SCALING AND SCALE BREAKING PHENOMENA IN QCD JETS*

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The perturbative QCD approach to multiparticle production assuming Local Parton Hadron Duality (LPHD) and some recent results are discussed. Finite asymptotic scaling limits are obtained for various observables, after an appropriate rescaling, in the Double Logarithmic Approximation (DLA). Non-asymptotic corrections are also known in some cases. The DLA applies also to very soft particle production where energy conservation constraints can be neglected. In this region the particle density follows rather well a scaling behaviour over the full energy range explored so far in e^+e^- annihilation.

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1. Introduction

The study of the intrinsic structure of particle jets produced in hard collisions continues to be an active field of research. The interest is directed in elaborating and testing specific predictions of perturbative QCD on the parton cascade evolution and secondly, to investigate the hadronization process which cannot be treated within a perturbative scheme. A reduction of the flexibility of the models involved and a deeper understanding of the phenomenological aspects of the confinement process is an important aim of this research.

The most popular models for particle production in hard collision processes are based on a primary hard partonic sub-process which is accompanied by gluon bremsstrahlung. The evolution of the partonic jets is derived

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in perturbation theory and is terminated at a scale of around 1 GeV; thereafter, non-perturbative processes take over and the final hadronic particles, often through intermediate resonances, are produced, for example, by a string mechanism [1] or through cluster formation [2].

Another approach is based on the concept of "Local Parton Hadron Duality" (LPHD) [3]. It has been observed at first that the hadronic energy spectra are rather well represented by the parton spectra themselves — without an additional hadronization phase — provided the cut-off of the parton cascade is lowered to a value around 250 MeV, of the order of the hadronic masses. This general idea has been applied to various other observables; the theoretical calculations are based in the simplest case on the Double Log Approximation (DLA) [4, 5], which provides the high energy limit, or the Modified Leading Log Approximation (MLLA) [6] which includes finite energy corrections which are usually essential to obtain quantitative agreement with experiment at present energies [7]. Recent experimental results from LEP, HERA and TEVATRON gave further support to this approach [8]. Although a justification of the model is not yet available at a fundamental level the related phenomenology is quite attractive because of its intrinsic simplicity with very few parameters. Also the analytical computations allow the derivation of scaling laws and the systematics of their violation which provides an important insight into the structure of the theory. On the other hand, it is clear that this model cannot compete with the standard hadronization models in the description of the various details of the final state like the production of different particle species or resonances. It has so far been applied successfully for suitably averaged quantities.

In this presentation we summarize, how scaling and scale breaking predictions obtained from analytical calculations compare with experiment.

2. Basic ingredients of analytical calculations

We consider high energy collisions which involve a hard subprocess. The colour charges of the primarily produced partons are the sources of subsequent gluon bremsstrahlung which leads to the partonic jets. The subprocess is described by the corresponding matrix element. The gluon bremsstrahlung at small angles δ with energy E off the primary hard parton of type A $(A = q, g)$ with momentum P is given by

$$dn_A = \frac{C_A}{N_C}\gamma_0^2(k_T)\frac{d\delta}{\delta}\frac{dE}{E}, \quad \gamma_0^2(k_T) = \frac{2N_C\alpha_s(k_T)}{\pi} = \frac{\beta^2}{\ln(k_T/\Lambda)}, \quad k_T \geq Q_0,$$
$$\tag{1}$$

where $k_T \approx E\delta$, $\beta^2 = 4N_C/b$, $b \equiv (11N_C - 2n_f)/3$ with N_C, n_f the numbers of colours and flavours, also $C_g = N_C$, $C_q = 4/3$. Inside the cascade the soft

gluons are coherently produced from all harder partons. For azimuthally averaged quantities the consequences of the coherence effect can be taken into account by the angular ordering prescription [9] which requires the angles of subsequent gluon emissions to be in decreasing order.

The multiparticle properties of the jet can be discussed conveniently by using the generating functional [10] $Z_A(P, \Theta; u(k))$. Here P and Θ denote the initial parton momentum and opening angle of the jet, and $u(k)$ is a profile function for particle momentum k. The functional is constructed from all the exclusive final states. Then the inclusive densities can be obtained by functional differentiation with respect to the profile function $u(k)$

$$\rho^{(n)}(k_1, ..., k_n) = \delta^n Z\{u\}/\delta u(k_1)...\delta u(k_n)\,|_{u=1} \ . \tag{2}$$

Properties of these densities can be obtained from the evolution equation for Z which relates the functional at scales P, Θ to the one at lower scales according to the "decay" $A \to BC$. In MLLA accuracy this evolution equation is given by [6]

$$\frac{d}{d \ln \Theta} Z_A(P, \Theta) = \frac{1}{2} \sum_{B,C} \int_0^1 dz \, \frac{\alpha_s(k_\perp^2)}{2\pi} \, \Phi_A^{BC}(z)$$

$$\times \left[Z_B(zP, \Theta) Z_C((1-z)P, \Theta) - Z_A(P, \Theta) \right], \tag{3}$$

where $\Phi_A^{BC}(z)$ denotes the DGLAP splitting functions. The initial condition of the evolution is given by

$$Z_A(P, \Theta; \{u\})|_{P\Theta = Q_0} = u_A(k = P), \tag{4}$$

i.e. at threshold there is only the primary parton.

These equations take into account energy conservation by choosing the proper arguments of Z. One can obtain for various observables O analytic solutions which correspond to the summation of the perturbative series in leading double logarithmic order, *i.e.* the summation of the terms $\alpha_s^n L^{2n}$ with a large logarithm L. Also results with resummed next-to-leading order terms $\alpha_s^n L^{2n-1}$ are available in some cases. At high energies $O \sim \exp \int^Y \gamma(\alpha_s(y)) \, dy$ where the anomalous dimension γ has the expansion $\gamma \sim \sqrt{\alpha_s} + \alpha_s + \ldots$ The leading term is referred to as the DLA, the next-to-leading one as the MLLA result. The evolution equation (3) yields the complete results for the first two terms in this $\sqrt{\alpha_s}$ expansion. These leading terms are not sufficient, however, to satisfy the initial condition (4), this is only possible by using the full result from the summed perturbative series. From (3) one can obtain the evolution equations for particle densities by appropriate differentiation (2). Therefore this equation is the basic tool for deriving the multiparticle properties of a jet analytically.

For very high energies the small z contributions dominate (fixed scales Q_0 and Λ), then one can neglect the recoil effects and approximate $1 - z \approx 1$ in the argument of Z in (3). Furthermore, in the high energy limit it is sufficient to include the most singular terms of the splitting functions $\Phi_A^{Ag} \sim 1/z$, in particular, one can neglect the production of quark pairs in the cascade with nonsingular splitting function. In this case Eq. (3) simplifies and can be integrated using the initial condition (4) to

$$Z_A(P, \Theta, u) = u(P) \exp \left(\int_\Gamma dn_A [u(E) Z_g(E, \delta) - 1] \right) \qquad (5)$$

with integration measure from (1) and boundary Γ which takes into account the angular ordering constraint $\delta < \Theta$ and the k_T cutoff $E\delta > Q_0$. This is the evolution equation in DLA accuracy appropriate for the high energy asymptotics.

For the energy spectra [11] and a large class of angular correlations [12] one can derive from (5) by functional differentiation and appropriate partial integration an evolution equation of the type

$$h_n(\delta, \Theta, P) = d_n(\delta, P) + \int \frac{dK}{K} \int \frac{d\psi}{\psi} \, \gamma_0^2 \, h_n(\delta, \psi, K) \,, \qquad (6)$$

where h_n denotes generically one such distribution or correlation of order n and d_n the appropriate initial condition. The singularities in the kernel are regularized by the k_T cut-off. A nonsingular evolution equation is obtained by changing to logarithmic momentum and angular variables.

3. High energy asymptotics

The high energy behaviour can be obtained from (5). As will be shown in several cases, the observable quantities, after appropriate rescaling, approach a finite scaling limit.

3.1. Multiplicity distribution

A well known example of such behaviour is the "KNO-scaling" [13, 14] of the multiplicity distribution

$$\langle n \rangle P_n(s) = f(\psi), \qquad \psi = n/\langle n(s) \rangle. \qquad (7)$$

Here $f(\psi)$ is the high energy limit of the probability P_n to produce n particles at cms energy $\sqrt{s}$, rescaled by the average multiplicity $\langle n \rangle$. This scaling law has been derived in the DLA for the partons of QCD [16] but

holds more generally for a large class of branching processes [14, 15]. Specifically for QCD one obtains also an explicit prediction for the function $f(\psi)$ or the normalized factorial moments $F^{(k)} = \langle n(n-1)...(n-k+1) \rangle / \langle n \rangle^k$ of the multiplicity distribution. For example, for e^+e^- annihilation one finds $F^{(2)} = \frac{11}{8}$. This prediction is infrared safe, *i.e.* independent of the cutoff Q_0. Furthermore QCD predicts the approach to the asymptotic limit, so for the same quantity one obtains with inclusion of the next-to-leading correction (MLLA) [17]

$$F^{(2)} = \frac{11}{8}(1 - \frac{4255}{1782\sqrt{6\pi}}\sqrt{\alpha_s}) \tag{8}$$

(for $n_f = 5$) which turns out to be large (about 30%); if yet higher order corrections are included the result fits the experimental data [18]. The energy dependence of $F^{(2)}$ in (8) is very weak ($\sim 1/\sqrt{\ln s}$) thus simulating the scaling behaviour observed at present energies. The ultimate KNO scaling function according to (7) or (8) is broader and will be approached only at much higher energies than available today and in the near future.

3.2. Momentum spectra

Spectra in the rescaled Bjorken or Feynman momentum variables $x = p/P$ do not scale in QCD. Rather the distributions of certain rescaled logarithmic variables approach a finite asymptotic limit in the DLA. Such scaling properties have been discussed recently in some detail for angular correlations [12] (see below). For the energy spectra a scaling limit of this type has been suggested already some time ago [11] and one obtains in logarithmic variables after rescaling

$$\frac{\ln dn/d\xi}{\ln\langle n \rangle} = f(\zeta), \qquad \zeta = \frac{\xi}{Y+\lambda}, \tag{9}$$

where $\xi = \ln(P/E) = \ln(1/x)$ for a particle of energy E in a jet with primary parton momentum P, $Y = \ln(P\Theta/Q_0)$ and $\lambda = \ln(Q_0/\Lambda)$. The function $f(\zeta)$ has an approximately Gaussian shape, the so-called "humpbacked plateau" [4, 19]. Again, the approach to this limit is rather slow, for example, the maximum of the spectrum occurs at (for $Q_0 \approx \Lambda$)

$$\xi^* = Y(\frac{1}{2} + \sqrt{\frac{c}{Y}} - \frac{c}{Y}) + 0.1 \tag{10}$$

where $c = 0.2915$ for $n_f = 3$. The leading DLA term gives the asymptotic limit $\zeta^* \to \frac{1}{2}$, the next two terms the high energy corrections [20], the last term a numerical estimate of the remaining contributions applicable

in the present energy range [21]. According to the LPHD hypothesis this prediction at the parton level can be compared directly to the hadronic observable. The rescaled quantity $\zeta^* = \xi^*/Y$ for charged hadrons is shown in Fig. 1 up to LEP-1.5 energies in comparison with the prediction (10). The cut-off parameter $Q_0 = 0.270$ GeV is taken from a global fit to the moments of the distribution from *cms* energies 3 to 91 GeV in e^+e^- annihilation [22]. The data in Fig. 1 closely follow the MLLA prediction (10) which very slowly approaches the asymptotic DLA limit $\zeta^* = \frac{1}{2}$.
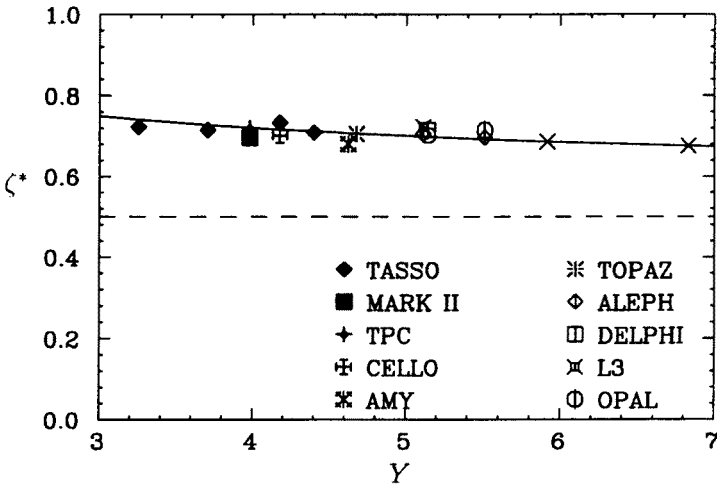

Fig. 1. Maximum of the rescaled inclusive momentum distribution $\zeta^* = \xi^*/Y$ as a function of $Y = \ln \frac{\sqrt{s}}{2\Lambda}$; comparison between experimental data from e^+e^- annihilation and theoretical prediction in MLLA numerically extracted from the shape of the Limiting Spectrum (solid line) for the cut-off parameter $Q_0 = \Lambda = 270$ MeV. Crosses mark the predictions at the *cms* energies 200 GeV and 500 GeV. Asymptotically, the leading DLA result $\zeta^* = \frac{1}{2}$ is approached (see [21, 23]).

3.3. Angular correlations

There was a lot of interest in the last years in the study and interpretation of angular correlations [24], which was triggered by the suggestion [25], such correlations could be power behaved at high resolution ("intermittency"). Such power behaviour is expected, for example, for selfsimilar branching processes, so it applies to QCD to the extent that the running of the coupling is neglected [28].

The observables which are considered in the analytical QCD calculations [12, 26, 27] are the two particle correlation density between two particles

$\rho^{(2)}(\vartheta_{12}, \Theta)$ in the forward cone of half angle Θ, the factorial or cumulant multiplicity moments $F^{(n)}$ and $C^{(n)}$ for particles between two cones at angles $\Theta - \delta$ and $\Theta + \delta$ around the jet axis or in a cone of angular size δ in direction Θ with respect to the jet axis. According to the volume of phase space δ^D these two configurations are referred to by their dimensions $D = 1$ und $D = 2$. One finds the results on these correlations by first deriving the integral equation of the n-particle correlation function from (5) and (2), and then integrates over the remaining variables; the resulting evolution equation is of the type (6) which can be solved approximately for running α_s (for fixed α_s one can get often exact results).

The correlation functions of order n are conventionally normalized by a power of the multiplicity $\langle n \rangle \sim \exp(2\beta\sqrt{\ln(P\Theta/\Lambda)})$. These normalized correlations, after removal of certain known kinematic phase space factors follow, after rescaling, the asymptotic angular scaling law [12]

$$\frac{\ln H^{(n)}(\delta, \Theta, P)}{2n\beta\sqrt{\ln P\Theta/\Lambda}} \to (1 - \omega(\varepsilon, n)/n), \qquad \varepsilon = \frac{\ln\frac{\Theta}{\delta}}{\ln\frac{P\Theta}{\Lambda}} \tag{11}$$

in the rescaled logarithmic variable ε with $0 \lesssim \varepsilon \lesssim 1$. So the rescaled observables do not depend on the variables δ, Θ and P separately but only through the variable ε. The normalization of the l.h.s of (11) corresponds to $(\ln\langle n \rangle)^n$. The function ω is known analytically, for large n one finds

$$\omega(\varepsilon, n) = n\sqrt{1 - \varepsilon}(1 - \frac{1}{2n^2}\ln(1 - \varepsilon) + \ldots) \tag{12}$$

which turns out to be a good approximation already for $n = 2$.

An interesting feature of these results is their universality, *i.e.* the same limit is obtained for quite different observables: the correlations $\hat{r}(\vartheta_{12}) = \rho^{(2)}(\vartheta_{12}, \Theta)/\langle n(\Theta) \rangle^2$ and the normalized moments of any order in one or two dimensions. These correlation functions refer actually to particles in quite different regions of phase space. As an example we show in Fig. 2 the rescaled normalized two particle density $\hat{r}(\vartheta_{12}, \Theta)$ as obtained from the DELPHI collaboration [29] which is rather well approximated by the asymptotic prediction from the DLA. The data are in good agreement with the Monte Carlo calculations at the same energy at either parton or hadron level. Monte Carlo results at a much higher energy show a similar behaviour in agreement with the scaling prediction (11). An observable which projects out the genuine 2-particle correlations more effectively from the uncorrelated background is the "correlation integral" [30, 12] $r(\vartheta_{12}) = \rho^{(2)}(\vartheta_{12})/\rho^{(2)}_{\text{norm}}(\vartheta_{12})$ where the normalization corresponds to the density of relative angles ϑ_{12} of particles from different jets. This quantity has been measured as well [29] and the predicted angular scaling law (11) for $r(\vartheta_{12})$ has been verified for different jet opening angles Θ.
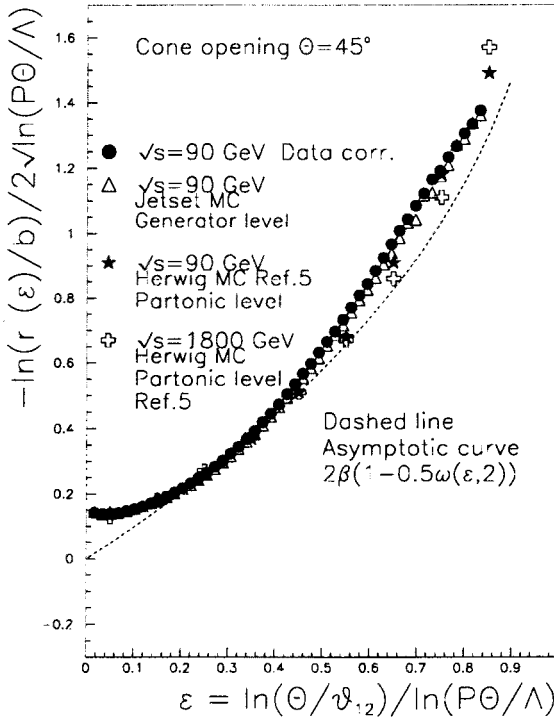
Fig. 2. The rescaled 2-particle angular correlation $\hat{r} = \rho^{(2)}(\vartheta_{12})/\langle n \rangle$ in the forward cone with half-opening Θ as function of the scaling variable ϵ as measured by DELPHI [29]. Also shown are the results from the JETSET and HERWIG Monte Carlo's at the parton and hadron levels at different energies. The data show the predicted scaling behaviour and the approach to the asymptotic DLA prediction (for $\Lambda = 0.15$ GeV, with $b = 2\beta\sqrt{\ln(P\Theta/\Lambda)}$).

An uncertainty in these comparisons, which is hard to quantify, comes from the choice of the jet axis (taken usually as the sphericity axis). An improvement of both theoretical and experimental results could be obtained by using the Energy-Multiplicity-Multiplicity (EMM) definition as applied already to the 2-particle azimuthal angle correlations [31].

The predictions for moments have been derived for cumulants [12] or for factorial moments [26, 27] which approach the same limit asymptotically. It turns out that the factorial moments at present energies are much closer to the asymptotic predictions; such moments are shown in Fig. 3 after appropriate rescaling by kinematic factors again as function of the ε-variable. The data show the same trend as the asymptotic DLA predictions of (11).

The approach of the cumulant moments to the asymptotic results is much slower.
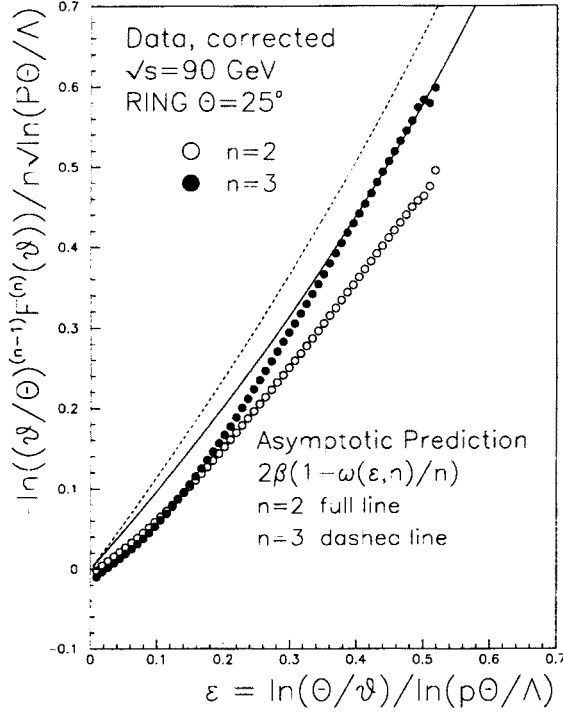

Fig. 3. Rescaled factorial multiplicity moments for particles in the ring around the jet axis with polar angles between $\Theta - \vartheta$ and $\Theta + \vartheta$ as measured by DELPHI [29] (momentum $P = 45$ GeV, $\Lambda = 0.15$ GeV) in comparison with the asymptotic DLA prediction. Note that the curve for $n = 2$ is the same as in Fig. 2 for the rescaled correlation $\hat{r}$.

Note that the curve in Fig. 3 for $n = 2$ is the same as the one in Fig. 2 according to the universality property of (11). Also the differences between the moments of orders $n = 2$ and $n = 3$ are largely removed after rescaling; they are of relative order $1/n^2$ according to (11) and (12).

The various angular regions have quite different characteristics. For small ε (large relative angles) the function $\omega(\varepsilon, n) \approx n - \frac{1}{2} \frac{n^2-1}{n} \varepsilon$, which yields a power behaviour of the moments $M^{(n)}$ (either $C^{(n)}$ or $F^{(n)}$)

$$M^{(n)}(\vartheta, \delta) \sim \left(\frac{\vartheta}{\delta}\right)^{\phi_n}, \qquad \phi_n = D(n-1) - (n - \frac{1}{n})\gamma_0(P\vartheta). \qquad (13)$$

This is the asymptotic power law ("intermittency") for the QCD cascade. It applies for large relative angles where the cascade is fully developed and reflects the selfsimilarity of the branching process; the intermittency exponent ϕ_n depends on the scale through the running α_s. In case of fixed α_s the same result is obtained with universal γ_0 parameter. Moving to larger ε the observables show the angular scaling law (11) with nonlinear function $\omega(\varepsilon, n)$. In this region the results are infrared safe (*i.e.* do not depend on Q_0), they apparently are also not much dependent on hadronization effects (see Fig. 2).

Moving to yet larger ε one comes to a critical angle $\varepsilon_{\text{crit}}$ which separates two kinematic regimes of quite different characteristics [28, 31, 32]. The correlation functions have a discontinuous second derivative at this angle. In the new region $\varepsilon > \varepsilon_{\text{crit}}$ (small relative angles) the correlation functions do depend on the cut-off Q_0 and they become independent of the order n, contrary to the behaviour for $\varepsilon < \varepsilon_{\text{crit}}$; they are given in terms of the one-particle inclusive spectrum. As a consequence, one expects in this region a dependence of the particle type ($\pi\pi, KK, pp$ correlations) if the particle mass is related to the cut-off Q_0.

For fixed α_s this angle is given by $\varepsilon_{\text{crit}} = n^2/(n^2 + 1)$ at order n [31, 32]. For running α_s a new scale Λ appears and one considers [32] the double scaling limit of the function $\omega_n(\varepsilon, \rho)$ with $\rho = \sqrt{\lambda/(Y + \lambda)}$ for asymptotic Y at fixed ε and ρ. Then again a critical behaviour at a certain angle $\varepsilon_{\text{crit}}$ is found. This limit requires also an increasing Q_0 and therefore does not correspond to the usual fixed k_T cut-off. Therefore, for finite, physical Q_0 the separation of the two regions is not expected to be complete. It would be interesting to verify the characteristics of these two regimes experimentally.

4. Scaling law for soft particles

The prediction of the hump-backed shape of the inclusive energy spectrum in the $\xi = \ln 1/x$ variable and its subsequent observation was an important success of QCD in its application to multiparticle physics. The coherence of the soft gluon emission from all harder partons forbids the multiplication of the soft particles and one expects nearly an energy independence of the soft particle rate [3]. Such a property has been pointed out to be present indeed in the data [33, 22].

This problem has been studied recently in more detail [21]. The analytical calculations both in DLA and MLLA converge towards the same limits independent of the *cms* energy for small particle energies. In this limit the energy conservation effects and large z corrections from the splitting functions which make up the differences between the approximations (3) and (5) can be neglected. If LPHD is valid towards these low energies one

expects also a scaling behaviour for the invariant density I_0 of hadrons in the soft limit where the particle momentum p or rapidity y and transverse momentum k_T become small:

$$I_0 = \lim_{y \to 0, p_T \to 0} E \frac{dn}{d^3 p} = \frac{1}{2} \lim_{p \to 0} E \frac{dn}{d^3 p}. \tag{14}$$

The factor $\frac{1}{2}$ in this definition takes into account that both hemispheres are
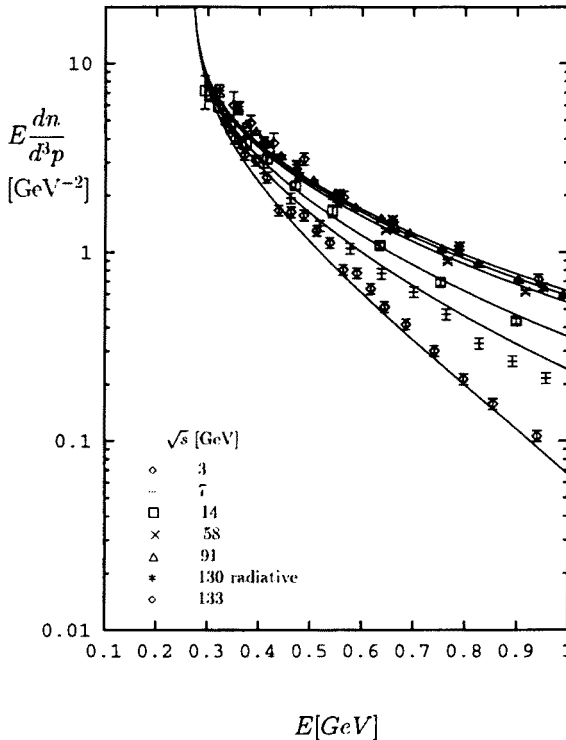

Fig. 4. Invariant density $E dn/d^3 p$ of charged particles in $e^+ e^-$ annihilation as a function of the particle energy $E = \sqrt{p^2 + Q_0^2}$ at $Q_0 = 270$ MeV. Data at various *cms* energies are compared to MLLA predictions with the overall normalization adjusted (from [34]).

included in the limit $p \to 0$. This scaling behaviour is a direct consequence of the coherence of the gluon emission: The emission rate for the gluon of large wavelength does not depend on the details of the jet evolution at smaller distances; it is essentially determined by the colour charge of the hard initial partons and is energy independent. The energy independent contribution comes from the single gluon bremsstrahlung of order α_s, the higher order

contributions generate the energy dependence but do not contribute in the soft limit. In Fig. 4 we show the experimental results on the invariant density of charged particles for *cms* energies from 3 to 130 GeV in e^+e^- annihilation. An approximate energy independence of the soft limit (within about 20%) is indeed observed; the same is true for identified particles π, K and p [34]. The curves in Fig. 4 represent the MLLA results, where also a particular prescription is employed to relate the different parton and hadron kinematics near the boundary $E \approx Q_0$ (for more details, see [34, 35]). The theoretical curves show the approach to the scaling limit and describe well the different slopes at larger particle energies. An important role here is played by the running α_s which provides the strong rise towards small energies for $E < 1$ GeV, for fixed α_s this rise would be much weaker [22, 34].

A crucial test of the QCD-LPHD interpretation of this scaling result is the verification of the dependence of the limiting densitiy I_0 on the primary colour charge. This can be obtained from $e^+e^- \rightarrow 3$ jets, deep inelastic scattering or semihard hadronic processes with gluon exchange [34].

5. Summary

The perturbative approach to multiparticle production in connection with the LPHD assumption represents a very economic description of the phenomena which involves only the parameters Q_0 and Λ apart from the normalization.

The analytical treatment singles out the logarithmic momentum and angular variables which are appropriate to the description of bremsstrahlung processes and absorb the collinear and soft divergent behaviour. Therefore the finite asymptotic limits of various observables in the rescaled logarithmic variables are a direct consequence of the parton branching process generated by bremsstrahlung type emissions. These scaling laws are then more specific to QCD than the KNO multiplicity scaling which holds for a wide class of branching processes, not necessarily of bremsstrahlung type. These results are obtained in the DLA where energy conservation is neglected. A noteworthy feature of angular correlations not met in energy spectra is the occurrence of a critical angle which separates two scaling regimes with quite different characteristics.

Another scaling prediction from DLA is obtained in the soft particle limit at finite energies where energy conservation effects can be neglected as well. It is remarkable that perturbative QCD predictions work even in such an extreme limit and this requires further investigations with different partonic antenna patterns for confirmation.

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