DYNAMICS OF HEAVY MESONS IN THE INSTANT AND THE FRONT FORM*

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We investigate the dynamics of bound $\overline{q}q$ systems in a covariant quasi-potential framework. In the spirit of the heavy baryon effective theory we restrict heavy quarks as spectators on their mass shell and evaluate multigluon exchange contributions up to α_S^3. We suggest a systematic extension of the spectator picture to massless particles (light quarks and gluons) and discuss the connection between the instant and the front form in the limit of heavy quarks.

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Though Quantum chromodynamics (QCD) is well accepted as the fundamental theory for strong interactions, attempts for its solution for bound hadronic systems so far are — except for lattice calculations – restricted to rather phenomenological quark-gluon models, which incorporate the general features of QCD — asymptotic freedom and confinement — via the one-gluon exchange (OGE), supplemented by a phenomenological confining potential [1]. Restricting ourselves to $q\overline{q}$ states as the simplest hadrons, there exists, in principle at least, a rigorous, covariant approach via a selfconsistent solution of the corresponding $q\overline{q}$ Bethe–Salpeter (BSE) and Schwinger–Dyson equations, given the full interaction kernel as the sum of all irreducible quark and gluon exchange contributions.

In practice, however, such a program is presently far from being achieved for mesonic systems in general. The only promising exception — in the spirit of recent progress in the Heavy Quark Effective Theory (HQETh) — are the investigation of heavy mesons, *i.e.* $q\overline{q}$ systems, which involve at least

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one heavy quark. Heavy constituents not only quite naturally reduce in the spectator limit the 4-dimensional BSE to a technically simpler 3-dimensional Quasi-Potential equation (QPE), but also provide better convergence in the expansion of the irreducible kernel in powers of the running coupling constant $\alpha_S(q^2)$.

In this note, starting from the BS amplitude

$$\psi(P,p) = \phi(P,p)\chi_{S,F,C}(q,\overline{q}),$$

(where χ contains the spin, flavour and colour content of the meson) we investigate for

$$\phi(P,p) = G_{BS}(P,p)\int K(P,p,k)\phi(P,k)dk$$

the expansion of the kernel K up to $\alpha_S^3(q^2)$ for heavy-light $q\overline{q}$ systems (Fig. 1) (an extention to mesons with two heavy constituents follows conceptionally the same line).

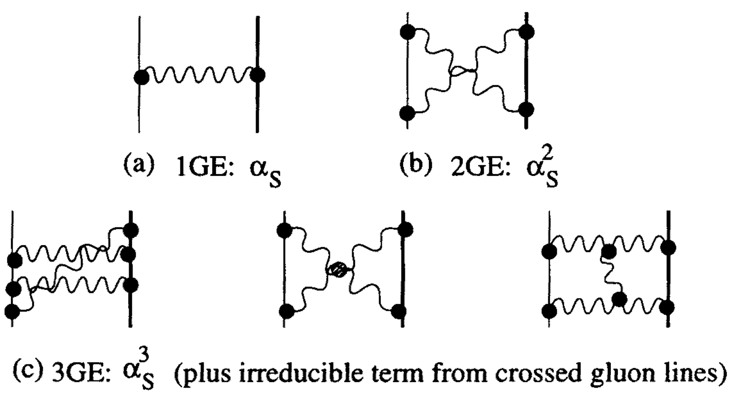

Fig. 1. Leading gluon-exchange contributions to the BS kernel

In detail we perform the following steps:

• We restrict in the spectator limit the heavy (anti)quark to its mass shell, to reduce for $m/M \to 0$ the BS propagator [2]

$$G_{BS}(P,p) = \left[\Big((\eta P+p)^2 - m^2 + i\varepsilon\Big)\Big(((1-\eta)P-p)^2 - M^2 + i\varepsilon\Big)\right]^{-1}$$

to the QP propagator

$$G_{QP}(P,p) = -\frac{i\pi}{2M}\frac{\delta(p_0-\varepsilon)}{\varepsilon^2-(\boldsymbol{p}^2+m^2)}$$

with ε related to bound state energy $\varepsilon = E - M$.

• We define the 3-dimentional QP amplitude

$$\phi(\varepsilon, \boldsymbol{p}) = \int \phi(P, p)\, dp_0$$

and the QPE

$$\Big(\varepsilon^2 - (\boldsymbol{p}^2 + m^2)\Big)\phi(\varepsilon, \boldsymbol{p}) = \int V(\boldsymbol{p}, \boldsymbol{k}, \varepsilon)\, \phi(\varepsilon, \boldsymbol{k})\, d\boldsymbol{k}$$

with a – due to its energy dependence – strictly nonlocal QP kernel $V(\boldsymbol{p}, \boldsymbol{k}, \varepsilon)$, given according to the diagramatic expansion in Fig. 1 as

$$V(\boldsymbol{p}, \boldsymbol{k}, \varepsilon) = \sum_n \alpha_S^n V_n(\boldsymbol{p}, \boldsymbol{k}, \varepsilon)\,.$$

• We calculate the 1GE upon decoupling the spin of the heavy quark in the heavy mass limit. With the running coupling constant parametrized as [1]

$$\alpha_{\mathrm{S}}(\boldsymbol{q}) = \frac{\alpha_{\mathrm{S}}(q_0^2)}{\ln(1 + \boldsymbol{q}^2/\Lambda_{\mathrm{QCD}}^2)};$$

we obtain in the Feynman gauge, upon transition to coordinate space

$$V_{1\mathrm{GE}}(r) = \frac{2}{3}\,\alpha_{\mathrm{S}}(q_0^2)\ln\Big(1 + \frac{q_0^2}{\Lambda_{\mathrm{QCD}}^2}\Big)\Big(-\frac{1}{r} + \Lambda_{\mathrm{QCD}}^2 r + f(r)\Big)$$

with $\boldsymbol{r} = \boldsymbol{r}_q - \boldsymbol{r}_{\overline{q}}$, which exhibits the pure one-gluon exchange a linear confinement term; smooth radial corrections are contained in $f(r)$.

• Along the same line we derive the irreducible two-gluon exchange and, corresponding to the first diagram in Fig. 1(c), the three-gluon exchange contribution.

Without detailing the full structure of the multi-gluon exchanges explicitly, it is evident that the full potential involves nonlocalities

$$V_{n\mathrm{GE}}(r, r') \sim \frac{\mathrm{e}^{-m|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

from the propagation of the light quark. Only in the limit $m^2 \gg k_{\mathrm{loop}}^2$ for the dominant loop momenta k_{loop} we obtain the local limit

$$V_{n\mathrm{GE}}(r) = c_n \frac{V_{1\mathrm{GE}}^n(r)}{m^{n-1}}$$

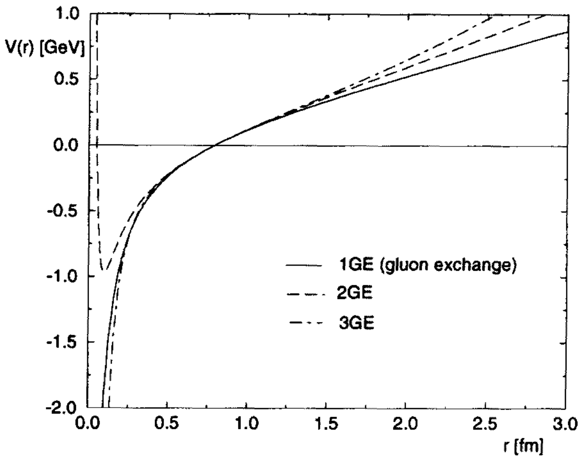

Fig. 2. Radial dependence of the localized heavy-light $q\bar{q}$ potential up to $\alpha_S^3$for the parameters $\alpha_S(q_0^2) = 0.20$ and $\Lambda_{\rm QCD} = 250$MeV [3].

with the coefficients $c_2 = \pi/4$ and $c_3 = 3\pi^2/8$. The radial dependence of the 1, 2 and 3-gluon exchange potential for a typical set of parameters is shown for a constituent quark mass $m = 300$ MeV in Fig. 2.

We extend the approach above in two directions:

• Evidently, the second and third diagram in Fig. 1(c) involve loops which contain only light quarks or the gluons, and thus still exhibit the full nonstatic energy dependence. We explore the structure of these contributions by pursuing the spectator picture to its limits in the spirit of the Blankenbecler-Sugar [4]: we restrict each pair of the interacting light particle symmetrically off their mass shell, *i.e.*for the n-particle propagator

$$G_n = \frac{1}{(D_1 + i\varepsilon)\dots(D_n + i\varepsilon)} \equiv (-i\pi)^{n-1} \frac{\prod_{k=2}^{n} \delta(D_1 - D_k)}{D_1} .$$

Thus we not only restrict the relative energies but also the momenta of the interacting particles.

• Though all derivations above are given in the instant form, they are easily extended to the front (or light cone) form in the heavy quark limit. The key relation is that for $P = (E^2, 1, \mathbf{0})$ and $p = (p_-, x, \boldsymbol{p}_\perp)$ the BS propagator

$$\begin{aligned} G_{\rm BS}(P,p) = \Big[\Big((\eta + x)(\eta E^2 + p_-) - (m^2 + \boldsymbol{p}_\perp^2) + i\varepsilon \Big) \\ \times \Big((1 - \eta - x)((1-\eta)E^2 + p_-) - (M^2 + \boldsymbol{p}_\perp^2) + i\varepsilon \Big) \Big]^{-1} \end{aligned}$$

reduces in the spectator limit to the QP propagator

$$G_{\rm QP}(M,\varepsilon,x,\boldsymbol{p}_\perp) = -\frac{i\pi\delta\Big(p_- + M^2 - m^2 - (1-x-2\eta)E^2\Big)}{x(1-x)E^2 - (1-x)(m^2+\boldsymbol{p}_\perp^2) - x(M^2+\boldsymbol{p}_\perp^2)}\,.$$

As an example, we establish the connection with the instant form in the heavy mass limit for the one-gluon exchange: with the LC projection above we find

$$G_{\rm gluon}((p-k)^2) = -\Big[M^2(x-y)^2 + (\boldsymbol{p}_\perp - \boldsymbol{k}_\perp)^2 + i\varepsilon\Big]^{-1}\,,$$

which reduces in the scaled variables $p_z = M\Big(x - \frac{1}{2}\Big)$, $k_z = M\Big(y - \frac{1}{2}\Big)$ to

$$G_{\rm gluon}((p-k)^2) = -\Big[(p_z - k_z)^2 + (\boldsymbol{p}_\perp - \boldsymbol{k}_\perp)^2 + i\varepsilon\Big]^{-1}$$

with $-\frac{M}{2} \leq p_z \leq \frac{M}{2}$. Thus in the limit $M \to \infty$ we recover the standard gluon propagator. The full gluon-exchange contributions on the light cone as well as corrections to the extreme heavy mass limit are then easily derived. (A detailed presentation of the results above is in preparation [5].)

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REFERENCES

[1] W. Lucha, F.F. Schobert, D. Gromes, *Phys. Rep.* **C200**, 127 (1991).
[2] F. Gross, *Relativistic Quantum Mechanics and Field Theory*, John Wiley & Sons Inc., New York 1993.
[3] S. Bethke, Proc. XXVI Int. Conf. on High Energy Physics, Dallas 1992.
[4] R. Blankenbecler, R. Sugar, *Phys. Rev.* **142**, 1051 (1966).
[5] A.A. Bel'kov, M. Dillig, F. Pilotto, (in preparation); M. Schmidt, Thesis, Univ. Erlangen 1996.