

# LIGHTCONE QUARK MODEL STUDIES OF MESON STRUCTURE\*

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Two main fundamental issues in contrasting the lightcone field quantization to the canonical equal time quantization are the change of the boost problem to the rotation problem and realization of nontrivial vacuum phenomena on the lightcone. Motivated by the outcome of these issues, a lightcone quark model is constructed and various meson structure properties are calculated. The model predictions are compared with the experimental data including the meson radiative decay widths.

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The distinguished features in the lightcone approach are the dynamical property of the rotation operators [1] and the simplicity of the vacuum except the zero modes [2]. Motivated by these outcomes, we construct a simple lightcone quark model to calculate various meson properties. In our point of view, the complicated nontrivial vacuum properties can be replaced by the constituent quark masses in this model. It is quite amusing to notice that the recent lattice QCD results indicate that the topological charge contribution to the flavor singlet axial vector current can be traded off by the constituent quark masses [3]. Thus, the key approximation in this approach is the mock-hadron approximation [4] to truncate the expansion by retaining only the lowest Fock state and treat the lowest Fock state as a free state as far as the spin-orbit part is concerned while the radial part is given by the ground state of the harmonic oscillator wavefunction. Then, the assignment of the quantum numbers such as angular momentum, parity and charge conjugation can be given to the light-cone wavefunctions by the Melosh transformation [5]. This process is necessary due to the fact that the light-cone time  $\tau$  changes under the rotation except the  $z$ -axis and the space inversion making the corresponding transverse rotation operator [1]

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and the parity operator [6] be dynamical and change the number of quanta inside the hadron. For example, the meson state  $|M\rangle$  is represented by

$$|M\rangle = \psi_{q\bar{q}}^M |q\bar{q}\rangle, \quad (1)$$

where  $|q\bar{q}\rangle$  is the two-body Fock state for a quark  $q$  and an antiquark  $\bar{q}$ . The model wavefunction is given by

$$\psi_{q\bar{q}}^M = \psi_M(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i}) \chi_M(x_i, \vec{k}_{\perp i}, \lambda_i), \quad (2)$$

where the radial wavefunction is given by

$$\Phi(x_i, \vec{k}_{\perp i}) = A \exp \left[ - \sum_{i=1}^2 \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} / 8\beta^2 \right] \quad (3)$$

and the spin-orbit wavefunction  $\chi_M(x_i, \vec{k}_{\perp i}, \lambda_i)$  is obtained by the interaction independent Melosh transformation [5] from the ordinary equal-time static spin-orbit wavefunction assigned by the quantum numbers  $J^{PC}$ . These wavefunctions represented by of the Lorentz-invariant variables  $x_i = p_i^+ / p^+$ ,  $\vec{k}_{\perp i} = \vec{p}_{\perp i} - x_i \vec{p}_{\perp}$ , and  $\lambda_i$ , where  $p^\mu = (p^+, p^-, \vec{p}_{\perp}) = (p^0 + p^3, m_M^2 + \vec{p}_{\perp}^2) / p^+$ ,  $\vec{p}_{\perp}$  is the momentum of the meson  $M$ , and  $p_i^\mu$  and  $\lambda_i$  are the momentum and the helicity of constituent quarks, respectively. Then, the light-cone spin-orbit wavefunction corresponding to the specific quantum number  $J^{PC}$  is given by

$$\chi_M = \chi_M(x_i, \vec{k}_{\perp i}, \lambda_i) = \bar{u}_{\lambda_1} \Gamma_{M,\mu} v_{\lambda_2}, \quad (4)$$

where the operators  $\Gamma_{M,\mu}$  are given by

$$\begin{aligned} J^{PC} = 0^{-+} ; \Gamma_p &= (M_p + \not{p}) \gamma_5 \\ 1^{--} ; \Gamma_{v,\mu} &= M_v \not{\epsilon}(\mu) + \frac{[\not{p}, \not{\epsilon}(\mu)]}{2} \\ 1^{++} ; \Gamma_{a,\mu} &= (M_a + \not{p}) \left[ \frac{k \cdot p}{M_a} \not{\epsilon}(\mu) + \frac{[\not{p}, \not{\epsilon}(\mu)]}{2} \right] \gamma_5. \end{aligned} \quad (5)$$

Except the well-known constituent quark masses of (u,d,s) quarks and the spin-averaged meson masses, the only parameter in the model is the gaussian parameter  $\beta$  which determines the broadness( or sharpness) of radial wavefunction. Our model provided a remarkably good description of static properties for the pion and K-mesons and reproduces the basic features of the QCD sum-rule results for  $\pi$ , K,  $\rho$  and  $a_1$  mesons. Detailed numerical results are shown in Refs. [7] and [8]. Very recently [9], we have also investigated the radiative decays of pseudoscalar( $\pi, K, \eta, \eta'$ ),

vector( $\rho, K^*, \omega, \phi$ ) and axial vector( $A_1$ ) mesons using a simple relativistic constituent quark model. The computed decay widths with  $\beta = 0.32$  GeV are as follows;  $\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma) = 63$  keV,  $\Gamma(\omega \rightarrow \pi \gamma) = 682$  keV,  $\Gamma(K^{*\pm} \rightarrow K^\pm \gamma) = 54$  keV,  $\Gamma(K^{*0} \rightarrow K^0 \gamma) = 126$  keV,  $\Gamma(\rho \rightarrow \eta \gamma) = 62$  keV,  $\Gamma(\omega \rightarrow \eta \gamma) = 9.1$  keV,  $\Gamma(\eta' \rightarrow \rho \gamma) = 60$  keV,  $\Gamma(\eta' \rightarrow \omega \gamma) = 4.3$  keV,  $\Gamma(\phi \rightarrow \eta \gamma) = 51$  keV,  $\Gamma(\phi \rightarrow \eta' \gamma) = 0.11$  keV,  $\Gamma(A_1^+ \rightarrow \pi^+ \gamma) = 695$  keV,  $\Gamma(\pi^0 \rightarrow \gamma \gamma) = 7.68$  eV,  $\Gamma(\eta \rightarrow \gamma \gamma) = 0.32$  keV,  $\Gamma(\eta' \rightarrow \gamma \gamma) = 2.3$  keV. We also calculated the transition form factors of  $\rho, \omega \rightarrow \pi(\eta) \gamma^*$ ,  $K^* \rightarrow K \gamma^*$  and  $A_1 \rightarrow \pi \gamma^*$  at  $0 \leq Q^2 \leq 5$  GeV<sup>2</sup> and  $\pi^0(\eta) \rightarrow \gamma^* \gamma$  at  $0 \leq Q^2 \leq 3$  GeV<sup>2</sup>. Our predictions of vector and pseudoscalar radiative decay processes are in a remarkably good agreement with the experimental data and the result for  $A_1^+ \rightarrow \pi^+ \gamma^*$  transition is quite consistent with the experiments of pion scattering on a nucleus using Primakoff effect.

Thus, the model presented here has a predictive power and more experimental data should be compared with this model. We conclude that the lightcone quark model presented here should be distinguished from the so called naive quark model. The challenge of the lightcone field quantization to derive the quark model from QCD is still left for further investigations.

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