TOP HIGGS

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Dedicated to Wojciech Królikowski in honour of his 70th birthday

For $m_{\rm top} > 150$ GeV there exists an $(m_{\rm top}, m_{\rm Higgs})$-area in which both the SM and the MSSM are excluded. But the requirement of vacuum stability in a 2-Higgs model fits into this region. With supersymmetric SU(5) one can overcome by part the socalled hierarchy problem. Further, SU(5) and SO(10) supersymmetric GUT models with fixed point solutions and non-linear realizations of SUSY belong also to this area. Finally one can show that the SM originates from its minimal supersymmetric extension which is explicitely broken due to soft SUSY breaking terms at scale $M_{\rm SUSY}$, and that the NMSSM has roughly no influence on the previous predictions. So far the discovery of Top at FNAL "localizes" Higgs.

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Within as well as well-defined theoretical models and applied theoretical methods [1] the discovery of Top at FNAL [2] "localizes" Higgs.

To establish this Top-Higgs idea lets start from the beginning with 2-Higgs doublets [3], a model which covers an (m_t, m_H)-area which whether belongs to the SM nor MSSM [4]. But, using the RGE-improved EP for these 2-Higgs doublets, the requirement of vacuum stability establishes a lower bound on the Higgs mass, briefly illustrated here in Fig. 1 (see Ref. [3]).

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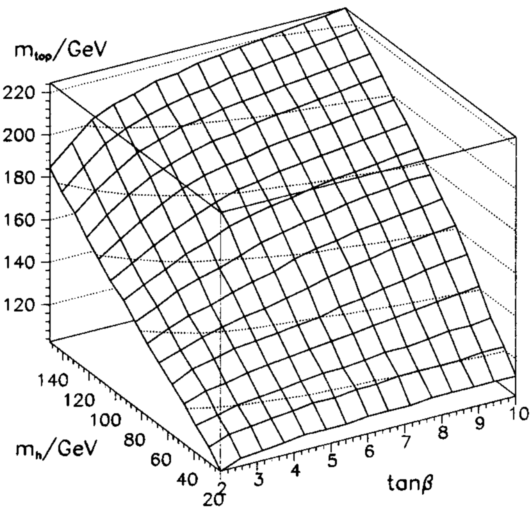

Fig. 1. Upper bound on $m_{\rm top}$ as a function of m_h and $\tan\beta$. The bound is valid for $\tan\beta > 4$ and will be about 10 GeV lower than indicated here for smaller values. (Other input values: $m_l = 0$GeV, $m_\pm = m_P = 100$GeV)

Below these bounds the Higgs can be composite [5] in this instabil region.

To overcome by part the scale dependence — the socalled hierarchy problem — we use supersymmetric SU(5) with standard kinetic terms and with the superpotential

$$
\begin{aligned}
W = {} & \sqrt{2}\lambda_1 T_{\alpha\beta}\left(\overline{F}^\alpha \overline{H}^\beta - \overline{F}^\beta \overline{H}^\alpha\right) \\
& + \frac{1}{4}\lambda_2 \epsilon^{\alpha\beta\gamma\delta\epsilon} T_{\alpha\beta} T_{\gamma\delta} H_\epsilon \\
& + \lambda_3 \operatorname{Tr}\phi^3 + \lambda_4 \overline{H}\phi H + M_1 \operatorname{Tr}\phi^2 + M_2 \overline{H} H \,. \qquad (1)
\end{aligned}
$$

Here we consider only the third (top quark) supermatter generation and neglect all Yukawa coupling constants of the first and second generations. The model contains the matter supermultipletts $\overline{F}^\alpha(\overline{\mathbf{5}})$ and $T_{\alpha\beta}(\mathbf{10})$ which correspond to the third generation and the superhiggses $\phi(\mathbf{24}), \overline{H}^\alpha(\overline{\mathbf{5}})$ and $H_\alpha(\mathbf{5})$.

The nonzero vacuum solution $\langle\phi\rangle$ = Diag(2,2,2,-3,-3) breaks the SU(5) gauge group to the electroweak SU(3)×SU(2)×U(1) gauge group. For energies higher than the grand unified scale $M_{\rm GUT}$ we have effective restoration of the SU(5) gauge symmetry. We shall neglect all Yukawa coupling constants in the effective potential, except that one for the top quark Yukawa coupling constant λ_2 . The corresponding renormalization group equations in one-loop approximation have the form (for the case of three generations)

$$\frac{d\bar{h}_t^2}{dt} = A\bar{h}_t^4 - B\bar{h}_t^2\bar{g}_5^2\,, \tag{2}$$

$$\frac{d\bar{g}_5^2}{dt} = -b\bar{g}_5^4\,. \tag{3}$$

Here g_5 is the SU(5) gauge coupling, $16\pi^2 t = \ln(M/\mu)$, $b = 6$, $A = 18$, $B = 192/5$, and $h_t = \lambda_2$. The solution of the Eqs (2), (3) has the form

$$\bar{g}_5^2 = \frac{g_5^2}{1 + g_5^2 bt}\,, \tag{4}$$

$$\frac{1}{\bar{h}_t^2} = A(g_5^2(B-b))^{-1}(1+g_5^2 bt) + \left(\frac{1}{h^2} - A\left((B-b)g_5^2\right)^{-1}\right)(1+g_5^2 bt)^{B/b}\,. \tag{5}$$

We shall use the numerical value $\alpha_{\mathrm{GUT}} = g_5^2/4\pi = 1/24$ for the SU(5) gauge coupling constant at the unification scale $M_{\mathrm{GUT}} = 10^{16}\,\mathrm{GeV}$. From the requirement of the absence of the Landau pole singularity for the effective top quark coupling constant $\bar{h}_t$ up to the Planck scale $M_{PL} = 1.2\times 10^{19}\,\mathrm{GeV}$ we find that the top quark Yukawa coupling constant $\bar{h}_t(M_{\mathrm{GUT}}) \leq 1.3$. For the renormalization group equations (2), (3) the infrared fixed point solution [6] is

$$\bar{h}_t^2 = k\bar{g}_5^2, \quad k = \frac{B-b}{A} = 1.8\,. \tag{6}$$

For the infrared fixed point solution (6) we find that $\bar{h}_t^2(M_{\mathrm{GUT}}) = 0.94$. It should be noted that in our analysis we neglected all Yukawa coupling constants in the superpotential (1) except that one for the top quark constant. However the neglected Yukawa coupling constants give positive contribution to the β-functions that can only make the bound $\bar{h}_t(M_{\mathrm{GUT}}) \leq 1.3$ more stringent.

The renormalization group equations for the SU(3)×SU(2)×U(1) electroweak supersymmetric gauge theory allow to connect the top quark Yukawa coupling constant at grand unified scale with the Yukawa coupling constant at the scale of the supersymmetry breaking M_{SUSY}; to relate the Yukawa coupling constant at the M_{SUSY} scale with the observable Yukawa coupling constant at the electroweak scale M_W we have to use the renormalization group equations for the standard Weinberg–Salam model. The renormalization group equations have the form [7, 8]

$$\frac{d\overline{h}_t^2}{dt} = A_1\overline{h}_t^4 - \overline{h}_t^2\left(B_1\overline{g}_1^2 + B_2\overline{g}_2^2 + B_3\overline{g}_3^2\right), \tag{7}$$

$$\frac{d\overline{g}_i^2}{dt} = -b_i\overline{g}_i^4 \tag{8}$$

with $b_1 = -22$, $b_2 = -2$, $b_3 = 6$, $A_1 = 12$, $B_1 = 26/9$, $B_2 = 6$, $B_3 = 32/3$ for the supersymmetric case and with $b_1 = -41/3$, $b_2 = 19/3$, $b_3 = 14$, $A_1 = 9$, $B_1 = 17/6$, $B_2 = 9/2$, $B_3 = 16$ for the standard case. The solution of the equation (7) can be represented in the form

$$\frac{1}{\overline{h}^2(M)} = \prod_{i=1}^{3}\left(1 + g_i^2 b_i t\right)^{B_i/b_i}\left(\frac{1}{\overline{h}^2(\mu)} - A_1K(t)\right),$$

$$K(t) = -\int_0^t dx\prod_{i=0}^{3}(1 + g_i^2 b_i x)^{-B_i/b_i}. \tag{9}$$

Numerically, for $M = 0.1\text{TeV}$ we find

$$\frac{1}{\overline{h}_t^2}(0.1\text{TeV}) = \frac{0.088}{\overline{h}_t^2(M_{\text{GUT}})} + 0.815. \tag{10}$$

Using our previously derived bound $\overline{h}_t(M_{\text{GUT}}) \leq 1.3$ we find that $\overline{h}_t(0.1\text{TeV}) \leq 1.08$. The top quark mass is equal to $m_t = \overline{h}(m_t)\langle H\rangle$, where $v^2 = (174\text{GeV})^2 = \langle H^2\rangle + \langle\overline{H}^2\rangle$ and $\langle H\rangle = v\cos\beta$. So our bound on the top quark mass reads

$$m_t = \overline{h}(m_t)v\cos\beta \leq 187\text{GeV}. \tag{11}$$

Note that from the requirement of the absence of the Landau pole singularity for the effective top quark Yukawa coupling constant up to GUT scale we find that $m_t \leq 193\text{GeV}$. For the fixed point solution the Yukawa coupling constant $\overline{h}_t(0.1\text{TeV}) = 1.05$ and $m_t = 183\text{GeV}$ for $\langle H\rangle = 174\text{GeV}$.

To estimate the Higgs boson mass we have to solve the renormalization group equation for the selfinteraction constant $\overline{\lambda}$ of the light Higgs doublet which at one loop level reads [9]

$$\frac{d\overline{\lambda}}{dt} = 12\left(\overline{\lambda}^2 + \left(\overline{h}_t^2 - \tfrac{1}{4}\overline{g}_1^2 - \tfrac{3}{4}\overline{g}_2^2\right)\overline{\lambda} - \overline{h}_t^4 + \tfrac{1}{16}\overline{g}_1^4 + \tfrac{1}{8}\overline{g}_1^2\overline{g}_2^2 + \tfrac{3}{16}\overline{g}_2^4\right). \tag{12}$$

Here λ is the Higgs doublet selfinteraction coupling constant. Besides we have to solve the eqs. (7/8) for the energies between m_t and $M_{\rm SUSY}$. We have found that for $M_{\rm SUSY} = 1$TeV the value of the $\overline{h}_t(m_t)$ (for 120GeV $< m_t <$ 200GeV) is increased approximately by 3% compared to the case $M_{\rm SUSY} = 0.1$TeV. We have solved the renormalization group equation (12) together with the boundary condition

$$\overline{\lambda}\Big|_{p^2=M^2_{\rm SUSY}} = \frac{\overline{g}_1^2 + \overline{g}_2^2}{4}\cos^2 2\beta\,. \tag{13}$$

Remember that $\cos\beta = m_t/(\overline{h}_t(m_t)v)$ and the knowledge of the $\overline{h}_t(m_t)$ allows to relate the top quark mass with $\overline{\lambda}(m_t^2)$ and hence to determine the Higgs boson mass

$$m_{\rm H}^2 = 2\overline{\lambda}(m_t^2)v^2\,. \tag{14}$$

The results of our calculations for the top quark masses in the interval 165GeV $\leq m_t \leq$ 180GeV are presented in the Table I.

TABLE I

The dependence of the Higgs mass on the top quark mass in the assumption that the supersymmetriy breaking scale is $M_{\rm SUSY} = 1$ TeV. The uncertainty in the calculation of m_t reflects the assumed 5% uncertainty in determination of $\overline{h}_t(m_t)$.

m_t [GeV]	$m_{\rm H}$ [GeV]
165	89^{-7}_{+10}
170	97^{-8}_{+11}
180	105^{-9}_{+11}

It is wellknown that SUSY can be realized nonlinearly [10] and the question, whether SUSY is realized in nature linearly or nonlinearly, is still open. Samuel and Wess have shown [11] how to extend the SM in a nonlinear supersymmetric way.

A characteristic feature of the nonlinear realization is that no supersymmetric partners are required. In global nonlinear SUSY the only additional field which has to be introduced is the Akulov–Volkov field (AVfield [10]), a Goldstone fermion (Goldstino). Experimentally no Goldstino has been observed. A possibility to avoid a massless physical Goldstino is to go to the curved space, to supergravity. In supergravity the Goldstino can be "gauged" away; the massless gravitino absorbs via the super Higgs mechanism [12] the Goldstino and becomes massive, whereas the graviton remains massless. In flat limit where the supergravity multiplet decouples from the ordinary matter the fermionic particle spectrum is the same as in the Standard Model without SUSY. The only reminiscence of SUSY manifests itself in the Higgs sector. The Higgs sector has to be extended. Now an extention of the Higgs sector is also necessary in linearly realized supersymmetric models. Both realizations need at least two Higgs doublets. This is a common aspect of the two realizations. There are however differences. A difference is that on the superfield level two Higgs doublets superfields suffice in the linear SUSY models, whereas in the nonlinear case an additional singlet Higgs field is required, either as auxiliary field or as dynamical one or as both. If the singlet field is an auxiliary one, the Physical Higgs boson spectrum of the nonlinear models are the same as that of the linear minimal SUSY model called the Minimal Supersymmetric Standard Model (MSSM). They differ, however, in general in the structure of the Higgs potentials in the component fields.

If the singlet Higgs field is both auxiliary and dynamical, the physical Higgs boson spectrum resembles to that of the linear model with one Higgs singlet, the next to the Minimal Supersymmetric Standard Model (NMSSM) [13], whereas there are typical differences in the structure of the Higgs potentials. Samuel and Wess considered an illustrative simple example for the Higgs sector assuming the Higgs doublets to be dynamical fields and the Higgs singlet to be auxiliary [11]. Kim [14] investigated the general case where the Higgs doublets and singlet fields are both dynamical and auxiliary. Upper bounds for the lightest Higgs scalar mass and mass relations between various Higgs bosons were derived.

So far starting with the Lagrangian of the nonlinear supersymmetric extension of the SM in curved space [11] and requirering that there are no physical singlet Higgs bosons, but only Higgs doublets, the Higgs potential in flat limit is given as

$$\begin{aligned} V = {} & \tfrac{1}{8}(g_1^2 + g_2^2)\left(|H_1|^2 - |H_2|^2\right) + \tfrac{1}{2}|H_1^\dagger H_2|^2 \\ & + \lambda_0^2 |H_1^{\mathrm{tr}} \epsilon H_2|^2 + \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 \end{aligned} \tag{15}$$

and the masses of the two neutral physical Higgs scalars $m_{S_1} < m_{S_2}$, the pseudoscalar m_P and the charged Higgs boson m_C are given by

$$\begin{aligned} m_{S_1,S_2}^2 = {} & \tfrac{1}{2}\Big(m_Z^2 + m_P^2 \\ & \pm \sqrt{(m_Z^2 - m_P^2)^2 \cos^2 2\beta + [m_Z^2(4\lambda_g^2 - 1) - m_P^2]^2 \sin^2 2\beta}\Big) \end{aligned} \tag{16}$$

$$m_C^2 = m_W^2 + m_P^2 - 2\lambda_g m_Z^2 = m_W^2 + \mu_1^2 + \mu_2^2 \tag{17}$$

$$m_P^2 = \frac{2\lambda_0 M^2}{\sin 2\beta} \tag{18}$$

$$\lambda_g^2 = \frac{\lambda_0^2}{g_1^2 + g_2^2}$$

Further, the following relations hold

$$m_{S_1}^2 \le m_Z^2 \left[\cos^2 2\beta + 2\lambda_g^2 \sin^2 2\beta\right] , \tag{19}$$

$$m_{S_1}^2 + m_{S_2}^2 = m_P^2 + m_Z^2 , \tag{20}$$

$$m_C^2 + 2\lambda_g^2 m_Z^2 = m_W^2 + m_P^2 , \tag{21}$$

where Eqs (19), (21) are different from those of MSSM, whereas Eq. (20) is the same as in MSSM.

In the case of $\mu_1 = \mu_2 = 0$ this model reduces to the model of Ref. [11] and one obtains

$$m_C^2 = m_W^2 \,,\; m_P^2 = 2\lambda_g^2 m_Z^2 \,,\; M_{S_1}^2 = 2\lambda_g^2 m_Z^2 \text{ and } m_{S_2}^2 = m_Z^2 \text{ for } \lambda_g \le \frac{1}{\sqrt{2}}$$

and

$$m_{S_1}^2 = m_Z^2 \quad \text{and} \quad m_{S_2}^2 = 2\lambda_g^2 m_Z^2 \quad \text{for} \quad \lambda_g > \frac{1}{\sqrt{2}} .$$

So far again nonlinear realizations of SUSY belong also to the $(m_t, m_{\rm H})$ area as mentioned above.

Now let us turn to the dependence of Higgs boson mass on the scale of SUSY breaking in the MSSM. So our main assumption is that the standard Weinberg–Salam model originates from its minimal supersymmetric extension which is explicitly broken due to soft supersymmetry breaking terms at a scale $M_{\rm SUSY}$. The tree level Higgs potential in the minimal supersymmetric Weinberg–Salam model with general soft supersymmetry breaking terms is given by

$$\begin{aligned} V(H_1, H_2) = {} & m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 \left(H_1^{\rm tr} \epsilon H_2 + \text{h.c.}\right) \\ & + \frac{g_1^2 + g_2^2}{8} \left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \frac{g_2^2}{2} \left(H_1^\dagger H_2\right)^2 . \end{aligned} \tag{22}$$

Here g_1 and g_2 are the U(1) and SU(2) gauge coupling constants and the Higgs doublets H_1 and H_2 couple with $q = 1/3$ and $q = 2/3$ quarks respectively. We assume that one of the combinations of the H_1 and H_2

$$H_{\rm light} = H_2 \cos\beta + i\sigma_2 H_1 \sin\beta \tag{23}$$

is relatively light, $m_{\rm light} \approx O(m_Z)$, whereas the other orthogonal combination

$$H_{\rm heavy} = -H_2 \sin\beta + i\sigma_2 H_1 \cos\beta \tag{24}$$

acquires a mass $m_{\rm heavy} \approx O(M_{\rm SUSY})$. We also assume that the masses of the superpartners of ordinary particles are of the order of $O(M_{\rm SUSY})$. It is clear that in our scenario for $M_{\rm SUSY} \gtrsim O(1{\rm TeV})$ it is necessary to have fine tuning among the soft supersymmetry breaking terms, however at present we do not have any idea how to realize fine tuning in a natural way.

At scales lower than the supersymmetry breaking scale $M_{\rm SUSY}$ we have the standard Weinberg–Salam model with the single light Higgs isodoublet $H = H_{\rm light}$. The crucial point is that from the explicit formula for the effectie potential (22) we find that the selfinteraction effective coupling constant λ for the light Higgs doublet H at scale $M_{\rm SUSY}$ is

$$0 \le \overline{\lambda}(M_{\rm SUSY}) = \left(\overline{g}_1^2(M_{\rm SUSY}) + \overline{g}_2^2(M_{\rm SUSY})\right) \frac{\cos^2 2\beta}{4}$$
$$\le \tfrac{1}{4}\left(\overline{g}_1^2(M_{\rm SUSY}) + \overline{g}_2^2(M_{\rm SUSY})\right) . \quad (25)$$

So the assumption that standard Weinberg–Salam model originates from its supersymmetric extension with the supersymmetry broken at scale $M_{\rm SUSY}$ allows us to obtain nontrivial information about the low energy effective Higgs selfcoupling constant in the effective potential $V = m^2 H^\dagger H + \lambda/2(H^\dagger H)^2$ and hence to obtain nontrivial information about the Higgs boson mass. To relate the high energy value (25) of $\overline{\lambda}(M_{\rm SUSY})$ with the low energy value of $\overline{\lambda}(M_Z)$ we use the renormalization group equations which at one loop read

$$\frac{d\overline{g}_3}{dt} = -7\overline{g}_3^2 , \quad \frac{d\overline{g}_2}{dt} = -\tfrac{19}{6}\overline{g}_2^3 , \quad \frac{d\overline{g}_1}{dt} = \tfrac{41}{6}\overline{g}_1^2 ,$$
$$\frac{d\overline{h}_t}{dt} = \left(\tfrac{9}{2}\overline{h}_t^2 - 8\overline{g}_3^2 - \tfrac{9}{4}\overline{g}_2^2 - \tfrac{17}{12}\overline{g}_1^2\right)\overline{h}_t ,$$
$$\frac{d\overline{\lambda}}{dt} = 12\left(\overline{\lambda}^2 + \left(\overline{h}_t^2 - \tfrac{1}{4}\overline{g}_1^2 - \tfrac{3}{4}\overline{g}_2\right)\overline{\lambda} - \overline{h}_t^4 + \tfrac{1}{16}\overline{g}_1^4 + \tfrac{1}{8}\overline{g}_1^2\overline{g}_2^2 + \tfrac{3}{16}\overline{g}_2^2\right) \quad (26)$$
$$t = \frac{1}{16\pi^2}\ln\frac{\mu}{m_Z} .$$

Here $\overline{g}_3, \overline{g}_2, \overline{g}_1$ are the SU(3), SU(2) and U(1) gauge coupling constants, respectively, and $\overline{h}_t$ is the top quark Yukawa coupling constant. In our analysis we neglected all Yukawa coupling constants except top quark coupling constant. We use the following values for the gauge coupling constants at electroweak scale (see Refs [15]):

$$\overline{\alpha}_3(M_Z) = 0.120 \pm 0.01 ,$$
$$\overline{\alpha}_{\rm em}^{-1}(m_Z) = 127.9 \pm 0.2 ,$$
$$\sin^2\theta_W(M_Z) = 0.2327 \pm 0.0008 . \quad (27)$$

These value correspond to electroweak gauge couplings of

$$\overline{\alpha}_1^{-1} = 58.59 \pm 0.11 , \quad \overline{\alpha}_2^{-1} = 29.75 \pm 0.11 . \quad (28)$$

In our numerical analysis we took the central values for $\overline{\alpha}_1(m_Z)$ and $\overline{\alpha}_2(m_Z)$ since the uncertainty in the determination of $\overline{\alpha}_1(m_Z), \overline{\alpha}_2(m_Z)$ practically does not change the allowed regions for the Higgs boson mass. The results of our numerical analysis for different values of top quark mass are summarized in Figs 2(a)–2(d). The curve 1 correspond to the initial value $\overline{\lambda}(M_{\rm SUSY}) = +\infty$ and it is the upperbound on the Higgs boson mass which comes from the requirement of the absence of the Landau pole singularity for the Higgs selfcoupling constant $\overline{\lambda}$ up to scala $M_{\rm SUSY}$. The curve 2 corresponds to the boundary condition $\overline{\lambda}(M_{\rm SUSY}) = (\overline{g}_1^2(M_{\rm SUSY}) + \overline{g}_2^2(M_{\rm SUSY}))/4$ and it is the upper bound on the Higgs boson mass provided the standard Weinberg-Salam model originates from the minimal supersymmetric Weinberg-Salam model. The curve 3 correspond to the boundary condition $\overline{\lambda}(M_{\rm SUSY}) = 0$ and it is lower bound on the Higgs boson mass. So the allowable values of the Higgs boson mass lie between the curves 2 and 3. Note that solid curves 2 and 3 correspond to the initial value $\overline{\alpha}_3(m_Z) = 0.120$ and the curves 2a(b), 3a(b) to the initial values $\overline{\alpha}_s(m_Z) = 0.110, (0.130)$. The curve 3 coincides with the vacuum stability bound which comes from the requirement that $\overline{\lambda}(\mu) \geq 0$ for $\mu \leq M_{\rm SUSY}$. For $M_{\rm SUSY} \leq 10^4$TeV the allowed values for the Higgs boson mass practically don't depend on the value of supersymmetry breaking scale $M_{\rm SUSY}$. We used the current top quark mass definition $m_t(m_t) = \overline{h}_t(m_t)\langle H\rangle$ with $\langle H\rangle = 174$GeV. The relation between the pole top quark mass and the current top quark is [16] (see also Ref. [5] in Ref. [16]):

$$m_t^{\rm pole} = \left(1 + \frac{4}{3\pi}\alpha_3(m_t)\right) m_t(m_t) \approx 1.05 m_t(m_t)\,. \qquad (29)$$

It should be noted that in nonminimal supersymmetric electroweak models, say in the model with an additional gauge singlet supermultiplet σ we have due to the $k\sigma H_1\epsilon H_2$ term in the superpotential an additional term $k^2|H_1\epsilon H_2|^2$ in the potential and as a consequence our boundary condition for the Higgs selfcoupling constant has to be modified, namely

$$\overline{\lambda}(M_{\rm SUSY}) = \tfrac{1}{4}\left(\overline{g}_1^2(M_{\rm SUSY}) + \overline{g}_2^2(M_{\rm SUSY})\right)\cos^2 2\beta + \tfrac{1}{2}\overline{k}^2(M_{\rm SUSY})\sin^2 2\beta\,. \qquad (30)$$

The boundary condition (30) depends on unknown coupling constant $\overline{k}^2(M_{\rm SUSY})$, so in general the allowed Higgs boson mass interval is between curves 1 and 3. It is very important to stress that for all nonminimal supersymmetric models broken to standard Weinberg-Salam model at scale $M_{\rm SUSY}$ the effective Higgs selfcoupling constant $\overline{\lambda}(M_{\rm SUSY})$ is nonnegative that is direct consequence of the nonnegativity of the effective potential in supersymmetric models. So the curve 3 describes the lower bound on the Higgs boson mass in general case. The single condition is that the supersymmetry is broken at scale $M_{\rm SUSY}$ and at lower scales we have standard Weinberg-Salam model with the single Higgs isodoublet. To conclude, we have found the allowed regions for the Higgs boson mass in standard Weinberg-Salam model in the assumption that it originates from the minimal supersymmetric model with the supersymmetry broken at some scale $M_{\rm SUSY}$. The allowed Higgs boson mass region is between curves 2 and 3. In arbitrary case when standard Weinberg–Salam model originates from arbitrary supersymmetric model with the supersymmetry broken at scale $M_{\rm SUSY}$ the allowed Higgs boson mass region is between curves 1 and 3, an area belonging also again to the $(m_t, m_{\rm H})$-region mentioned above.

Finally, one should mention, that the NMSSM has roughly no influence on the previous prediction [17]. This can demonstrated briefly as follows:

$$\begin{aligned} V_{\rm SUSY} = V_F + V_D = |\lambda|^2 &\Big(\Big(H_1^\dagger H_1 + H_2^\dagger H_2\Big) N^* N \\ &+ (H_1^\dagger H_1)(H_2^\dagger H_2) - (H_1^\dagger H_2)(H_2^\dagger H_1)\Big) + |k|^2 (N^* N)^2 \\ &+ \Big(\lambda k^* H_1^{\rm tr} \epsilon H_2 N^{*2} + {\rm h.c.}\Big) + \frac{g_1^2 + g_2^2}{8}\Big(H_1^\dagger H_1 - H_2^\dagger H_2\Big)^2 \\ &+ \frac{g_2^2}{2}(H_1^\dagger H_2)(H_2^\dagger H_1) \end{aligned} \tag{31}$$

and

$$\begin{aligned} V_{\rm soft} &= m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + m_N^2 N^* N \\ &\quad - \lambda A_\lambda H_1^{\rm tr} \epsilon H_2 N - \frac{k A_k}{3} N^3 + {\rm h.c.} \end{aligned} \tag{32}$$

where the quartic terms are always positive and $\lambda, k, A_\lambda, A_k$ are in general complex, the charge conserving minimum configuration

$$\langle H_1 \rangle_0 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}; \quad \langle H_2 \rangle_0 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}; \quad \langle N \rangle_0 = x \tag{33}$$

with real and positive v_1, v_2 and x results into

$$\begin{aligned} {\rm Im}\, \lambda A_\lambda &= x\, {\rm Im}\, \lambda k^* \\ {\rm Im}\, k A_k &= \frac{3v^2 \sin 2\beta}{2x} {\rm Im}\, \lambda k^* \end{aligned} \tag{34}$$

so that a possible CP-violation in the Higgs sector can now be described with an independent parameter.

With the help of the minimum conditions the soft breaking parameters can be expressed in terms of $v^2 = v_1^2 + v_2^2$, $\tan\beta$ and x, resulting

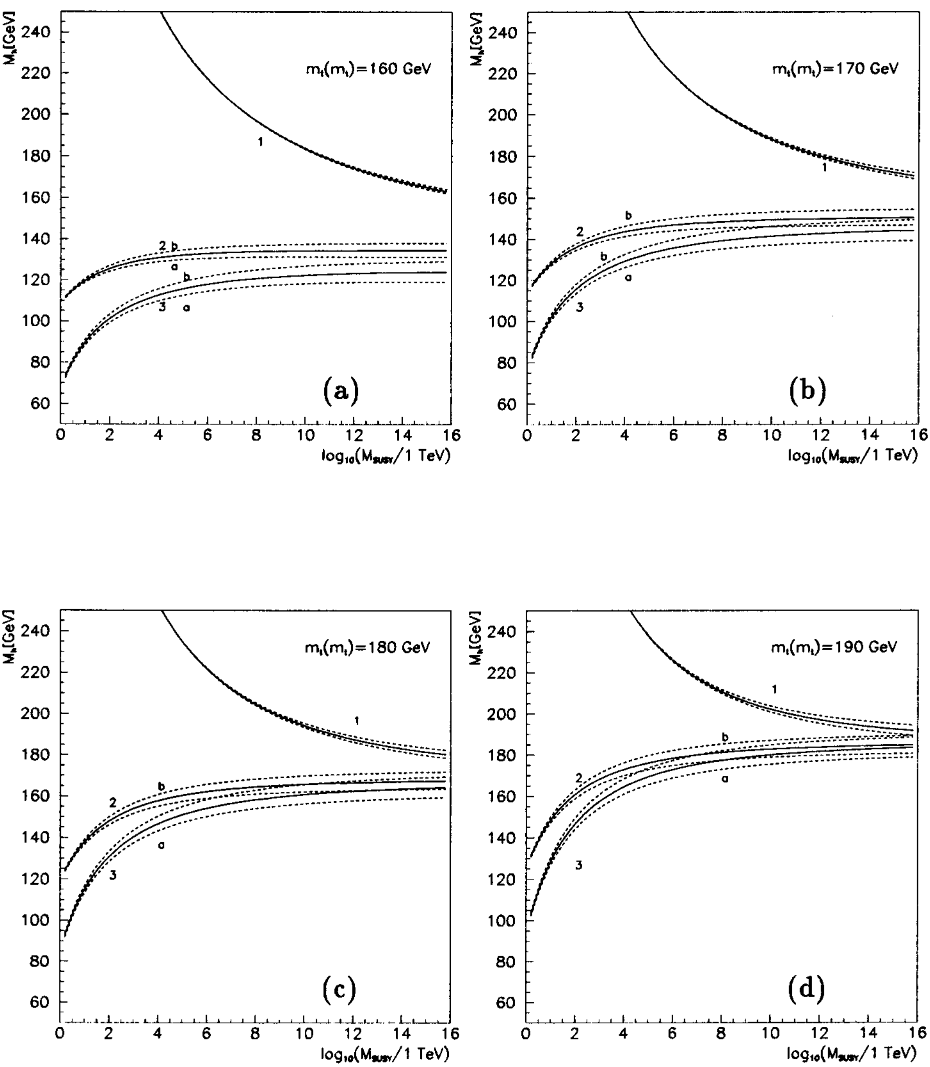

Fig. 2. Allowed regions for the Higgs boson mass for different values of the current top quark mass $m_t(m_t) = h_t(m_t)\langle H\rangle$. Solid curve 1 describes the upper bound obtained from the requirement of the absence of Landau pole singularity for $\overline{\lambda}(\mu)$ up to scale $M_{\rm SUSY}$. Solid curve 2 describes the upper bound on the Higgs boson mass which corresponds to the boundary condition $\overline{\lambda}(M_{\rm SUSY}) = (\overline{g}_1^2(M_{\rm SUSY}) + \overline{g}_2^2(M_{\rm SUSY}))/4$ and $\alpha_3(m_Z) = 0.120$. Solid curve 3 describes the lower bound on the Higgs boson mass and corresponds to the boundary condition $\overline{\lambda}(M_{\rm SUSY}) = 0$ and $\alpha_s(M_{\rm SUSY}) = 0.120$. Curves 2a, 3a and 2b, 3b correspond to $\alpha_s(m_Z) = 0.130$ and $\alpha(m_Z) = 0.110$.

into $m^2_{H_1}, m^2_{H_2}, m^2_N$. So far on the tree-level the Higgs sector contains $\tan\beta, x, A_\lambda, A_k, \lambda, k$ as free parameters and in case of including CP-violation further a complex phase as free parameter too.

Model dependent upper bounds for the coupling constant λ gives us an upper bound for the lightest scales. So far varying x and A_t in the regions $x < 10\text{TeV}$ and $-3\text{TeV} \leq A_t \leq 3\text{TeV}$ in Fig. 3 the upper bound for the lightest scalar mass in the NMSSM is demonstrated.

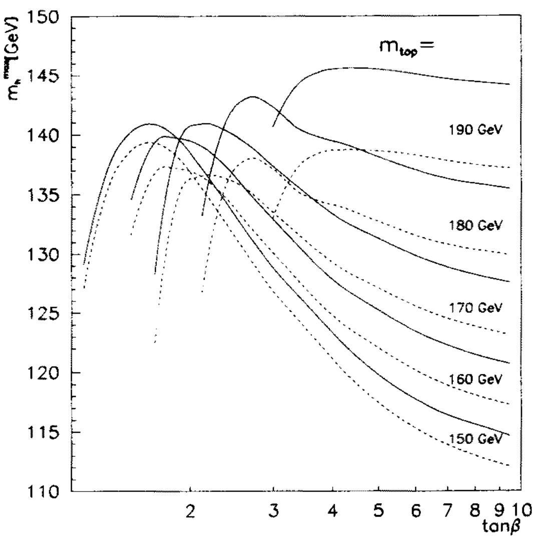

Fig. 3. Upper bounds on the mass of the lightest scalar in the NMSSM. Straight lines: $M_{\text{SUSY}} = 1000\text{GeV}$. Dotted lines: $M_{\text{SUSY}} = 500\text{GeV}$.

Further, the maximal mass of the lightest scalar with the relavant parameters is given in Table II.

TABLE II

Upper bounds on the mass of the lightest scalar and parameters reponsible for it. Further parameters: $m_t = 174\text{GeV}$, $m_{\tilde{t}} = 1\text{TeV}$.

$\tan\beta$	1.7	2	2.2	2.5	3	5	10
A_λ [GeV]	1444	9504	2037	6724	8201	2863	8536
A_k [GeV]	987	2925	62	2544	1226	3712	2071
A_t [GeV]	3.6	143	2755	1	3553	3420	2404
k	0.186	0.161	0.053	0.183	0.217	0.12	0.37
λ	0.195	0.576	0.633	0.648	0.664	0.71	0.617
x [GeV]	19984	8553	1372	4612	4791	867	1602
m_h [GeV]	109.2	135.7	138.3	137.9	136.6	133.2	129.6

Fig. 3 indicates, that for large $\tan\beta$ the upper bounds agree with the predictions of the MSSM. So far again (m_t, m_H)-area will be covered again by part as mentioned above.

So far all these above mentioned well-defined theoretical models and applied methods localize, — after the discovery of Top at FNAL — now the Higgs.

Finally we should not forget to mention, that all these previously quoted treatments are certainly more or less embedded completely in the Higgs-Kibble mechanism, usually denoted as the socalled "Higgs-Phenomenon" [18], indicating all the completely not understood and "hidden" problems and troubles due to this procedure. As long as we cannot quantize gravity completely, a possible way out of these difficulties just to avoid, roughly spoken, the socalled "zero-length", might be "Strings". Toward to these ideas and so far to get a chance to overcome in this context perhaps step by step the existing and not understood problems, a comparison of the "running coupling constants", *e.g.*, calculated via SUGRA with those obtained via "Superstrings and Quantum Gravity", might indicate a "way out" of all these problems [19].

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