SCALAR FIELD FLUCTUATIONS IN THE RADIATION DOMINATED POSTINFLATIONARY UNIVERSE*

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(Received February 9, 1996)

Dedicated to Wojciech Królikowski in honour of his 70th birthday

It is shown that quantum fluctuations due to a nontrivial gravitational background in the flat radiation dominated universe can play an important cosmological role generating nonvanishing cosmological global charge, *e.g.* baryon number, asymmetry. The explicit form of the fluctuations at vacuum and at finite temperature is given. Implications for particle physics are discussed.

PACS numbers: 98.80.Cq

1. Introduction

Since the early eighties it has been widely recognized that quantum fluctuations of scalar matter fields may play an important role in cosmology, especially in the context of the inflationary de Sitter epoch [1]. The reason is that in the de Sitter space fluctuations of the light fields ($m^2/H_I^2 \ll 1$, where H_I is the de Sitter Hubble parameter) grow linearly with time assuming finally a significantly large value of the order of $\mathrm{H}_\mathrm{I}^4/m^2$. It is believed that fluctuations produced at the time of inflation are seen during subsequent stages of the evolution of the universe as energy density inhomogeneities responsible for the formation of the large scale structure. It is also argued that those fluctuations set initial conditions for the classical evolution of fields in subsequent epochs.

* Work partially supported by the Polish KBN Grant.

In contrast to the above, it is usually assumed that gravitationally induced fluctuations in spatially flat radiation dominated (RD) and matter dominated (MD) epochs are irrelevant for particle physics. We would like to point out that this assumption is not properly discussed in the literature. On one hand, one observes that in the RD universe the fluctuations (as explained in this letter) decrease in time. On the other hand, they may in principle be large enough to control violation of some symmetries or to alter the evolution of some fields. This problem becomes particularly important in view of the ongoing search for a reliable mechanism for production of the baryon asymmetry in the Universe, the need of better understanding of the scenarios for late phase transitions and discussions of the possible lepton number nonconservation.

In this letter we address the problem of quantum fluctuations of a massive scalar field during the RD epoch. This epoch covers most of the history of the Universe, and the temperature range from, say, 10^{14} GeV down to 10 eV. On that energy scale one can find a lot of interesting phenomena in popular extensions of the standard model such as its supersymmetric version or string inspired models.

The paper is organized as follows. In Section 2 we set our notation and subsequently evaluate fluctuations of a massive scalar field in the RD flat Robertson–Walker space at vacuum and at finite temperature. In Section 3 we apply our formulae to a generic field theoretical model with particular attention paid to two specific examples resembling the Affleck–Dine model [2] and the so-called spontaneous baryogenesis scenario [3].

2. Scalar field fluctuations in the RD universe

The RD Universe is the solution to the Einstein's equations with the energy-momentum tensor in the form $T^\mu_\nu = \mathrm{diag}(\rho, -p, -p, -p)$. Tracelessness of the T^μ_ν implies equation of state for the content of the RD Universe: $\rho = 3p$. In this letter we assume a flat RD space endowed with the metric $g_{\mu\nu} = \mathrm{diag}(1, -a^2(t), -a^2(t), -a^2(t))$ where $a(t)$ is the RW scale factor given by $a(t) \equiv (t/t_0)^{1/2}$, t_0 being the beginning of the RD epoch.

The results presented in this paper can be easily generalized to the matter dominated era.

We couple a massive scalar field to gravity in the minimal way (note that in the RD epoch the curvature scalar R vanishes identically)

$$S[\phi] = \int d^4x\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right) . \tag{1}$$

As usual in this type of analysis we assume that there is no "back-reaction" of the scalar field on the metric, *cf.* [4]. The equation resulting from Eq. (1) is ($k = |\vec{k}|$)

$$\left(\frac{d^2}{dt^2} + 3\frac{\dot{a}}{a}\frac{d}{dt} + \frac{k^2}{a^2} + m^2\right)\phi_k(t) = 0\,, \tag{2}$$

where $\phi_k(t)$ is the spatial Fourier transform of the field $\phi(\vec{x},t)$,

$$\phi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3 2k}\left(\phi_k(t)e^{i\vec{k}\vec{x}}a_k^\dagger + \text{h.c.}\right) \tag{3}$$

with $a^\dagger$ and its Hermitian conjugate denoting standard creation and annihilation operators respectively.

A general solution of the equation (2) for $a(t) = (t/t_0)^{1/2}$ is given by confluent hypergeometric functions

$$\begin{aligned}\phi_k(t) = {} & A_1(k,m)2ikt_0 e^{-imt}{}_1F_1\left(\frac{3}{4} + \frac{ik^2t_0}{2m}, \frac{3}{2}, 2imt\right) \\ & + A_2(k,m)\frac{\sqrt{t_0}}{\sqrt{t}}e^{-imt}{}_1F_1\left(\frac{1}{4} + \frac{ik^2t_0}{2m}, \frac{1}{2}, 2imt\right).\end{aligned} \tag{4}$$

The two undetermined coefficients A_1 and A_2 , which may in principle depend on both m and k, are not independent if one takes into account quantization condition imposed on a field ϕ

$$[\phi(\vec{x},t), \partial_t\phi(\vec{y},t)] = \frac{i}{\sqrt{-g}}\delta^{(3)}(\vec{x}-\vec{y}) \tag{5}$$

$$\left[a_k, a_{k'}^\dagger\right] = (2\pi)^3 2k\delta^{(3)}(\vec{k}-\vec{k}') \tag{6}$$

From these equations and the decomposition (3) we get a normalization condition

$$\text{Im}(\bar{\phi}_k(t)\partial_t\phi_k(t)) = \frac{k}{\sqrt{-g}} \tag{7}$$

which translates into the constraint on A_1 and A_2 (we confine them to be real)

$$A_1(k,m)A_2(k,m) = 1\,. \tag{8}$$

In most general case there are several ways of fixing both coefficients. One possibility is to use initial conditions set at the timelike surface $t = t_0$ for ϕ and $\partial_t\phi$. This is the proper procedure if one knows for example the explicit solution for ϕ in the epoch preceding the RD one. We do not assume such a detailed knowledge, hence we use an alternative approach instead. We demand that the "correct" mode functions we choose, which will define our Fock space, should approach at short distances ($k \to \infty$) the massless positive frequency solution,

$$\phi_k(t) \to \frac{\sqrt{t_0}}{\sqrt{t}} e^{2ik\sqrt{tt_0}} . \tag{9}$$

In this way we obtain the asymptotic behaviour of both coefficients

$$A_{1,2}(k,m) \to 1 . \tag{10}$$

Here we assume that $A_1 = A_2 = 1$, what completes the definition of our Fock space.

We set out to calculate the fluctuations of the field ϕ *i.e.* $\langle 0|\phi^2|0\rangle$. This quantity is divergent and needs renormalization. We calculate the Green's function $\langle \phi(\vec{x},t)\phi(\vec{x}',t)\rangle$, separate out the piece divergent when $|\vec{x}-\vec{x}'| \to 0$ and define the renormalized fluctuations as the remaining piece when $|\vec{x}-\vec{x}'| = 0$.

$$\begin{aligned}\langle \phi(\vec{x},t)\phi(\vec{x}',t)\rangle &= \int \frac{d^3k}{(2\pi)^3 2k}\left(|\phi_k(t)|^2 e^{i\vec{k}(\vec{x}-\vec{x}')}\right)\\ &= \frac{1}{4\pi^2}\int dk\ k\ \frac{\sin(k|\vec{x}-\vec{x}'|)}{k|\vec{x}-\vec{x}'|}\ |\phi_k(t)|^2 . \end{aligned} \tag{11}$$

Introducing the physical distance $\sigma = |\vec{x}-\vec{x}'|a(t)$ we may write ($y = k\sqrt{tt_0}/(mt)$)

$$\begin{aligned}\langle \phi(\vec{x},t)\phi(\vec{x}',t)\rangle &= \frac{m^2}{4\pi^2}\int_0^\infty dy\ y\frac{\sin(my\sigma)}{my\sigma}|f(y,mt)|^2\\ &= \frac{m^2}{4\pi^2}\int_0^\infty dy\ y\frac{\sin(my\sigma)}{my\sigma}\frac{y}{\sqrt{y^2+1}}\\ &+ \frac{m^2}{4\pi^2}\int_0^\infty dy\ y\frac{\sin(my\sigma)}{my\sigma}\left(|f(y,mt)|^2 - \frac{y}{\sqrt{y^2+1}}\right), \end{aligned} \tag{12}$$

where

$$f(y,mt) = {}_1F_1\left(\frac{1}{4}+\frac{iy^2mt}{2},\frac{1}{2},2imt\right) + 2iymt\,{}_1F_1\left(\frac{3}{4}+\frac{iy^2mt}{2},\frac{3}{2},2imt\right) . \tag{13}$$

The first term in (12) contains all the short distance divergencies of the Green function:

$$G_{\rm div} = \frac{m^2}{4\pi^2}\frac{K_1(m\sigma)}{m\sigma} \underset{a\to\infty}{\longrightarrow} \frac{m^2}{4\pi^2}\left(\frac{1}{m^2\sigma^2} + \frac{1}{2}\ln(m\sigma) + {\rm const} + O(\sigma)\right). \tag{14}$$

The second factor in (12) is finite when $\sigma \to 0$ and defines $\langle\phi^2\rangle_R$ (up to a constant which can always be added)

$$\langle\phi^2\rangle_R = \frac{m^2}{4\pi^2}\int\limits_0^\infty dy\; y\left(|f(y,mt)|^2 - \frac{y}{\sqrt{y^2+1}}\right). \tag{15}$$

Unfortunately, $\langle\phi^2\rangle_R$ cannot be explicitly evaluated in its most general form. However, it is possible to write down the systematic expansion of the mode functions (4) and the integral (15) in terms of mt. Using such an expansion we will be able to discuss reliably fluctuations in the regime of small mass and to control the passage to the massless limit. For the region $mt > 1$ we apply the WKB method.

In the case $mt < 1$, the relevant expansion of modes is given by (*cf.* [5])

$$f(y,mt) = \sum_{n=0}^{\infty}\left[p_n^{(-1/2)}(2mt)j_{n-1}(2ymt) + ip_n^{(1/2)}(2mt)j_n(2ymt)\right]\frac{1}{(2ymt)^{n-1}}. \tag{16}$$

The j_n is the n-th spherical Bessel function and coefficient $p_n^{(\mu)}$ can be read from

$$\sum_{n=0}^{\infty} p_n^{(\mu)}(z)w^n = \exp\left(\frac{z}{2}\left(\cot(zw) - \frac{1}{zw}\right)\right)\left(\frac{zw}{\sin(zw)}\right)^{1-\mu}. \tag{17}$$

One easily finds that

$$|f(y,mt)|^2 = \left[1 + (mt)^2\left(\frac{\sin^2(2ymt)}{8(ymt)^4} - \frac{1}{2(ymt)^2}\right) + \ldots\right]. \tag{18}$$

On the basis of the expansion we see that indeed $\langle\phi^2\rangle_R$ is the ultraviolet-finite quantity. It is also infrared finite, since the modes are perfectly regular functions for $k \to 0$ (*i.e.* $y \to 0$).

In the region $mt < 1$, with the help of the expansion (18), we get the following formula for the leading behaviour:

$$\langle\phi^2\rangle_R = \frac{m^2}{8\pi^2}\left(-\ln(mt) + {\rm const} + O((mt)^2)\right). \tag{19}$$

One should note that the fluctuations vanish as m approaches zero and grow with m if we keep mt constant. This agrees with the earlier result for an exactly massless field reported in Ref. [6]. The interesting feature of the formula (19) is its non-analyticity in mt and the appearance of the logarithmic singularity at $t = 0$ which is related to the singularity of the RD Universe at $t = 0$.

In the regime $mt > 1$ we use the WKB expansion to get the amplitude of fluctuations ([4]). When we change time t to the conformal time η defined as

$$d\eta = \frac{dt}{a(t)} \tag{20}$$

and introduce

$$u_k(\eta) = a(\eta)\phi_k(\eta) \tag{21}$$

then the equation (2) is transformed to

$$\epsilon^2 \frac{d^2 u_k}{d\eta^2} + \omega^2(\eta) u_k(\eta) = 0 \tag{22}$$

with $\epsilon = 1$ and

$$\omega^2(\eta) = k^2 + m^2 a^2 \,. \tag{23}$$

Now we substitute WKB expression

$$u_k(\eta) = \frac{\sqrt{k}}{\sqrt{f(\eta)}} \exp\left(i \int \sqrt{f(\eta)} d\eta \right) \tag{23}$$

and expand

$$f(\eta) = \frac{\omega(\eta)}{\epsilon} + \epsilon f_1(\eta) + \epsilon^3 f_3(\eta) + \dots . \tag{25}$$

The solution to (22) is given by

$$f_1(\eta) = \frac{3(\omega')^2 - 2\omega\omega''}{8\omega^3} \tag{26}$$

$$f_3(\eta) = \frac{-297(\omega')^4 + 396\omega(\omega')^2\omega'' - 52\omega^2(\omega'')^2 - 80\omega^2\omega'\omega''' + 8\omega^3\omega''''}{128\omega^7} . \tag{27}$$

The amplitude of fluctuations is given by

$$\langle \phi^2 \rangle_R = \frac{1}{4\pi^2 a^2} \int_0^\infty dk\ k^2 \left(\frac{1}{f(\eta)} - \frac{\epsilon}{\omega(\eta)} \right) \tag{28}$$

$$\frac{1}{4\pi^2 a^2} \int_0^\infty dk\ k^2 \left(-\frac{\epsilon^3 f_1}{\omega^2} - \frac{\epsilon^5 f_3}{\omega^2} + \frac{\epsilon^5 (f_1)^2}{\omega^3} + O(\epsilon^7) \right) . \tag{29}$$

The procedure outlined above remains valid in any era — we now restrict ourselves to the radiation epoch. In this era

$$a(\eta) = \eta/(2t_0)\,, \qquad \omega(\eta) = \sqrt{k^2 + \frac{m^2\eta^2}{4t_0^2}} \tag{30}$$

and the result of integration is

$$\langle\phi^2\rangle_R = \frac{16t_0^4}{240\pi^2 m^2 \eta^8} = \frac{1}{16 * 240\pi^2 m^2 t^4} = \frac{H(t)^4}{240\pi^2 m^2}\,. \tag{31}$$

Up to now we have been calculating curved space vacuum expectation value of ϕ^2. However, if we were to take into account that the Universe is "hot", *i.e.* it is in fact in a mixed state to which many-particle states may contribute significantly, we should better calculate a thermal average of ϕ^2, with finite temperature effects included. Assuming thermal equilibrium of the content of the Universe we have

$$\langle\phi^2\rangle|_{T\geq 0} = \int \frac{d^3k}{(2\pi)^3 2k} |\phi_k|^2 (1 + 2n_k)\,, \tag{32}$$

where ϕ_k are modes given by (4), (10), and n_k is the occupation number for the particles with the comoving momentum k. As n_k we take

$$n_k = \frac{1}{\exp\left(\sqrt{k^2/T_i^2 + m^2/T^2}\right) - 1} \tag{33}$$

which is correct for sufficiently large k in view of the choice (9) (T_i is the temperature at the onset of the radiation era, close to the reheating temperature). The (32) is again divergent. However, as usual in finite-temperature calculations, it may be divided into $T = 0$ part and the temperature correction, among which only the former is UV divergent. Hence, we can use the renormalization procedure (11) to get meaningful results even at $T > 0$

$$\langle\phi^2\rangle|_{\mathrm{renormalized},T\geq 0} \to \langle\phi^2\rangle_R + \langle\phi^2\rangle_R^T, \tag{34}$$

where

$$\langle\phi^2\rangle_R^T = 2\int \frac{d^3k}{(2\pi)^3 2k} |\phi_k|^2 \frac{1}{\exp\left(\sqrt{k^2/T_i^2 + m^2/T^2}\right) - 1}\,. \tag{35}$$

This expression may be approximated analytically in two limiting cases: a) $m/T \gg 1$, and b) $m/T \ll 1$. In the case b) one easily gets

$$\langle\phi^2\rangle_R^T = \frac{T^2}{12}\,, \tag{36}$$

exactly as in the flat Minkowski case. In the case a) one can see that in the region which dominates the integral, $k < mT_i/T$, the second term in (4) is unimportant. Hence we obtain

$$\langle\phi^2\rangle_R^T \sim \mathrm{e}^{-m/T} \tag{37}$$

which is exponentially suppressed.

3. Implications for particle physics in the expanding Universe

Let us consider a global U(1) symmetry realized in a single complex scalar field model. If Q is the charge of the field χ, the Noether current associated with that symmetry is

$$j^\mu = iQ\{\bar{\chi}\partial^\mu\chi - \chi\partial^\mu\bar{\chi}\}\,, \tag{38}$$

(we put Q≡1 in what follows) and the conservation law for j^μ in the expanding Universe reads

$$\partial_\mu(a^3(t)j^\mu) = -ia^3(t)\left\{\bar{\chi}\frac{\partial V}{\partial\bar{\chi}} - \chi\frac{\partial V}{\partial\chi}\right\}. \tag{39}$$

One can see that a symmetry is broken once the rhs of (39) is nonvanishing. One can see also that when a symmetry is broken explicitly, the net cosmological charge density gets generated according to the formula

$$a^{-3}\frac{d}{dt}(j^0a^3(t)) \approx -i\left\{\bar{\chi}\frac{\partial V}{\partial\bar{\chi}} - \chi\frac{\partial V}{\partial\chi}\right\}. \tag{40}$$

Let us assume that the term violating the symmetry is

$$\delta V = \frac{\lambda}{2n\Lambda^{2n}}\phi^{2n}\,, \tag{41}$$

where $\phi \equiv \mathrm{Re}(\chi)$ (in this section we assume the absence of derivative couplings, they will be discussed later). Suppose that the initial conditions and the shape of the potential are such that the $\mathrm{Im}(\chi)$ and its fluctuations are negligible when compared with $\mathrm{Re}(\chi)$ at any time t (this situation may be easily realized in the Affleck–Dine model, *cf.* [8]). Hence

$$a^{-3}\frac{d}{dt}(j^0a^3(t)) \approx i\frac{\lambda}{\Lambda^{2n}}\phi^{2n}. \tag{42}$$

We can see that the magnitude of the symmetry violation is proportional to a coupling λ, inverse powers of some scale Λ if $n > 2$, and to some power of the scalar field ϕ. One can say that, λ and Λ being fixed in a given theory, the expectation value of ϕ determines the amount of symmetry breaking. Here the quantum fluctuations of the field ϕ come into play. In the quasiclassical picture one can describe the evolution of the quantum field, lets call it Φ, writing it down as the superposition of the quasiclassical field ϕ which obeys essentially classical (perhaps perturbatively corrected) equation of motion and quantum fluctuations $\delta\phi$, the dispersion squared of which we identify as $\langle\phi^2\rangle_R$. If the potential drives the quasiclassical field to zero, then the magnitude of the symmetry breaking term is determined by the dispersion of $\delta\phi$. Using $\langle\phi\rangle^{2n} = (\langle\phi^2\rangle_R)^n$ one gets an estimate

$$a^{-3}\frac{d}{dt}(j^0 a^3(t)) = i\frac{\lambda}{\Lambda^{2n}}\left(\frac{H(t)^4}{240\pi^2 m^2}\right)^n . \tag{43}$$

In general the field ϕ has some additional couplings to light particles, which facilitate its decay with the decay width Γ_ϕ. This changes the behaviour of the classical field ϕ, namely $\phi^2 \to \exp(-\Gamma t)\ \phi^2$. Actually, as pointed out by several authors in the context of the Affleck–Dine mechanism (which corresponds to our toy model when $n = 2$) the Γ_ϕ should be reasonably large in order to avoid an unobservable excess of the net charge produced during symmetry violation [8]. We want to stress that in such a case, the $\langle\phi^2\rangle_R$, decaying accordingly to the power law, dominates the divergence of the Nother current and the net cosmological charge density even at the late times.

As next example let us consider models where a massive scalar ϕ is derivatively coupled to other particle species. This situation corresponds for instance to models possessing pseudogoldstone bosons with nonvanishing masses. The relevant scenario is similar to that of the "spontaneous baryogenesis" described in Ref. [5]. If a Lagrangian has a coupling of the form $L_\phi = -\frac{1}{f}\phi\partial_\mu j^\mu$ (f being some, presumably large, mass scale) where $\partial_\mu j^\mu$ is a divergence of a current corresponding to some explicitly broken symmetry, the baryon number symmetry for instance. Then, as we have shown, there are fluctuations in the field ϕ with dispersion $\sqrt{\langle\phi^2\rangle_R}$. We may represent them as the effective term in the Lagrangian

$$L_{\delta\phi} = -\frac{1}{f}\sqrt{\langle\phi^2\rangle_R}\partial_\mu j^\mu . \tag{44}$$

Up to the total divergence (44) is equivalent to

$$L_{\delta\phi} = \frac{1}{f}\partial_0\sqrt{\langle\phi^2\rangle_R}j^0 . \tag{45}$$

This produces an effective chemical potential $\mu = -(1/f)\partial_0\sqrt{\langle\phi^2\rangle_R}$ for the charge density j^0 which means a nonzero cosmological charge density generated in thermal equilibrium. Explicitly, *cf.* [9],

$$j^0 \approx -\frac{1}{f}\partial_t\sqrt{\langle\phi^2\rangle_R}T^2\,, \tag{46}$$

or charge to entropy ratio

$$\frac{j^0}{s} \approx -\frac{1}{fg_*T}\partial_t\sqrt{\langle\phi^2\rangle_R}\,, \tag{47}$$

where g_* is the number of relativistic degrees of freedom at temperature T. The above estimate gives in the case of our toy model

$$\frac{j^0}{s} \approx \frac{m}{4g_*fT}\frac{1}{t\sqrt{\ln(1/mt)}}\,, \tag{48}$$

when $mt \ll 1$, *i.e.* at very high temperatures T, and

$$\frac{j^0}{s} \approx \frac{O(10^{-2})}{g_*fT}\frac{1}{mt^3}\,, \tag{49}$$

for $mt > 1$, hence at late times — low temperatures. One can see that both expressions fall off as time elapses, as $\sim T^5$ at late times and as $\sim T$ at the beginning of the RD epoch. If there is no phase transition in the model before the end of RD epoch, then the final charge to entropy ratio produced will be equal to (48) or (49) taken at the "decoupling" temperature T_D. This is the temperature at which symmetry violating interactions fall off from equilibrium or the one which corresponds to the end of RD stage, when the shape of the fluctuations changes qualitatively *i.e.* at $T_D \approx T_f$ close to 10 eV. As previously, the numerical values predicted depend on various details of a model under investigation. For example, let us take $T_D = 10$ eV and $g_* = 100$. Then if we require the charge-to-entropy ratio to be equal to 10^{-10}, as it should be for the baryonic charge, then we get the condition $m = f^4 \times 10^{-77}$GeV, which gives $m = 10^{-17}$GeV for $f = 10^{15}$GeV and $m = 1$GeV for $f = 10^{19}$GeV.

4. Conclusions

In this letter we have given explicit expressions for a massive scalar field fluctuations in the flat radiation dominated universe. It turns out that in the region of small mt, *i.e.* shortly after the beginning of the RD epoch or for very light fields, the fluctuations decrease with time only logarithmically and are proportional to the square of the mass of the field in question. For large mt, *i.e.* very late or for a heavy field, the time dependence is stronger, $1/t^4$. As far as finite temperatures are concerned, we have concluded that the "radiation-dominated" background modifies Minkowski space results very weakly. In general, fluctuations vanish when one takes the limit $m \to 0$.

Given all that, the postinflationary fluctuations can still play a significant role in particle physics models, which has been illustrated in the second part of this note. The case when our parameter n equals 2 corresponds precisely to the Affleck–Dine model, and the higher n terms are often encountered in the important class of string inspired supergravities. The late fluctuations constitute the phenomenon which is relevant when some cosmological charge density, first of all the baryonic charge density, is supposed to be generated during the radiation dominated epoch. The results presented in this paper can be easily generalized to the matter dominated era.

REFERENCES

[1] J.M. Bardeen, P.J. Steinhardt, M.S. Turner, *Phys. Rev.* **D28**, 679 (1983); A.D.Linde, *Phys. Lett.* **116B**, 335 (1982).
[2] I. Affleck, M. Dine, *Nucl. Phys.* **B249**, 361 (1985).
[3] A. Cohen, D. Kaplan, *Phys. Lett.* **199B**, 251 (1987).
[4] N. Birrel, P.C. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge 1984.
[5] H. Buchholz, *Die Konfluente Hypergeometrische Funktion*, Springer-Verlag, Berlin 1953.
[6] C. Pathinayake, L.H. Ford, *Phys. Rev.* **D37**, 2099 (1988).
[7] M. Abramowitz,I. Stegun, *Handbook of Mathematical Functions*, Dover Publications, Inc., New York.
[8] A.D. Dolgov, Yukawa Institute preprint YITP/K-940, 1991.
[9] E.W. Kolb, M.S. Turner, *The Early Universe*, Addison-Wesley Publishing Company, 1990.