THE WARD IDENTITY OF SUPERSYMMETRY: A CHALLENGE FOR PHENOMENOLOGY*

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The renormalization of supersymmetric gauge theories requires solution of either the problem of handling canonical fields of vanishing dimensions or the problem of non-linear symmetry transformations. Since the supersymmetric extension of the standard model has become a realistic option the use of the respective supersymmetry Ward identity represents a relevant issue in phenomenology.

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1. Introduction

More than twenty years after its invention supersymmetry ([1], textbook [2]) has become a realistic candidate for the extension of the standard model (s.[3] for review and references). Since in the latter radiative corrections are undoubtedly measured [4] for any supersymmetric model too its consistent renormalization is mandatory. For this to achieve it is not sufficient to calculate say one-loop corrections in a general gauge theory having susy field content and then to specialize to supersymmetric values of couplings and masses. Rather one has to establish at the same order a Ward-identity expressing supersymmetry itself. The reason for this is the non-existence of an invariant regularization scheme maintaining both gauge invariance and supersymmetry which would permit naive multiplicative renormalization. Since γ_5 occurs not only in vertices (like in the standard model), but also in the transformation law the lack of naive invariance of dimensional renormalization is much more dangerous than in the standard model. This necessity understood one faces immediately the following alternative:

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— Supersymmetry transformations can be formulated linearly in terms of superfields; then one has to deal with fields of canonical dimension zero which lead (in the massless case) to infrared problems already off-shell and require managing at every dimension infinitely many terms.

— Supersymmetry is formulated in terms of ordinary fields of canonical dimensions ("component formulation"). Then the transformations become non-linear, do only close after use of equations of motion and lead to an open gauge algebra. Whereas the first route has been followed in the years 1978-1986 [5], the second was opened only fairly recently [7,8]. A comparison of the results obtained seems thus to be adequate and is the subject of the present review. Following Ref. [5] we first describe supersymmetric theories as formulated in superspace. The necessary ingredients for this are the supersymmetry algebra, superfields and invariant actions. In chapter 2 perturbation theory in superspace is shortly sketched and applied to all relevant types of models. In chapter 3 we turn to the component treatment and present the results as they have been obtained in [8]. We finish with some conclusions.

2. Susy-algebra, superfields and models

2.1. Susy-algebra and superfields

The fundamental relation defining supersymmetry is the algebra

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \,, \tag{2.1}$$

$$\{Q_\alpha, Q_\beta\} = 0 = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} . \tag{2.2}$$

In

$$\sigma^\mu = (\mathbf{1}, \underline{\sigma}) \,, \qquad \bar\sigma^\mu = (\mathbf{1}, -\underline{\sigma}) \tag{2.3}$$

one has collected the Pauli-matrices $\underline{\sigma}$ and the 2x2 unit matrix. P_μ is the total momentum operator of the system and generates space-time translations. The operators $Q_\alpha(\bar{Q}_{\dot\alpha})$ transform as Weyl-spinors $(\alpha, \dot\alpha = 1, 2)$

$$[M_{\mu\nu}, Q_\alpha] = -\tfrac{1}{2}\sigma_{\mu\nu\alpha}{}^\beta Q_\beta \,, \tag{2.4}$$

$$[M_{\mu\nu}, \bar{Q}^{\dot\alpha}] = -\tfrac{1}{2}\bar\sigma_{\mu\nu}{}^{\dot\alpha}{}_{\dot\beta}\bar{Q}^{\dot\beta} \,, \tag{2.5}$$

$$\sigma_{\mu\nu} = \frac{i}{2}(\sigma_\mu\bar\sigma_\nu - \sigma_\nu\bar\sigma_\mu) \,, \tag{2.6}$$

$$\bar\sigma_{\mu\nu} = \frac{i}{2}(\bar\sigma_\mu\sigma_\nu - \bar\sigma_\nu\sigma_\mu) \,, \tag{2.7}$$

and generate the supersymmetry transformations. In order to guarantee the respective Jacobi identities we require

$$[Q_\alpha, P_\mu] = 0 = [\bar{Q}_{\dot\alpha}, P_\mu]\,, \tag{2.8}$$

and deduce immediately

$$[Q_\alpha, \underline{P}^2] = 0 = [\bar{Q}_{\dot\alpha}, \underline{P}^2]\,. \tag{2.9}$$

The simplicity of this result contrasts remarkably with its importance: as long as supersymmetry is not broken superymmetry multiplets are degenerate in mass – the nightmare of every application. In field theory we want to represent $Q_\alpha, \bar{Q}_{\dot\alpha}, P_\mu, M_{\mu\nu}$ as charges of conserved Noether currents, to find the algebra as a consequence of field commutators and to have multiplets on which the symmetry is linearly realized. In formulas this means

$$i[X, \phi] = \delta^X \phi \qquad \text{for} \qquad X \in \{Q, \bar{Q}, P, M\} \tag{2.10}$$

with closure off-shell

$$[\delta^{X_1}, \delta^{X_2}] = \delta^{X_3} \tag{2.11}$$

(independent of ϕ *i.e.* without use of eqns. of motion). In order to find such representations one uses the analogy to the Poincaré-group, writes down a "group element"

$$G(a, \xi, \bar{\xi}) = e^{i(a^\mu P_\mu + \xi^\alpha Q_\alpha + \bar{\xi}_{\dot\alpha}\bar{Q}^{\dot\alpha})} \tag{2.12}$$

and studies the group action

$$G(a, \xi, \bar{\xi})G(x, \theta, \bar{\theta}) = G(a + x + i\xi\sigma\bar{\theta} - i\theta\sigma\bar{\xi}, \theta + \xi, \bar{\theta} + \bar{\xi})\,. \tag{2.13}$$

Here we have used

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]} \tag{2.14}$$

which is true since the higher commutators vanish. The law (2.13) represents a motion in parameter space

$$(x, \theta, \bar{\theta}) \to (x + a + i\xi\sigma\bar{\theta} - i\theta\sigma\bar{\xi}, \theta + \xi, \bar{\theta} + \bar{\xi}) \tag{2.15}$$

which can be reproduced on functions ϕ defined over this parameter space by differential operators

$$\begin{aligned} P_\mu \phi &= -i\partial_\mu \phi\,, \\ Q_\alpha \phi &= -i\left(\frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_\mu\right)\phi\,, \\ \bar{Q}_{\dot\alpha}\phi &= -i(-\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu)\phi\,, \\ \phi &= \phi(x, \theta, \bar{\theta})\,. \end{aligned} \tag{2.16}$$

Hence the desired variations δ^X read

$$\begin{aligned} \delta^P_\mu &= \partial_\mu \,, \\ \delta^Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial \,, \\ \delta^{\bar Q}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma_{\alpha\dot\alpha}\partial \,, \end{aligned} \tag{2.17}$$

and satisfy

$$\left\{\delta^Q_\alpha, \delta^{\bar Q}_{\dot\alpha}\right\} = -2i\sigma^\mu_{\alpha\dot\alpha}\delta^P_\mu \,, \tag{2.18}$$

$$0 = \left\{\delta^Q_\alpha, \delta^Q_\beta\right\} = \left\{\delta^{\bar Q}_{\dot\alpha}, \delta^{\bar Q}_{\dot\beta}\right\} = [\delta^Q_\alpha, \delta^P_\mu] = [\delta^{\bar Q}_{\dot\alpha}, \delta^P_\mu] \,. \tag{2.19}$$

Identifying the parameters of translations with usual space-time, we may interprete the fermionic parameters assotiated to $Q_\alpha, \bar Q^\alpha$ as fermionic coordinates, consider $(x, \theta, \bar\theta)$ as a point in *superspace*, the functions $\phi(x, \theta, \bar\theta)$ as *superfields* provided they transform correctly:

$$\begin{aligned} i[Q_\alpha, \phi] &= \delta^Q_\alpha \phi \,, \\ i[\bar Q_{\dot\alpha}, Q] &= \delta^{\bar Q}_{\dot\alpha} \phi \,, \\ i[P_\mu, \phi] &= \delta^P_\mu \phi \,, \end{aligned} \tag{2.20}$$

(δ^X as given above).
The anticommutativity of the fermionic coordinates

$$\theta_\alpha\theta_\beta = -\theta_\beta\theta_\alpha \qquad \alpha, \beta = 1, 2 \tag{2.21}$$

restricts the maximal power of θ's occurring in a product to four:

$$\theta^\alpha\theta_\alpha\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha} \equiv \varepsilon_{\alpha\beta}\theta^\alpha\theta^\beta\varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} \,. \tag{2.22}$$

Hence one can expand every superfield in components

$$\phi(x, \theta, \bar\theta) = \phi^{(0,0)}(x) + \theta^\alpha\phi^{(1,0)}_\alpha(x) + \ldots + \theta^2\bar\theta^2\phi^{(2,2)}(x) \tag{2.23}$$

whose transformation law can be deduced from (2.20) by expanding l.h.s. and r.h.s. in θ and equating equal powers. As examples we reproduce supersymmetry transformations of

$$\begin{array}{ll}
\textit{chiral field} & \textit{anti-chiral field} \\
A = A_1(x,\theta) = \mathcal{A} + \theta\psi + \theta^2 F & \bar{A} = \bar{A}_2(x,\bar\theta) = \bar{\mathcal{A}} + \bar\theta\bar\psi + \bar\theta^2 \bar{F} \\
\delta_\alpha \mathcal{A} = \psi_\alpha & \delta_\alpha \bar{\mathcal{A}} = 0 \\
\delta_\alpha \psi_\beta = -2\varepsilon_{\alpha\beta} F & \delta_\alpha \bar\psi_{\dot\alpha} = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \bar{\mathcal{A}} \\
\delta_\alpha F = 0 & \delta_\alpha \bar{F} = -i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \bar\psi^{\dot\alpha} \\
\bar\delta_{\dot\alpha} \mathcal{A} = 0 & \bar\delta_{\dot\alpha} \bar{\mathcal{A}} = 0\bar\psi_{\dot\alpha} \\
\bar\delta_{\dot\alpha} \psi_\alpha = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \mathcal{A} & \bar\delta_{\dot\alpha}\bar\psi_{\dot\beta} = 2\varepsilon_{\dot\alpha\dot\beta}\bar{F} \\
\bar\delta_{\dot\alpha} F = i\partial_\mu \psi^\alpha \sigma^\mu_{\alpha\dot\alpha} & \bar\delta_{\dot\alpha}\bar{F} = 0
\end{array} \tag{2.24}$$

real field

$$\begin{aligned}
\phi(x,\theta,\bar\theta) \;=\; & C + \theta\chi + \bar\theta\bar\chi + \tfrac{1}{2}\theta^2 M + \tfrac{1}{2}\bar\theta^2 \bar{M} + \theta\sigma^\mu\bar\theta v_\mu + \tfrac{1}{2}\bar\theta^2\theta\lambda \\
& + \tfrac{1}{2}\theta^2\bar\theta\bar\lambda + \tfrac{1}{4}\theta^2\bar\theta^2 D
\end{aligned} \tag{2.25}$$

$$\begin{array}{ll}
\delta_\alpha C = \chi_\alpha & \bar\delta_{\dot\alpha} C = \bar\chi_{\dot\alpha} \\
\delta_\alpha \chi_\beta = -\varepsilon_{\alpha\beta} M & \bar\delta_{\dot\alpha}\bar\chi_{\dot\beta} = \varepsilon_{\dot\alpha\dot\beta}\bar{M} \\
\delta_\alpha \bar\chi_{\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha}(v_\mu + i\partial_\mu C) & \bar\delta_{\dot\alpha}\chi_\alpha = -\sigma^\mu_{\alpha\dot\alpha}(v_\mu - i\partial_\mu C) \\
\delta_\alpha M = 0 & \bar\delta_{\dot\alpha}\bar{M} = 0 \\
\delta_\alpha \bar{M} = \lambda_\alpha - i(\sigma^\mu\partial_\mu\bar\chi)_\alpha & \bar\delta_{\dot\alpha} M = \bar\lambda_{\dot\alpha} + i(\partial_\mu\chi\sigma^\mu)_{\dot\alpha} \\
\delta_\alpha v_\mu = \tfrac{1}{2}(\sigma_\mu\bar\lambda)_\alpha - \tfrac{i}{2}(\sigma^\nu\bar\sigma_\mu\partial_\nu\chi)_\alpha & \bar\delta_{\dot\alpha} v_\mu = \tfrac{1}{2}(\lambda\sigma_\mu)_{\dot\alpha} + \tfrac{i}{2}\sigma^\nu_{\alpha\dot\alpha}\bar\sigma^{\dot\beta\alpha}_\mu\partial_\nu\bar\chi_{\dot\beta} \\
\delta_\alpha \lambda_\beta = -2\varepsilon_{\alpha\beta}(D + i\partial v) + \tfrac{1}{2}\sigma^{\mu\nu}_{\alpha\beta}v_{\mu\nu} & \bar\delta_{\dot\alpha}\bar\lambda_{\dot\beta} = \varepsilon_{\dot\alpha\dot\beta}(D - i\partial v) - \tfrac{1}{2}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}v_{\mu\nu} \\
\delta_\alpha \bar\lambda_{\dot\alpha} = i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu M & \bar\delta_{\dot\alpha}\lambda_\alpha = i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \bar{M} \\
\delta_\alpha D = -i(\sigma^\mu\partial_\mu\bar\lambda)_\alpha & \bar\delta_{\dot\alpha} D = i(\partial_\mu\lambda\sigma^\mu)_{\dot\alpha}\,.
\end{array}$$

In these examples we have already introduced superfields which are constrained in a way which is compatible with supersymmetry:

$$\begin{array}{rcll}
\phi^*(x,\theta,\bar\theta) & = & \phi(x,\theta,\bar\theta) & \text{“real superfield”}\,, \\
D_\alpha\phi & = & 0 & \phi \text{ anti-chiral superfield}\,, \\
\bar{D}_{\dot\alpha}\phi & = & 0 & \phi \text{ chiral superfield}\,.
\end{array}$$

Here

$$\begin{aligned}
D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \\
\bar{D}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu
\end{aligned} \tag{2.26}$$

are derivatives which are covariant with respect to supersymmetry *i.e.* if ϕ was a superfield $D\phi$ or $\bar{D}\phi$ is also a superfield.

They satisfy relations which are very similar to those of the variations

$$\{D_\alpha, \bar{D}_{\dot\alpha}\} = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\,, \tag{2.27}$$

$$\{D_\alpha, D_\beta\} = 0 = \left\{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\right\}\,. \tag{2.28}$$

As a consequence follows

$$D_\alpha D_\beta D_\gamma = 0 \,. \tag{2.29}$$

The construction of susy invariant actions is greatly simplified with their help. We shall need in particular

$$D\bar{D}\bar{D}D = \bar{D}DD\bar{D}\,, \tag{2.30}$$

$$D\bar{D}\bar{D}D - \tfrac{1}{2}[DD, \bar{D}\bar{D}] = 8\Box\,, \tag{2.31}$$

$$\begin{aligned}[DD, \bar{D}\bar{D}] &= -16\Box - 8i\bar{D}\bar{\sigma}D\partial \\ &= +16\Box + 8iD\sigma\bar{D}\partial\,. \end{aligned} \tag{2.32}$$

2.2. Models

For the construction of supersymmetric models in terms of superfields one starts from the observation that sums and products of superfields of one and the same type (chiral, anti-chiral, real) lead again to superfields of the same type. The only other needed ingredient is the observation that the highest θ-components of every type transform into total divergences hence their space-time integrals are invariant. On the basis of this fact one defines covariant measures in superspace which project on the desired components.

$$\int dS \equiv \int d^4x \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha} = \int d^4x D^\alpha D_\alpha\,, \tag{2.33}$$

$$\int d\bar{S} \equiv \int d^4x \frac{\partial}{\partial\bar{\theta}^{\dot\alpha}}\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}} = \int d^4x \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}\,, \tag{2.34}$$

$$\int dV \equiv \int d^4x \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}}\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}} = \int d^4x \bar{D}\bar{D}DD\,. \tag{2.35}$$

The simplest model is made up from one chiral field and its anti-chiral partner obtained by complex conjungation. When written as real fields they have the form

$$A = \mathcal{A} + \theta\psi + \theta^2 F - i\theta\sigma\bar{\theta}\partial\mathcal{A} - \frac{i}{2}\theta^2\bar{\theta}\bar{\sigma}\partial\psi - \frac{1}{4}\theta^2\bar{\theta}^2\Box\mathcal{A}\,, \tag{2.36}$$

$$\bar{A} = \bar{\mathcal{A}} + \bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F} + i\theta\sigma\bar{\theta}\partial\bar{\mathcal{A}} - \frac{i}{2}\bar{\theta}^2\theta\sigma\partial\bar{\psi} - \frac{1}{4}\theta^2\bar{\theta}^2\Box\bar{\mathcal{A}}\,. \tag{2.37}$$

The D-term of their product provides therefore an invariant kinetic term

$$\frac{1}{16}\int dV\bar{A}A = \int dx(\partial\bar{\mathcal{A}}\partial\mathcal{A} + \frac{i}{2}\bar{\psi}\bar{\sigma}\partial\psi + \bar{F}F)\,, \tag{2.38}$$

i.e. a complex scalar together with a Weyl spinor and an auxiliary field forms an invariant.

Mass and interaction terms can be constructed by taking products of fields with the same chirality

$$\frac{m}{8}\int dSA^2+\frac{m}{8}\int d\bar{S}\bar{A}^2=-\frac{m}{2}\int dx\left(2(\mathcal{A}F+\bar{\mathcal{A}}\bar{F})-\frac{1}{2}(\psi\psi+\bar{\psi}\bar{\psi})\right), \tag{2.39}$$

$$\frac{g}{48}\left(\int dSA^3+\int d\bar{S}\bar{A}^3\right)=-\frac{g}{4}\int dx\left(F\mathcal{A}^2+\bar{F}\bar{\mathcal{A}}^2-\frac{1}{2}(\mathcal{A}\psi\psi+\bar{\mathcal{A}}\bar{\psi}\bar{\psi})\right). \tag{2.40}$$

That these terms give indeed rise to conventional mass and interaction terms is seen by using the equation of motion for the auxiliary fields $F,\bar{F}$:

$$\frac{\delta\Gamma}{\delta F}=\bar{F}-m\bar{\mathcal{A}}-\frac{g}{4}\bar{\mathcal{A}}^2\,. \tag{2.41}$$

They lead to $\mathcal{A}\bar{\mathcal{A}},\mathcal{A}^2\bar{\mathcal{A}}^2$ and trilinear terms. The parameters of the model are: one common mass m, one common coupling g.

The generalization to several chiral fields reads

$$\Gamma_{\text{kin}}=\sum_i\int dV\bar{A}_iA_i\,, \tag{2.42}$$

$$\Gamma_{\text{chiral}}=\int dS(\lambda_iA_i+m_{ij}A_iA_j+g_{ijk}A_iA_jA_k)+\int d\bar{S}\overline{(\ldots)}\,. \tag{2.43}$$

For such models spontaneous breakdown of supersymmetry (and of internal symmetries) is possible and can be detected by studying the tree potential. The Ward-identities which express supersymmetry of the action Γ read

$$W_\alpha\Gamma\equiv\int dS\delta_\alpha A\frac{\delta\Gamma}{\delta A}+\int d\bar{S}\delta_\alpha\bar{A}\frac{\delta\Gamma}{\delta\bar{A}}=0\,, \tag{2.44}$$

$$\bar{W}_{\dot\alpha}\Gamma\equiv\int dS\bar{\delta}_{\dot\alpha}A\frac{\delta\Gamma}{\delta A}+\int d\bar{S}\bar{\delta}_{\dot\alpha}\bar{A}\frac{\delta\Gamma}{\delta\bar{A}}=0\,. \tag{2.45}$$

In the case when supersymmetry is spontaneously broken, the fields $A,\bar{A}$ are shifted into $A+\theta^2f$, resp. $\bar{A}+\bar{\theta}^2\bar{f}$.

The supersymmetric extension of QED can be constructed with the help of one real general superfield ("vector superfield") and two chiral multiplets. Gauge transformations and associated field content can be understood by supersymmetrizing transverse and longitudinal projection operators:

$$P_{L_1}=-\frac{DD\bar{D}\bar{D}}{16\Box}\,,\qquad P_{L_2}=-\frac{\bar{D}\bar{D}DD}{16\Box}\,,$$

$$P_T=\frac{D\bar{D}\bar{D}D}{8\Box}\,. \tag{2.46}$$

They satisfy:

$$P_i P_j = \delta_{ij} P_j \,, \qquad i,j = L_1, L_2, T \,, \tag{2.47}$$

and

$$P_{L1} + P_{L2} + P_T = 1 \,. \tag{2.48}$$

Hence P_{L1} projects to a chiral field, P_{L2} projects to an anti-chiral field, P_T projects to a real field.
This suggests

$$\begin{aligned} \delta\phi &= i(\Lambda - \bar{\Lambda}) \\ &= i(\mathcal{A} + \theta\psi + \theta^2 F - i\theta\sigma\bar{\theta}\partial\mathcal{A} - \frac{i}{2}\theta^2\bar{\theta}\bar{\sigma}\partial\psi + \frac{1}{4}\theta^2\bar{\theta}^2\Box\mathcal{A}) \\ &\quad -i(\bar{\mathcal{A}} + \bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F} + i\theta\sigma\bar{\theta}\partial\bar{\mathcal{A}} + \frac{i}{2}\bar{\theta}^2\theta\sigma\partial\bar{\psi} + \frac{1}{4}\theta^2\bar{\theta}^2\Box\bar{\mathcal{A}}) \end{aligned} \tag{2.49}$$

as gauge transformation and

$$\Gamma_{\text{kin}}^{\text{vector}} = \frac{1}{128}\int dV \phi D\bar{D}\bar{D}D\phi = \int dx(-\frac{1}{4}f^{\mu\nu}f_{\mu\nu} + \frac{i}{4}\lambda' \not{\partial}\bar{\lambda}' + \frac{1}{8}D'^2) \tag{2.50}$$

as gauge invariant kinetic term for the vector field. Here appear the gauge invariant combinations

$$\begin{aligned} \lambda' &= \lambda + i\sigma\partial\bar{\chi} \,, \\ D' &= D + \Box C \,, \\ F_{\mu\nu} &= \partial_\mu v_\nu - \partial_\nu v_\mu \,. \end{aligned} \tag{2.51}$$

Another useful form of $\Gamma_{\text{kin}}^{(\text{vector})}$ is given by

$$\Gamma_{\text{kin}}^{(\text{vector})} = \frac{1}{64}\int dS \bar{D}\bar{D}D^\alpha\phi\bar{D}\bar{D}D_\alpha\phi + \frac{1}{64}\int d\bar{S} DD\bar{D}_{\dot\alpha}\phi DD\bar{D}^{\dot\alpha}\phi \,. \tag{2.52}$$

A supersymmetric gauge fixing term takes the form

$$\Gamma_{\text{g.f.}} = -\frac{1}{2\alpha}\frac{1}{128}\int dV \phi \left\{DD, \bar{D}\bar{D}\right\} \phi \,. \tag{2.53}$$

A version with chiral Lagrange multiplier field B is given by

$$\Gamma_{g.f.} = \frac{1}{128}\int dV(\alpha B\bar{B} + \frac{1}{8}(BDD\phi + \bar{B}\bar{D}\bar{D}\phi)) \,. \tag{2.54}$$

A gauge invariant and supersymmetric interaction with matter fields turns out to be

$$\Gamma_{\text{kin}}^{\text{matter}} = \frac{1}{16}\int dV(\bar{A}_+ e^{g\phi} A_+ + A_- e^{-g\phi}\bar{A}_-) \tag{2.55}$$

for gauge transformations

$$\begin{aligned} \delta_\Lambda A_\pm &= \mp ig\Lambda A_\pm\,, & \delta_\Lambda \bar{A}_\pm &= 0\,, \\ \delta_{\bar{\Lambda}} A_\pm &= 0\,, & \delta_{\bar{\Lambda}} \bar{A}_\pm &= \pm ig\bar{\Lambda}\bar{A}_\pm\,, \end{aligned} \tag{2.56}$$

(which respect chirality).

The invariant mass term is then

$$\Gamma_m^{\text{matter}} = \frac{m}{4}\int dS A_+ A_- + \frac{m}{4}\int d\bar{S}\bar{A}_-\bar{A}_+\,. \tag{2.57}$$

Parity transformations are realized by

$$\phi \to -\phi, \quad A_+ \to \bar{A}_-, \quad \bar{A}_+ \to A_- \tag{2.58}$$

$$\Gamma_{\text{inv}} = \Gamma_{\text{kin}}^{(\text{vector})} + \Gamma_{\text{kin}}^{(\text{matter})} + \Gamma_m^{\text{matter}} \tag{2.59}$$

constitutes the action of supersymmetric QED, combining scalar and spinor QED, having one matter mass parameter and one coupling.

The supersymmetric extension of Yang–Mills theory can be found by first choosing a multiplet of chiral fields with non-abelian rigid transformations

$$\begin{aligned} \delta_\omega A_k &= -i\Lambda^a \tau^a_{kl} A_l & [\tau^a, \tau^b] &= i f^{abc}\tau^c \\ \delta_\omega \bar{A}_k &= i\Lambda^a \bar{A}_l \tau^a_{lk}\,. \end{aligned} \tag{2.60}$$

and then building up the finite transformations

$$\begin{aligned} A_k &\to (e^{-i\Lambda})_{kl} A_l & \Lambda &\equiv \Lambda^a \tau^a \\ \bar{A}_k &\to \bar{A}_l (e^{i\Lambda})_{lk} \end{aligned} \tag{2.61}$$

The interaction suggested from SQED becomes invariant

$$\bar{A} e^\phi A \to \bar{A} e^\phi A \tag{2.62}$$

if

$$e^\phi \to e^{-i\bar{\Lambda}} e^\phi e^{i\Lambda} = e^{\phi'} \tag{2.63}$$

is the transformation law for the non-abelian vector multiplet

$$\phi \equiv \phi^a \tau^a. \tag{2.64}$$

Infinitesimally

$$\phi' = \phi + \delta\phi \tag{2.65}$$

this law implies

$$\begin{aligned}\delta\phi &= i(\Lambda-\bar{\Lambda})+\frac{i}{2}[\phi,\Lambda+\bar{\Lambda}]+\frac{i}{12}[\phi,[\phi,\Lambda-\bar{\Lambda}]]+\mathcal{O}(\phi^3)\\ &\equiv iQ_s(\phi,\Lambda,\bar{\Lambda})\,. \end{aligned}\tag{2.66}$$

The non-abelian extension of (2.52) reads

$$-\frac{1}{128}Tr\int dSF^\alpha F_\alpha = Tr\int dx(-\frac{1}{4}(F^{\mu\nu})^2-\frac{1}{4}\lambda\, \not{D}\bar{\lambda}+\frac{1}{8}D^2+\mathcal{O}(\phi^3))\,, \tag{2.67}$$

where

$$F^\alpha \equiv \bar{D}\bar{D}(e^{-\phi}D^\alpha e^\phi)\,. \tag{2.68}$$

The most general action invariant under supersymmetry and local gauge transformations in thus given by

$$\Gamma_{\text{inv}} = \Gamma_{\text{YM}}+\frac{1}{16}\int dV\bar{A}e^{\tilde{\phi}}A+\Gamma_m+\Gamma_h\,, \tag{2.69}$$

$$\Gamma_m = m_{ab}\int dSA_aA_b+\bar{m}_{ab}\int d\bar{S}\bar{A}_a\bar{A}_b\,, \tag{2.70}$$

$$\Gamma_h = h_{abc}\int dSA_aA_bA_c+\bar{h}_{abc}\int d\bar{S}\bar{A}_a\bar{A}_b\bar{A}_c\,, \tag{2.71}$$

(2.72)

here

$$\tilde{\phi}\equiv\phi^aT^a\,, \tag{2.73}$$

for a general representation of matter fields transforming as

$$\delta A_k = i\Lambda^aT^a_{kl}A_l,\quad \delta\bar{A}_k = -i\bar{\Lambda}^a\bar{A}_lT^a_{lk} \tag{2.74}$$

and invariant terms Γ_m, Γ_h.

3. Perturbation theory in superspace

3.1. Propagators

The main motivation to formulate supersymmetric field theories in superspace originated from the fact that one can there also formulate perturbation theory and thereby embody the peculiar properties which supersymmetric theories indeed possess.

Extending functional derivatives and δ-functions from ordinary space to superspace[1]

$$\frac{\delta}{\delta\phi(1)}\phi(2) = \delta_V(1,2) \equiv \frac{1}{16}\theta_{12}^2\bar{\theta}_{12}^2\delta(x_1 - x_2)\,, \tag{3.1}$$

$$\frac{\delta}{\delta A(1)}A(2) = \delta_S(1,2) \equiv \bar{D}\bar{D}_1\delta_V(1,2) = \bar{D}\bar{D}_2\delta_V(1,2)\,, \tag{3.2}$$

$$\theta_{12} \equiv \theta_1 - \theta_2, \qquad \bar{\theta}_{12} \equiv \bar{\theta}_1 - \bar{\theta}_2 \tag{3.3}$$

one can as usual derive propagators and obtains

$$\langle TA(1)A(2)\rangle = i\frac{4m\delta_S(1,2)}{\Box + m^2}\,, \tag{3.4}$$

$$\langle TA(1)\bar{A}(2)\rangle = -i\frac{DD_2\delta_S(1,2)}{\Box + m^2}\,, \tag{3.5}$$

for chiral fields.

The explicit form of chiral δ-functions and propagators of chiral fields depends on the basis in which the fields are written and whose preference depends on the context. As an example we write them down in the chiral (resp. anti-chiral) basis:

$$\delta_S(1,2) = -\tfrac{1}{4}\theta_{12}^2\delta(x_1 - x_2)\,. \tag{3.6}$$

The Fourier-transforms of the propagators read

$$\langle TA(1)A(2)\rangle = \frac{m\theta_{12}^2}{k^2 - m^2}\,, \tag{3.7}$$

$$\langle TA(1)\bar{A}(2)\rangle = i\frac{e^{-2\theta_1\sigma\bar{\theta}_2 k}}{k^2 - m^2}\,, \tag{3.8}$$

wherefrom one can *e.g.* identify component propagators:

$$\begin{aligned}\langle TA(1)\bar{A}(2)\rangle = {} & \langle TA(x_1)\bar{A}(x_2)\rangle + \langle T\theta_1\psi(x_1)\bar{\theta}_2\bar{\psi}(x_2)\rangle \\ & + \langle T\theta_1^2 F(x_1)\bar{\theta}_2^2 F(x_2)\rangle\,.\end{aligned} \tag{3.9}$$

The free propagator for the vector field with mass M takes the form

$$\langle T\phi(1)\phi(2)\rangle = \frac{16i}{M^2}\left(1 - \frac{1}{8}\frac{D\bar{D}\bar{D}D}{\Box + M^2} - \frac{1}{16}\frac{\{DD, \bar{D}\bar{D}\}}{\Box + \alpha M^2}\right)\delta_V(1,2) \tag{3.10}$$

$$\langle T\phi(1)\phi(2)\rangle|_{\alpha=1} = \frac{8i}{\Box + M^2}\delta_V(1,2)\,. \tag{3.11}$$

[1] For conventions s. [5]

In the massless case one obtains

$$\langle T\phi(1)\phi(2)\rangle = \frac{i}{\Box^2}\left(D\bar{D}\bar{D}D - \frac{\alpha}{2}\{DD,\bar{D}\bar{D}\}\right)\delta_V(1,2)\,, \quad (3.12)$$

$$\langle T\phi(1)\phi(2)\rangle|_{\alpha=1} = \frac{8i}{\Box}\delta_V(1,2)\,. \quad (3.13)$$

The $\Box^2$ signals infrared difficulties already off-shell in all gauges but $\alpha = 1$ and goes along with the vanishing canonical dimension of the field C (cf.(2.24)), *i.e.* of the superfield ϕ. Dimension zero of ϕ permitted also the appearance of functions of ϕ like $e^{g\phi}$ and will in higher orders pose the problem of uniquely identifying such functions – a highly non-trivial task.

3.2. Diagrams, divergences

The propagators are the main ingredient for Feynman rules and diagrams in superspace. One reveals the fact that for supersymmetric theories ultraviolet divergences are softened compared to generic theories. We shall demonstrate this fact with the simplest examples, the one-loop contributions to the 2-point-functions of chiral fields.

$$\langle TA(1)A(2)\rangle^{(1)} \sim \frac{m\delta_S(1,2)}{\Box + m^2}\frac{m\delta_S(1,2)}{\Box + m^2} = 0 \quad (3.14)$$

because $\theta_{12}^2\theta_{12}^2 = 0$;

$$\langle TA(1)\bar{A}(2)\rangle^{(1)} \sim \frac{1}{(2\pi)^4}\int dp e^{i(x_1-x_2)p}\frac{1}{(2\pi)^4}\int dkI\,, \quad (3.15)$$

$$\begin{aligned} I &= \frac{iDD_2\delta_S(1,2;k)}{k^2 - m^2}\frac{iDD_2\delta_S(1,2;p-k)}{(p-k)^2 - m^2}\,, \\ I &= \frac{e^{-E_{12}p}}{(k^2 - m^2)((p-k)^2 - m^2)}\,, \\ E_{12} &\equiv \theta_1\sigma\bar{\theta}_1 + \theta_2\sigma\bar{\theta}_2 - 2\theta_1\sigma\bar{\theta}_2\,. \end{aligned} \quad (3.16)$$

We see that in the chiral-chiral example a logarithmic divergence is absent, whereas in the chiral-antichiral example a possible quadratical divergence is softened to a logarithmic one. This can in fact be generalized to all orders: purely chiral (or purely antichiral) vertex functions are better behaved by one in power counting (hence are finite); the chiral- antichiral vertex function is never worse than logarithmically divergent. Quadratical divergences can only occur in the 2-point-functions of vector fields (and are reduced there to logarithmic ones by gauge invariance).

The above chiral-antichiral example leads to one more conclusion. One can render the integral finite by replacing I by

$$R = (1 - t_p^2)I\,, \tag{3.17}$$

where t_p^2 is the Taylor operator up to and including power 2. Now the integral exists. Due to the exponential $exp(-E_{12}p)$ we find

$$R = e^{-E_{12}p}(1 - t_p^0)\frac{1}{(k^2 - m^2)((p-k)^2 - m^2)}\,, \tag{3.18}$$

which is a very important result: the exponential obviously contained the entire supersymmetry structure of this radiactive correction and appears untouched after performing subtractions. Hence momentum space subtractions maintain supersymmetry! Or generally stated: defining renormalized vertex functions by (BPHZ-) momentum space subtractions we do not violate supersymmetry – it is a naively invariant renormalization scheme. As a consequence the susy Ward-identities (2.44), (2.45) hold to all orders of perturbation theory, where Γ is the generating functional of vertex functions and starts with the classical action

$$\Gamma = \Gamma_{\rm cl} + \hbar\Gamma^{(1)} + \hbar^2\Gamma^{(2)} + \dots\,. \tag{3.19}$$

In SQED this naive invariance of the renormalization scheme still pays off because we can circumvent the infrared trouble of the massless vector field. With the help of the Zimmermann/Lowenstein convergence criteria one can show that
(1) UV-subtractions alone are sufficient for convergence (off-shell);
(2) the gauge $\alpha = 1$ is stable.
Hence susy is naively maintained (add $\int dV\delta_\alpha\phi\frac{\delta}{\delta\phi}$ resp. $\int dV\bar\delta_{\dot\alpha}\phi\frac{\delta}{\delta\phi}$ to the WI-operators in (2.44),(2.45)) and the gauge Ward identities hold:

$$w_\Lambda\Gamma \equiv \left(\bar D\bar D\frac{\delta}{\delta\phi} - gA_+\frac{\delta}{\delta A_+} + gA_-\frac{\delta}{\delta A_-}\right)\Gamma = \frac{1}{8}\Box\bar D\bar D\phi\,, \tag{3.20}$$

$$w_{\bar\Lambda}\Gamma \equiv \left(DD\frac{\delta}{\delta\phi} - g\bar A_+\frac{\delta}{\delta\bar A_+} + g\bar A_-\frac{\delta}{\delta\bar A_-}\right)\Gamma = \frac{1}{8}\Box DD\phi\,. \tag{3.21}$$

One derives as usual from the gauge WI that the longitudinal parts $\bar D\bar D\phi$, $DD\phi$ of the vector field ϕ are free, hence that the theory is unitary (in a formal sense for massless vector field, in a strict sense for massive vector field).

In SYM the off-shell infrared problem and the fact that to a given dimension infinitely many field monomials belong can no longer be circumvented, but must be solved. The latter is dealt with by expanding in the number of fields in addition to the perturbation in the number of loops: we arrive at a double expansion. The solution of the former proceeds along the following lines. One first observes that every function

$$\mathcal{F}(\phi) = \phi + a_2\phi^2 + a_3\phi^3 + ... \tag{3.22}$$

is as good a field as ϕ itself. And indeed this generalized wave function renormalization is required for taming all divergences [6]. Also Q_s of (2.66) can be generalized by this substitution, still leading to BRS invariance (which replaced gauge invariance). Our theory has infinetely many parameters! Fortunately it turns out that these parameters a_k are *gauge* parameters. And this then provides the clue for solving the off-shell infrared problem. Performing a field redefinition

$$\phi \to \phi\left(1 + \tfrac{1}{2}\mu^2\theta^2\bar{\theta}^2\right) + \mathcal{O}(\phi^2) \tag{3.23}$$

maintains BRS invariance, breaks susy – but only softly and removes the dangerous double poles in the ϕ-propagator:

$$\frac{1}{\Box^2} \to \frac{1}{\Box}\frac{1}{\Box + \mu^2} \tag{3.24}$$

(in the $\langle CC \rangle$ component). Hence μ^2 is an infrared regulator which maintains BRS and is a gauge parameter. Hence gauge invariant quantities which are also gauge parameter independent will exist infraredwise: for them the limit $\mu^2 \to 0$ exists trivially.

The soft breaking of supersymmetry can be controlled algebraically. This proceeds as follows. If in the classical approximation (softly broken: $\simeq 0$) supersymmetry holds

$$W_\alpha \Gamma_{\rm cl} \simeq 0 \qquad \bar{W}_{\dot\alpha} \Gamma_{\rm cl} \simeq 0\,, \tag{3.25}$$

then in one-loop the breaking is a local field polynominal

$$(W_\alpha \Gamma)^{(1)} \simeq \Delta_\alpha\,, \qquad (\bar{W}_{\dot\alpha} \Gamma)^{(1)} \simeq \bar{\Delta}_{\dot\alpha}\,, \tag{3.26}$$

and the breaking terms have to satisfy

$$\begin{aligned} W_\alpha \bar{\Delta}_{\dot\alpha} + \bar{W}_{\dot\alpha} \Delta_\alpha &\simeq 0\,, \\ W_\alpha \Delta_\beta + W_\beta \Delta_\alpha &\simeq 0\,, \\ \bar{W}_{\dot\alpha} \bar{\Delta}_{\dot\beta} + \bar{W}_{\dot\beta} \bar{\Delta}_{\dot\alpha} &\simeq 0\,, \end{aligned} \tag{3.27}$$

as a consequence of the algebra of the WI operators

$$\{W_\alpha, \bar{W}_{\dot\alpha}\} = 2\sigma^\mu W^P_\mu\,, \tag{3.28}$$

$$\{W_\alpha, W_\beta\} = 0 = \{\bar{W}_{\dot\alpha}, \bar{W}_{\dot\beta}\}\,, \tag{3.29}$$

and the translation invariance of Γ. Now it is a theorem(s.[5,9]) that the only solution of (3.29) is

$$\Delta_\alpha \simeq W_\alpha \hat{\Delta}\,, \tag{3.30}$$

$$\bar{\Delta}_{\dot\alpha} \simeq \bar{W}_{\dot\alpha} \hat{\Delta}\,, \tag{3.31}$$

where $\hat{\Delta}$ is a local field polynomial. Hence redefining $\Gamma^{(1)}$ by $-\hat{\Delta}$ removes this breaking in the susy WI and establishes susy at one-loop. By induction this works to all orders: supersymmetric theories of chiral, anti-chiral and vector fields are free of anomalies.

The susy WI has the form

$$W_\alpha \Gamma \equiv W^h_\alpha \Gamma - 2i \int dx \mu^2 \frac{\delta \Gamma}{\delta u^\alpha_\lambda} = 0\,. \tag{3.32}$$

Here W^h_α collects all homogeneous superfield transformations and the inhomogeneous term generates the breaking terms in form of derivative with respect to the λ-component of an external superfield u.

Next one has to study that identity which expresses BRS symmetry on the functional level. We shall refer to the literature (s.[5] sects. 5,15) and only quote the result. The Adler-Bardeen anomaly is absent if the representations of (chiral) matter superfields are chosen in a suitable form

$$r = \frac{1}{3 \cdot 2^{12}\pi^2} \frac{d^{ijk}}{d^2} TrT^iT^jT^k \overset{!}{=} 0 \tag{3.33}$$

($sA_a = c^j T^i_{ab} A_b$; $dijk$ is the totally symmetric tensor in the symmetry group; $d^2 \equiv d^{ijk} d_{ijk}$): the usual condititon. In this case the theory is unitary and Green functions of BRS invariant (and gauge parameter independent) operators exist and are supersymmetric as well as μ^2 independent.

4. Supersymmetry in the Wess-Zumino gauge and without auxiliary fields

As mentioned above one can formulate susy gauge theories also in terms of canonical fields of spin $0, \frac{1}{2}, 1$ with canonical dimensions $1, \frac{3}{2}, 1$. But then the susy and gauge transformations are non-linear, intertwined, do not close off-shell and form an open algebra, which makes itself felt on the gauge fixing term: it is part of an infinite chain of operators [10]. The solution [7,8] of these combined difficulties came with the introduction of $\phi\pi$-like fields for *all* transformations[2] (not only for the rigid gauge transformations):

$$\begin{aligned}
sA^i_\mu &= (D_\mu c)^i + \varepsilon^\alpha \sigma_{\mu\alpha\dot\beta}\bar\lambda^{i\dot\beta} + \lambda^{i\alpha}\sigma_{\mu\alpha\dot\beta}\bar\varepsilon^{\dot\beta} - i\xi^\nu\partial_\nu A^i_\mu\,,\\
s\lambda^{i\alpha} &= -f^{ijk}c^j\lambda^{k\alpha} - \tfrac{1}{2}\varepsilon^\gamma\sigma^{\mu\nu}{}_\gamma{}^\alpha F^i_{\mu\nu} - \tfrac{i}{2}g^2\varepsilon^\alpha\left(\bar\phi_a T^i_{ab}\phi_b\right) - i\xi^\mu\partial_\mu\lambda^{i\alpha} - \eta\lambda^{i\alpha}\,,\\
s\phi_a &= -T^i_{ab}c^i\phi_b + 2\varepsilon^\alpha\psi_{a\alpha} - i\xi^\mu\partial_\mu\phi_a - \tfrac{2}{3}\eta\phi_a\,,\\
s\psi^\alpha_a &= -T^i_{ab}c^i\psi^\alpha_b - i\sigma^{\mu\alpha}{}_{\dot\beta}\bar\varepsilon^{\dot\beta}(D_\mu\phi)_a + 2\varepsilon^\alpha\bar\lambda_{(abc)}\bar\phi_b\bar\phi_c - i\xi^\mu\partial_\mu\psi^\alpha_a + \tfrac{1}{3}\eta\psi^\alpha_a\,,\\
sc^i &= -\tfrac{1}{2}f^{ijk}c^jc^k - 2i\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\bar\varepsilon^{\dot\beta}A^i_\mu - i\xi^\mu\partial_\mu c^i\,,\\
s\bar c^i &= b^i - i\xi^\mu\partial_\mu\bar c^i\,,\\
sb^i &= -2i\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\bar\varepsilon^{\dot\beta}\partial_\mu\bar c^i - i\xi^\mu\partial_\mu b^i\,,\\
s\xi_\mu &= -2\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\bar\varepsilon^{\dot\beta}\,,\\
s\varepsilon^\alpha &= -\eta\varepsilon^\alpha\,,\\
s\eta &= 0\,.
\end{aligned}\qquad(4.1)$$

In this transformation law $c, \bar c$ and b are resp. the ghost, anti-ghost and Lagrange multiplier field; $\varepsilon^\alpha, \xi^\mu, \eta$ are the global ghosts associated to supersymmetry, translations, R-transformation. Like for ordinary BRS transformations the algebra of the transformation law is contained in the nilpotency of the transformations

$$s^2\phi = \text{field equation}\qquad(4.2)$$

which closes for spinors only on the equations of motion, but this is sufficient for having a manageable gauge fixing. So, in the Landau gauge one can choose

$$\Gamma_{\text{g.f.}} = s\int \bar c\partial A = \int\left(b^i\partial A^i + \partial^\mu\bar c^i((D_\mu c)^i + \varepsilon^\alpha\sigma_{\mu\alpha\dot\beta}\bar\lambda^{i\dot\beta} + \lambda^{i\alpha}\sigma_{\mu\alpha\dot\beta}\bar\varepsilon^{\dot\beta})\right)\qquad(4.3)$$

Like in ordinary YM theory on couples external fields to the non-linear field transformations. In addititon one introduces in this external field part terms which are quadratic in external fields. They render the problem of off-shell closure solvable and correct for terms going with field equations.

$$\Gamma_{\text{ext.f.}} = \int(A^{*i\mu}(sA^i_\mu) + \lambda^{*i\alpha}(s\lambda^i_\alpha) + \bar\lambda^{*i}_{\dot\beta}(s\bar\lambda^{i\dot\beta})$$

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[2] We follow Ref. [8].

$$+\phi^*_a(s\phi_a) + \bar{\phi}^*_a(s\bar{\phi}_a) + \psi^{*\alpha}_a(s\psi_{a\alpha}) + \bar{\psi}^*_{a\dot{\beta}}(s\bar{\psi}^{\dot{\beta}}_a)$$
$$+c^{*i}(sc^i) - \frac{g^2}{2}(\varepsilon^\alpha\lambda^{*i}_\alpha - \bar{\varepsilon}_{\dot{\beta}}\bar{\lambda}^{*i\dot{\beta}})^2 + 2\varepsilon^\alpha\psi^*_{a\alpha}\bar{\varepsilon}_{\dot{\beta}}\bar{\psi}^{*\dot{\beta}}) \,. \quad (4.4)$$

The complete action

$$\Gamma = \Gamma_{inv} + \Gamma_{\text{g.f.}} + \Gamma_{\text{ext.f.}} \quad (4.5)$$

then satisfies the Slavnov–Taylor identity

$$s(\Gamma) = 0\,, \quad (4.6)$$

where (s.[7] Eq.(2.13))

$$s(\Gamma) \equiv \int \frac{\delta\Gamma}{\delta A^{*i\mu}}\frac{\delta\Gamma}{\delta A^i_\mu} + \frac{\delta\Gamma}{\delta\lambda^{*i}_\alpha}\frac{\delta\Gamma}{\delta\lambda^{i\alpha}} + \frac{\delta\Gamma}{\delta\bar{\lambda}^{*i\dot{\beta}}}\frac{\delta\Gamma}{\delta\bar{\lambda}^i_{\dot{\beta}}}$$
$$+\frac{\delta\Gamma}{\delta\phi^*_a}\frac{\delta\Gamma}{\delta\phi_a} + \frac{\delta\Gamma}{\delta\bar{\phi}^*_a}\frac{\delta\Gamma}{\delta\bar{\phi}_a} + \frac{\delta\Gamma}{\delta\psi^*_{a\alpha}}\frac{\delta\Gamma}{\delta\psi^\alpha_a} + \frac{\delta\Gamma}{\delta\bar{\psi}^{*\dot{\beta}}_a}\frac{\delta\Gamma}{\delta\bar{\psi}_{a\dot{\beta}}}$$
$$+\frac{\delta\Gamma}{\delta c^{*i}}\frac{\delta\Gamma}{\delta c^i} + \left(b^i - i\xi^\mu\partial_\mu\bar{c}^i\right)\frac{\delta\Gamma}{\delta\bar{c}^i}$$
$$+\left(-2i\varepsilon^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\varepsilon}^{\dot{\beta}}\partial_\mu\bar{c}^i - i\xi^\mu\partial_\mu b^i\right)\frac{\delta\Gamma}{\delta b^i}$$
$$-2\varepsilon^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\varepsilon}^{\dot{\beta}}\frac{\delta\Gamma}{\delta\xi^\mu} - \eta\varepsilon^\alpha\frac{\delta\Gamma}{\delta\epsilon^\alpha} - \eta\bar{\varepsilon}_{\dot{\beta}}\frac{\delta\Gamma}{\delta\bar{\varepsilon}_{\dot{\beta}}}\,. \quad (4.7)$$

For the purposes of renormalization it is crucial that the linearized functional BRS operator, to which it gives rise, is off-shell nilpotent:

$$s_\Gamma \equiv \int\left(\frac{\delta\Gamma}{\delta A^{*i\mu}}\frac{\delta}{\delta A^i_\mu} + \frac{\delta\Gamma}{\delta A^{i\mu}}\frac{\delta}{\delta A^{*i}_\mu} + \ldots\right.$$
$$+\left(b^i - i\xi^\mu\partial_\mu\bar{c}^i\right)\frac{\delta}{\delta\bar{c}^i} + \left(-2i\varepsilon^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\varepsilon}^{\dot{\beta}}\partial_\mu\bar{c} - i\xi^\mu\partial_\mu b^i\right)\frac{\delta}{\delta b^i}$$
$$\left.-2\varepsilon^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\varepsilon}^{\dot{\beta}}\frac{\partial}{\partial\xi^\mu} - \eta\varepsilon^\alpha\frac{\partial}{\partial\varepsilon^\alpha} - \eta\bar{\varepsilon}_{\dot{\beta}}\frac{\partial}{\partial\bar{\varepsilon}_{\dot{\beta}}}\right), \quad (4.8)$$
$$s^2_\Gamma = 0\,. \quad (4.9)$$

It turns out that the ST identity can be extablished to all orders of perturbation theory by adding suitable counterterms in all fields, provided the usual anomaly is absent (cp.(3.33)).
By construction the ST identity contains all relevant symmetries of the model and it is very instructive to bring them to light individually. Since s_Γ contains all symmetries and s^2_Γ their algebra one expands in powers of

the ghosts and reads then off the desired relations. A tool to formalize this procedure is the *filtration* operator

$$\mathcal{N} \equiv \varepsilon^\alpha \frac{\partial}{\partial \varepsilon^\alpha} + \bar{\varepsilon}^{\dot\alpha} \frac{\partial}{\partial \bar{\varepsilon}^{\dot\alpha}} + \xi^\mu \frac{\partial}{\partial \xi^\mu} + \eta \frac{\partial}{\partial \eta} . \tag{4.10}$$

It counts the degree in the global ghosts:

$$s_\Gamma = \sum_{n\geq 0} (s_\Gamma)_n \qquad \text{with} \qquad [\mathcal{N}, (s_\Gamma)_n] = n (s_\Gamma)_n . \tag{4.11}$$

The generators of susy, translations and R-transformations (the "rigid symmetries") can now be identified by the expansion

$$\begin{aligned} (s_\Gamma)_1 &= \varepsilon^\alpha W_\alpha + \bar{\varepsilon}^{\dot\alpha} \bar{W}_{\dot\alpha} + \xi^\mu W_\mu + \eta W_R \\ &\quad -2\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha} \bar{\varepsilon}^{\dot\alpha} \frac{\partial}{\partial \xi^\mu} - \eta \varepsilon^\alpha \frac{\partial}{\partial \varepsilon^\alpha} + \eta \bar{\varepsilon}^{\dot\alpha} \frac{\partial}{\partial \bar{\varepsilon}^{\dot\alpha}} , \end{aligned} \tag{4.12}$$

where the transformation law of the ghosts has been separated from the ghost linear part. It is clear that these WI operators will in general be non-local. The susy WI can be isolated as follows

$$W_\alpha(\Gamma) \equiv \sum_u \int \left(\frac{\delta}{\delta u^*} \frac{\partial}{\partial \varepsilon^\alpha} \Gamma \right)_{|0} \frac{\delta \Gamma_0}{\delta u} + \frac{\delta \Gamma_0}{\delta u^*} \left(\frac{\delta}{\delta u} \frac{\partial}{\partial \varepsilon^\alpha} \Gamma \right)_{|0} = 0 . \tag{4.13}$$

Here the sum runs over $u = A_\mu, \lambda, \bar\lambda, \phi, \bar\phi, \psi, \bar\psi, c$ and Γ_0, $|_0$ indicates that all ghosts have to be put equal to zero. The conjugate WI $\bar{W}_{\dot\alpha}(\Gamma)$ has the analogous form with the replacement $\varepsilon^\alpha \to \bar{\varepsilon}^{\dot\alpha}$. The nilpotency of s_Γ (4.8) yields at the orders 0,1,2

$$(s_\Gamma)_0^2 = 0 , \tag{4.14}$$

$$\{(s_\Gamma)_0, (s_\Gamma)_1\} = 0 , \tag{4.15}$$

$$(s_{\Gamma_1})^2 + \{(s_\Gamma)_0, (s_\Gamma)_2\} = 0 , \tag{4.16}$$

$(s_\Gamma)_0$ coincides with the usual linearized ST operator and thus

$$(s_\Gamma)_0 (Q \cdot \Gamma) = 0 \tag{4.17}$$

characterizes Q as a (gauge-) BRS invariant insertion. (4.14) expresses the commutativity of the ordinary BRS transformations with $(s_\Gamma)_1$. (4.15) implies that on (gauge-) BRS invariant insertions $(s_\Gamma)_1$ is nilpotent

$$(s_\Gamma)_1^2 (Q \cdot \Gamma) = -(s_\Gamma)_0 (s_\Gamma)_2 (Q \cdot \Gamma) \sim 0 . \tag{4.18}$$

(The tilde indicates equivalence under (gauge-) BRS: Q can differ from an equivalent Q' by a $(s_\Gamma)_0\hat{Q}$.)
This relation implies now the algebra of the rigid transformations indentified in (4.12)

$$\{W_\alpha, \bar{W}_{\dot\alpha}\} \sim 2\sigma^\mu_{\alpha\dot\alpha} W_\mu \tag{4.19}$$

$$[W_R, W_\alpha] \sim W_\alpha \qquad [W_R, \bar{W}_{\dot\alpha}] \sim -\bar{W}_{\dot\alpha} \tag{4.20}$$

(other (anti) commutators ~ 0).
And we thus have the result: on Green functions of (gauge-) BRS invariant insertions supersymmetry, translations and R-symmetry is realized. One can even show that the susy transformations are linear (s. app. A of [8]).

5. Conclusions

The quantization of $N = 1$ supersymmetric gauge theories requires the solution of either one of two problems:
— in the case of linearly realized susy (superfields) one has to deal with the vanishing canonical dimension of the vector superfield; this poses infrared troubles and leads to infinitely many (gauge) parameters in the theory;
— in the case of realization by canonical fields (Wess-Zumino gauge, auxiliary fields eliminated) one has to deal with nonlinear symmetry transformations and an open algebra.
Both problems have been solved ([5, 7, 8]) and lead to qualitative agreement: the Green functions of gauge invariant (and gauge parameter independent) operators exist and realize supersymmetry linearly. Supersymmetry is in both cases expressed by a Ward identity: (3.32), (4.13). The formulation of susy in terms of superfields makes obvious the multiplet structure (*e.g.* for composite objects), whereas the component formulation permits an easier comparison with non-supersymmetric theories. Hence both are important and deserve further study.

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REFERENCES

[1] J. Wess, B. Zumino, *Nucl. Phys.* **B70**, 39 (1974).

[2] J. Wess, J. Bagger, *Supersymmetry and Supergravity*, Princeton Univ. Press 1983.

[3] W. Hollik, Electroweak theory, 5-th Hellenic summer school 1995 hep-ph/9602380.

[4] W. Hollik, Tests of the standard model, Intern. Conf. Europhysics, Brussels 1995 hep-ph/9512232.

[5] O. Piguet, K. Sibold, *Renormalized Supersymmetry*, Birkhäuser Boston 1986.

[6] J. W. Juer, D. Storey, *Nucl. Phys.* **B 216**, 185 (1983).

[7] P. L. White, *Class. Quantum Grav.* **9**, 413, 1663 (1992); N. Maggiore, hep-th/9412092, in *Int. J. Mod. Phys.* **A**, in press; hep-th/9501057, in *Int. J. Mod. Phys.* **A**, in press.

[8] N. Maggiore, O. Piguet, S. Wolf, *Nucl. Phys.* **B458**, 403 (1996).

[9] O. Piguet, M. Schweda, K. Sibold, *Nucl. Phys.* **B174**, 183 (1980).

[10] P. Breitenlohner, D. Maison in: *Renormalization of Quantum Field Theories with Nonlinear Field Transformations*, Proceedings Ringberg Workshop eds. P. Breitenlohner, D. Maison, K. Sibold; Springer 1988.