L}_1^{(B)} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle + \frac{D+F}{2} \langle \bar{B} \gamma^\mu \gamma_5 u_\mu B \rangle + \frac{D-F}{2} \langle \bar{B} \gamma^\mu \gamma_5 B u_\mu \rangle, \quad (12)$$

where  $B$  is the SU(3) matrix of the octet of baryon states

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} \quad (13)$$

$$\nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B], \Gamma_\mu = \frac{1}{2}(u^+ \partial_\mu u + u \partial_\mu u^+),$$
$$u_\mu = iu^+ \partial_\mu U u^+. \tag{14}$$

With these ingredients we generate the sort of Feynman diagrams of Figs 1, 2, 3

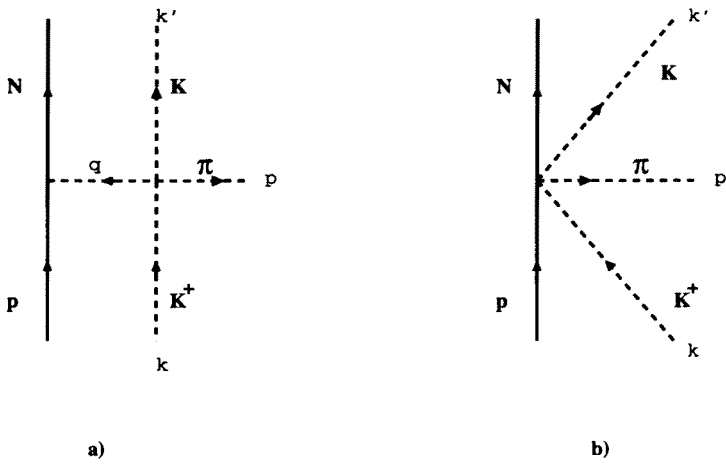


Fig. 1.

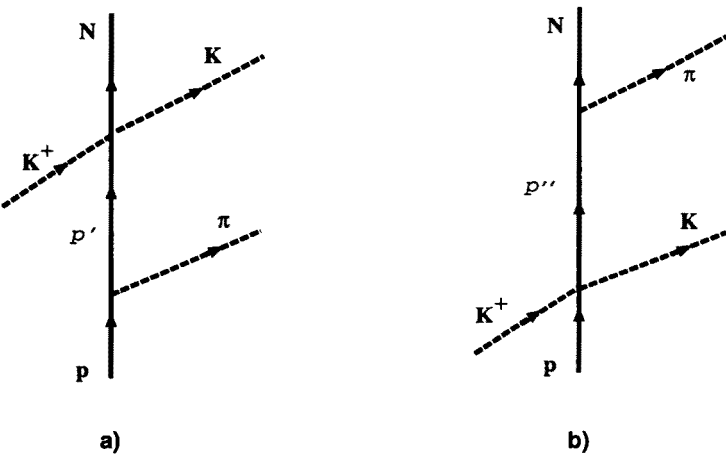


Fig. 2.

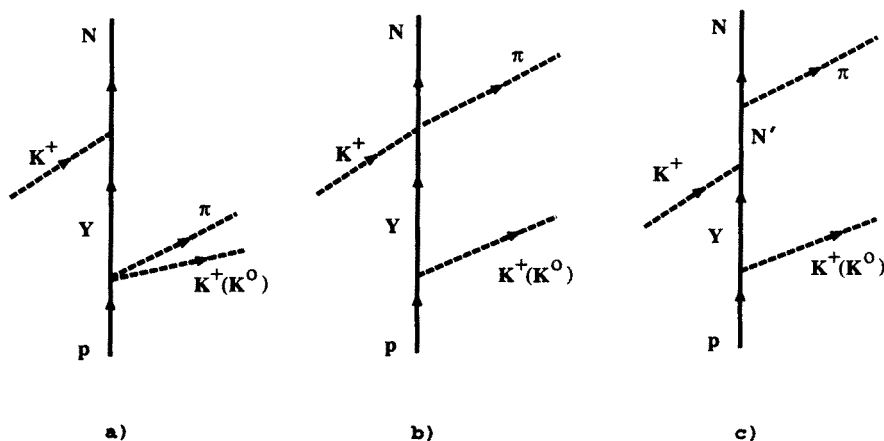


Fig. 3.

The terms in Fig. 3 give a negligible contribution (less than 10% contribution to  $\Sigma$  in all channels), but those of Fig. 1 and Fig. 2 are important and do not vanish at threshold.

However, even close to threshold there is a  $\Delta$  contribution coming from the term of Fig. 4

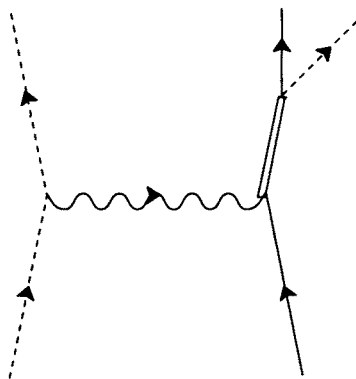


Fig. 4.

the diagram requires a new Lagrangian for the  $K^+K^- \rightarrow \rho$  coupling, which is given by

$$-i \delta H_{\rho^0 K^+ K^-} = -i \tilde{f}_\rho \epsilon_\rho^\mu [p_{K^+} - p_{K^-}]_\mu \quad (15)$$

(extra factor  $\sqrt{2}$  for charged  $\rho$ )

the coupling  $\tilde{f}_\rho$  is fitted to the data of  $\Delta$  excitation which are given in refs. [19, 20]. The value  $\tilde{f}_\rho = 4.2$  is obtained. For reference, the corresponding coupling for  $\rho\pi^+\pi^-$  is  $f_\rho = 6.1$ .

The  $\Delta$  term is important above  $p_{\text{lab}}(K^+) = 800$  MeV. Below this momentum the Chiral perturbation theory terms of Figs 1 and 2 are more important and dominate the amplitude.

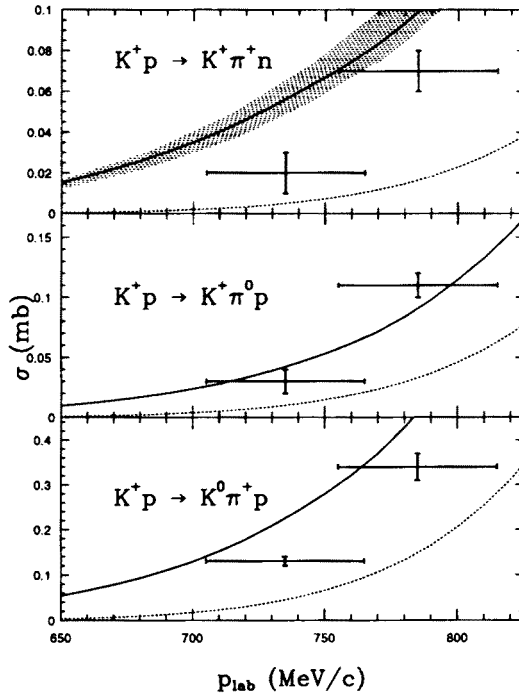


Fig. 5.

In Fig. 5 we show the results of the  $K^+p \rightarrow K^+\pi^+n$ ,  $K^+\pi^0p$ ,  $K^0\pi^+p$  reactions obtained with the present approach. The dotted lines correspond to the  $\Delta$  term contribution, while the solid line gives the total contribution. The data are from Refs. [19, 20]. The dashed area in the  $K^+p \rightarrow K^+\pi^+n$  results corresponds to accepting uncertainties for the  $K\pi \rightarrow K\pi$  amplitude as obtained in Ref. [21] when the  $K\pi$  amplitude is corrected to account for next to leading order contributions [21]. This gives us an idea of what kind of corrections we can expect if one goes to next to leading order in the  $K^+N \rightarrow K\pi N$  amplitudes. So far, as the figure shows, one obtains a qualitative agreement with the data in all the channels, but it would be very interesting to obtain new measurements with more precision and closer to threshold to allow for more quantitative tests of chiral perturbation theory.



## 6. Meson exchange currents with kaons

From Fig. 1, we can obtain exchange currents contributing to  $K^+$  nucleus scattering by producing the pion off shell and attaching it to a second nucleon. The same procedure with the diagrams of Fig. 2 leads to diagrams which are implicitly accounted for when one uses a wave function for the initial and final states, which incorporates the effect of pion exchange. The exchange currents obtained from the diagrams of Fig. 1 with the procedure discussed above have been evaluated in Ref. [22], where it was proved that they vanish exactly for  $T = 0$  nuclei in the forward direction. The relevance of such terms for  $T \neq 0$  nuclei and non forward angles is the object of present investigation in Ref. [23].

## 7. Conclusions

- a) We find a qualitative agreement for the partial decay widths of the  $f_0, a_0$  mesons assuming them to be bound states dominated by the  $K\bar{K}$  components and using Chiral perturbation theory in lowest order with only one unknown quantity for each meson. The results obtained for the decay into the  $K\bar{K}, \pi\pi, \pi\eta$  and  $\gamma\gamma$  channels give support to the basic  $K\bar{K}$  nature of the  $a_0, f_0$  mesons.
- b) The agreement with the data of the  $K^+N \rightarrow KN\pi$  channels is fair. This reaction should be a good testing ground of Chiral perturbation theory when more precise data are available. Effects from higher orders of ChPT seem moderate.
- c) The model constructed for the  $K^+N \rightarrow KN\pi$  reaction is sufficiently accurate to be used to generate meson exchange currents in the  $K^+A$  reaction, a field which is just beginning to be explored.

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