IDENTICAL ROTATIONAL BANDS IN THE $A \sim 130$ SUPERDEFORMED REGION ANALYSED IN TERMS OF THE PSEUDOSPIN SYMMETRY*

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Dedicated to Wojciech Królikowski in honour of his 70th birthday

Assignments for the configurations underlying the formation of identical bands in terms of the eigenstates of rotating harmonic oscillator are discussed in superdeformed nuclei. The method which is based on the pseudo-SU(3) symmetry is applied to the superdeformed bands in nuclei from the $A \sim 130$ region.

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1. Introduction

The occurrence of the identical bands in some rotational bands of atomic nuclei is an intriguing phenomenon that can shed some light on the unresolved problems in nuclear structure. Two rotational bands are defined as identical if their dynamical moments of inertia $\mathcal{J}^{(2)}$ are equal. Following this definition a rather large class of identical bands has been found [1].

In the present paper we shall be interested in a certain subset of the above class obeying more stringent relations. We aim at analysing certain linear relations between the gamma-ray energies E_γ deexciting the bands. Three types of these relations have been discovered and widely discussed in the literature (*cf.* Refs. [2–4]. First, there exist pairs called *twin bands* or *zero-point bands* where the corresponding gamma-ray energies are equal.

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Second, the bands called *mid-point bands* where the gamma-ray energies in one band are arithmetic averages of the gamma rays in the other band. Finally, the *three quarter-point* (*one quarter-point*) band relations are characterized by the weighted average energies with weights 3/4 and 1/4 (1/4 and 3/4). These three types have been analysed [3] (see also [4]) in terms of the pseudo-spin symmetry that seems to play an essential role in the classification of identical bands in nuclei. It has been also shown explicitly [5, 6] that the three types of identity relations correspond to the three possible orientations of the nucleon pseudo-spin relative to the pseudo-orbital angular momentum. In this way the pseudo-spin symmetry offers a simple possible classification scheme for the assignments of the identical bands in terms of the individual-particle nucleonic orbits in the rotating potential. However, in most publications so far these rotating orbits are labelled by the Nilsson asymptotic quantum numbers that do not include rotation [7]. This way of classifying the nucleonic orbits seems inadequate in the presence of fast nuclear rotation since the Nilsson orbits are strongly mixed due to fast rotation. Another representation has, therefore been suggested [5] that is based on the rotating harmonic oscillator (RHO) in the pseudo-orbital space. This representation gives, therefore, more physical insight into the structure of the orbits.

In the present paper we intend to employ the new representation and to draw some physical conclusions about the nature of the orbits. For the illustration of the procedure we have chosen the $A \sim 130$ region where several superdeformed rotational bands have been found [8–12] and interesting structure of identical bands has been observed.

2. Rotational motion in a pseudo-spin representation

We shall now describe a simple model that will be employed in the description of the fast nuclear rotation. It is well known that the pseudo-spin symmetry (and its more restrictive version of $\widetilde{\mathrm{SU}}(3)$, *i.e.* the pseudo-SU(3) symmetry) [13–16] plays an essential role in description of nucleonic motion [16] especially with large deformations and fast rotation. Various physical quantities such as for example the particle angular momentum $\boldsymbol{j}$, its orbital part $\boldsymbol{l}$ and spin part $\boldsymbol{s}$ ($\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$) should be then transformed to the pseudo-SU(3) picture [16]. Although the $\boldsymbol{j}$ operator remains unchanged in the pseudo-spin picture ($\tilde{\boldsymbol{j}} = \boldsymbol{j}$) it is not the case for its orbital and spin components $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} = \tilde{\boldsymbol{j}} = \tilde{\boldsymbol{l}} + \tilde{\boldsymbol{s}}$ with $\boldsymbol{l} \neq \tilde{\boldsymbol{l}}$ and $\boldsymbol{s} \neq \tilde{\boldsymbol{s}}$ In addition the strength of the new $(\tilde{\boldsymbol{l}}.\tilde{\boldsymbol{s}})$ becomes so small that it can be neglected. Thus in the pseudo-spin picture the pseudo-orbital motion is decoupled from its pseudo-spin part. In this situation the motion can be approximated by the pure harmonic oscillator. In principle, a term taking into account the

existence the flat bottom (like for example the familiar l^2 term in the Nilsson potential) should be also included. In the first approximation, however, this term will be neglected. Thus we shall deal with the pure rotating harmonic oscillator(RHO) only.

Within this approximation a nucleonic orbit can be characterized by the three quantum numbers , say n_1, n_2 and n_3 and the eigenvalues of the Routhian (*i.e.* the energy in a rotating frame) can be given by the Valatin formula ([17])

$$e^{\omega}_{\nu} = (n_1 + 1/2)\omega_1 + (n_2 + 1/2)\Omega_2 + (n_3 + 1/2)\Omega_3 \,. \tag{1}$$

In the above formula the tildas over all the symbols have been omitted and it has been assumed that $\hbar = 1$. Here the motion along the 1-axis remains unaffected by the rotation (since 1-is the rotation axis) while the remaining (normal mode) frequencies Ω_2 and Ω_3 are functions of the original (HO) frequencies ω_1, ω_2 and ω_3 as well as of the rotational frequency ω. More explicitly

$$\Omega_{2/3} = \sqrt{\frac{\omega_2^2 + \omega_3^2}{2} + \omega^2 \pm (1/2)\Gamma}\,, \tag{2}$$

with

$$\Gamma = \sqrt{(\omega_2^2 - \omega_3^2)^2 + 8\omega^2(\omega_2^2 + \omega_3^2)}\,. \tag{3}$$

The pseudo-orbital wave functions corresponding to (Eq. (1)) are simply the harmonic oscillator wave functions

$$\mid n_1, n_2, n_3\rangle = \frac{1}{\sqrt{n_1! n_2! n_3!}} (\Gamma_1^{\dagger})^{n_1} (\Gamma_2^{\dagger})^{n_2} (\Gamma_3^{\dagger})^{n_3} \mid 0\rangle \,, \tag{4}$$

where $\Gamma_1^{\dagger}, \Gamma_2^{\dagger}$ and $\Gamma_3^{\dagger}$ denote the creation operators for the (HO) gamma quanta in the three directions. It is interesting to analyse what is the behaviour of the wave function (4) under the rotation of the system through an angle π about the rotation axis 1. It is easy to see that such a rotation leads to the change of signs in the operators $\Gamma_2^{\dagger}$ and $\Gamma_3^{\dagger}$ while the sign of $\Gamma_1^{\dagger}$ remains unchanged. Thus acting with the rotation operator in the pseudo-orbital space on the state (Eq. (4)) we obtain

$$e^{-i\pi\tilde{I}_1} \mid n_1, n_2, n_3\rangle = (-1)^{n_2+n_3} \mid n_1, n_2, n_3\rangle \,. \tag{5}$$

Here the rotation operator

$$R = e^{-i\pi\tilde{I}_1} \tag{6}$$

denotes the pseudo-orbital part of the signature operator. The total signature operator is

$$e^{-i\pi j_1} = e^{-i\pi \tilde{l}_1} e^{-i\pi \tilde{s}_1} . \tag{7}$$

Let us now see which configurations (n_1, n_2, n_3) are favorable for the formation of the identical bands [5]. It follows from the pseudo-SU(3) symmetry the pseudo- spin $\tilde{s}$ is entirely decoupled from the orbital motion (due to the absence of the $(\tilde{l}.\tilde{s})$ term in the nuclear Hamiltonian as already mentioned above).Thus the pseudo-orbital part plays a decisive role in the dynamics of the rotational motion. Suppose we add a valence nucleon to the nuclear even core. Its single-particle Routhian e^{ω}_{ν} is given by Eq. (1). The addition of a single-particle Routhian e^{ω}_{ν} to the even core with angular momentum I results in the formation of identical bands only if the contribution of the odd particle to the nuclear alignment

$$i_{\nu} = -\frac{\partial e^{\omega}_{\nu}}{\partial \omega} , \tag{8}$$

and the contribution to the dynamical moment of inertia

$$\delta \mathcal{J}^{(2)} = \frac{\partial i_{\nu}}{\partial \omega} , \tag{9}$$

are both negligibly small. The examination of formula (1) leads to the conclusion that this may happen if the quantum numbers n_2 and n_3 obey the relation

$$n_2 = n_3 . \tag{10}$$

This can be seen from Fig. 1 of Ref. [5]. Orbits of this type have been called (SO) (=special orbits). Orbits lying immediately above (SO) orbits (for example orbits (1,1,0) in Fig. 1 of Ref. [5]) are probably also good candidates since the nuclear rotation can cause a slight polarization in the (HO) potential tending to increase nuclear elongation [18]. Thus orbits of this type may also become flat. These orbits will be referred to as the (NSO) *i.e.* neighboring special orbits. In this way the special orbits (SO) and their neighbors (NSO) with slightly negative alignment i_{ν} (*cf.* Eq. (8)) may both be good candidates that lead to the formation of identical bands in the pair of nuclei with mass numbers A and $(A+1)$.

Now let us add the pseudo-spin to our consideration. This procedure has been described in Ref. [5] (see also [3] and [4]). It consist on adding one valence nucleon to the even system with angular momentum I. One possible way to describe this procedure would be to apply the concept of a nuclear mean field rotating with a rotational frequency ω. This procedure is known as the cranking model. One can then calculate angular momenta

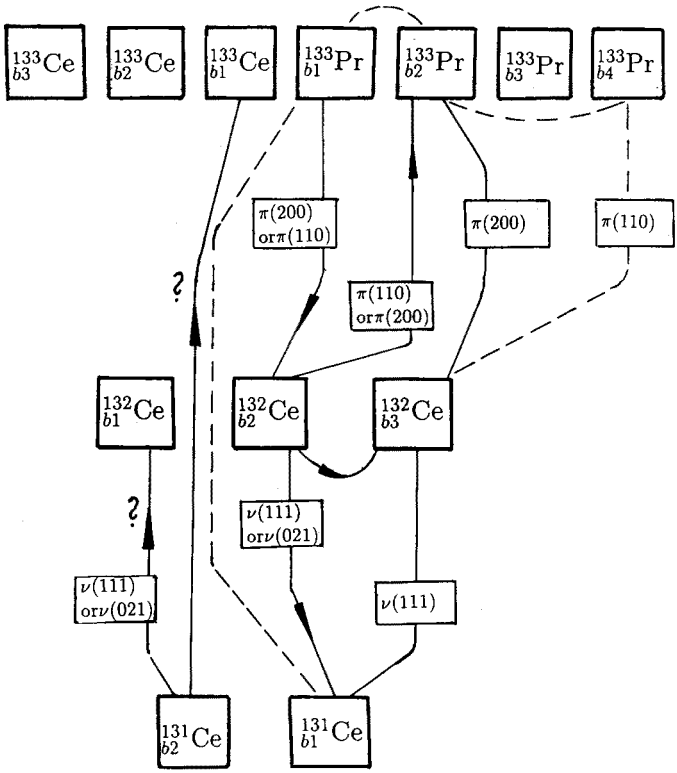

Fig. 1. Identical bands in the Ce-Pr region. Various superdeformed bands in one nucleus are labelled b1(yrast),b2,b3, *etc.* The zero-point (twin), mid-point, and coupled bands (quarter-point and three-quarter-point) are denoted by solid (no arrow), dashed (no arrow) and solid lines (with an arrow up or down), respectively (*cf.* Table I). Lines corresponding to pairs of bands in A, $A\pm1$ nuclei are labelled by the rotating harmonic oscillator quantum numbers for the relevant configurations. Only the one-particle states (*i.e.* the $A, A \pm 1$ pairs) are explicitly labelled in the figure. The bands in ^{132}Ce have been chosen as reference bands. Thus the bands in ^{133}Ce are considered as core plus one-neutron states, those in ^{133}Pr as one-proton states while states in ^{131}Ce as one-neutron holes. This figure is related to Fig. 16 in Ref. [17].

I' and I in the two bands as functions of ω. Their difference calculated for the same value of ω

$$i(\omega) = I'(\omega) - I(\omega) \tag{11}$$

is called relative alignment. This quantity can be understood as the contribution of the pseudo-spin added to the core. Thus we can expect that

$$i(\omega) = +\tfrac{1}{2} \text{ or } -\tfrac{1}{2}. \tag{12}$$

On the other hand, the simple results of the algebra of angular momenta tells us that when a 1/2 particle is added to the system with angular momentum I one obtains a system with angular momentum I' differing by one half integer unit of angular momentum. It follows from Eq. (5) to (7) that the difference in angular momenta $I' - I$ equals the sum of two signature exponents , the orbital one $\tilde{l}_1$ and the spin one $\tilde{s}_1$. Thus

$$I' - I = \tilde{l}_1 + \tilde{s}_1 \ (\mathrm{mod}\ 2)\,. \tag{13}$$

We have assumed here that both $\tilde{l}_1 and \tilde{s}_1$ are good quantum numbers. In this case it seems consistent to relate $\tilde{s}_1$ contribution to the alignment i

$$\tilde{s}_1 = i\,. \tag{14}$$

The relation between the difference $I' - I$ and the type of the identity relation has been discussed in [5] (*cf.* also [3] and [4]).

Taking into account all the above considerations we can summarize the above results in the first four rows of Table I *i.e.* in case when $\tilde{l}_1$ and $\tilde{s}_1$ are good quantum numbers.

TABLE I

Quantum numbers of valence orbit underlying the creation of identical bands. First column specifies the type of identity relation. Second column explains the suggested graphical notation employed in Fig. 1 (abbreviations: [sl no a.]= solid line with no arrow, [dl no a.]= dashed line with no arrow, [sl a. up]= solid line with an arrow pointing up and [sl a. dn]= solid line with an arrow pointing down). Third column exhibits the energy relations between E' (= energy of the core plus valence state) (E_l, E_u) denoting the lower and upper energies in the core band (sometimes the interval (E_l, E_u) may be shifted by an amount corresponding to the change of two units of angular momentum). Forth column gives the corresponding difference in angular momenta I' and I characterizing the core-plus-particle and the core system. This relation should be understood as an equation (mod 2). Fifth column shows two quantum numbers $\tilde{l}_1$ and $\tilde{s}_1$ entering eqs.(6) and (7). Finally, sixth column gives the value of the alignment (*cf.* Eq. (11)).

Type	Notation	Energy formula	$I' - I$	l,s	i
0-point	sl no a.	$E' = E_l$	+1/2	0,+1/2	+1/2
0-point	sl no a.	$E' = E_l$	-1/2	0,-1/2	-1/2
mid-point	dl no a.	$E' = (E_l + E_u)/2$	-1/2	1,+1/2	+1/2
mid-point	dl no a.	$E' = (E_l + E_u)/2$	+1/2	1,-1/2	-1/2
1/4-point	sl a. up	$E' = (3E_l + E_u)/4$	+1/2	0,+1/2	0
1/4-point	sl a. up	$E' = (3E_l + E_u)/4$	+1/2	1,-1/2	0
3/4-point	sl a. dn	$E' = (E_l + 3E_u)/4$	-1/2	1,+1/2	0
3/4-point	sl a. dn	$E' = (E_l + 3E_u)/4$	-1/2	0,-1/2	0

Examining the first four rows of Table I one can see that the twin bands occur for the valence orbits having the structure of special bands (SO) ($n_2 = n_3$) while the mid-point bands must be (NSO), for example with $n_2 = n_3 + 1$.

Let us now start the analysis of the coupled bands (*i.e.* the three quarter- or one quarter-point bands). As previously we assume that a single-nucleon orbit is added to the core with angular momentum I forming a composite system with angular momentum I'. It follows from the algebra of angular momentum that $I' - I$ must be half integer. Thus

$$I' - I = \tfrac{1}{2} \pmod 2, \tag{15}$$

or

$$I' - I = -\tfrac{1}{2} \pmod 2. \tag{16}$$

On the other hand, one has to assume that the relative alignments between the two bands vanishes. It has been argued in Ref. [5] that in this case the relation (Eq. (15)) implies the weighted average expression for the gamma-ray energy E' the relation:

$$E' = (3/4)E_l + (1/4)E_u. \tag{17}$$

Now the difference $I' - I$ is equal to the difference between the signature exponents (mod 2) *i.e.* the sum of $\tilde{l}_1 + \tilde{s}_1$. Finally, the pseudo-orbi- tal part $\tilde{l}_1$ of the signature exponent equals 0 for the (SO) or else 1 for the (NSO). In this way we obtain the full classification of all the possible states for the single particle added to the core (*cf.* rows 5 to 8 in Table I). The above explanation for the assignments does not seem to be entirely consistent. All the sets of quantum numbers given in rows 5 to 8 in Table I correspond to definite alignments different from zero. This is incompatible with the starting assumption $i = 0$ introduced above for the coupled case. Perhaps some mixed states composed out of these listed in Table I should be taken into account in the coupled case. Nevertheless, we shall apply the assignments exhibited in Table I as explained above with no further reservations.

3. Assignments for identical bands in the superdeformed region around $A \sim 130$

We have chosen the $A \sim 130$ region in order to illustrate the possible single-particle assignments in terms of the (RHO) quantum numbers and explain the consequences that follow from this procedure. Several SD bands have been observed in this region and numerous identical pairs of these bands have been suggested ([8–12]). The identity relations have been

discussed for the pairs of bands in (^{131}Ce, ^{132}Ce) , (^{132}Ce, ^{133}Ce) and (^{132}Ce, ^{133}Pr). Nuclear deformations in this region are generally of the order of $\epsilon = 0.34$ which is not far from $\epsilon = 3/8$ corresponding to the 3:2 ratio of axes This is a region of considerably lower deformations as compared to the 2:1 ratio observed in the $A \sim 150$ region. Nevertheless typical SD bands have been observed . The appearance of the superdeformation in this region is connected with the existence of some appreciable gaps that occur in the spectrum of the independent Routhians at the neutron number $N = 74$ and proton number $Z = 58$ (*cf.* Ref. [19]).

Let us now search for the more detailed analysis of the independent particle Routhians in connection with the existence of the identical bands. We shall assume that the pairing correlations are negligible. As follows from the considerations in Chapter 2 the occurrence of the identical bands requires the presence of the appropriate orbits for both neutrons and protons in this region.

The general chart of the existing identical bands in the $A \sim 130$ region is given in Fig. 1. Here, lines joining particular bands (solid line with no arrow, dashed line, solid line with an arrow pointing upward or downward) denote various types of identity relations which are taken from experiments with labels established in Table I. The chart has been drawn treating the ^{133}Ce and ^{133}Pr as one-particle states (neutron, or proton) while the ^{131}Ce — as one-hole states with respect to the ^{132}Ce nucleus treated as a reference. This statement is important since the whole system of lines and arrows in Fig. 1 corresponds to the notation treating ^{132}Ce as a reference. In the $A \sim 130$ region the relevant (SO) configurations are (2,0,0) and (1,1,0) (NSO) for protons and the (1,1,1) (SO) together with the (0,2,1) (NSO) for neutrons. We also assume the bands in ^{131}Ce as one-hole states with respect to the ^{132}Ce reference.

Now let us examine more closely some details of Fig. 1 Let us start with the triangle formed by band b2 in ^{132}Ce, band b3 in ^{132}Ce and band b1 (yrast) in ^{131}Ce. We suggest that ^{131}Ce (band b1)is a one-neutron hole with respect to the core ^{132}Ce (band b2) of the 3/4-point type. Thus the single-particle configuration for this hole state can be assigned as (SO) ν (1,1,1) according to consideration in Section 2 (*cf.* Table I). The other possible choice would be an (NSO) ν (0,2,1). On the other hand, the single-hole for the pair ^{132}Ce (b3) and ^{131}Ce (b1) which is of the zero-point type (*cf.* Fig. 1 and Table I) can be only an (SO) ν (1,1,1). No other assignment like the (NSO)) is possible for a zero-point type (*cf.* Table I). The third link of the triangle corresponds to b2 and b3 in ^{132}Ce. It is the 3/4-type and must be a one-particle plus one hole configuration (treating the b2 as a reference) connected with the change of the (SO) ν (1,1,1) (3/4-point) into the (SO) ν (1,1,1) (0-point) in ^{131}Ce (b3). The other possible choice

would be an (NSO) ν (0,2,1) into the (SO) ν (1,1,1) (zero point for the same pair of bands. This example hopefully illustrates well the principles for the assignment procedure. In a similar way one may assign the links between the (SD) bands in ^{132}Ce and ^{133}Pr nuclei shown in Fig. 1, or the (not so certain, marked with "?") links starting from ^{131}Ce (b2).

Obviously, the absence of some links in Fig. 1 implies that the corresponding bands are not identical so that they are based on configuration different from those mentioned up to now.

It should be stressed that all the above assignments are rather tentative. In fact they are not based on the detailed examination of the single-particle Routhians as functions of the rotational frequency ω. We believe, however, that such a detailed calculation may depend on many model-dependent parameters and it is rather difficult to confront them with experiment. The order of the single-particle Routhians has not been yet established by reliable experiments up to now. On the other hand the appearance of the identical bands which is rather known experimentally may perhaps help by providing some data on the location of some Routhian orbits and thus understand better the structure in a deformed fast rotating nucleus.

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