DETERMINATION OF POLARIZED PARTON DISTRIBUTIONS IN THE NUCLEON*

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Dedicated to Wojciech Królikowski in honour of his 70th birthday

A fit to proton, neutron and deuteron spin asymmetries is presented and polarized parton distributions in nucleon are given. These densities have their roots in the MRS fit for unpolarized case. The integrals of polarized distributions are compared with the experimental figures. The role of polarized gluons is also discussed.

PACS numbers: 14.20.Dh, 13.88.+e

There are a lot of new experiments which yield the data on spin structure of a nucleon. We have the newest experimental points from CERN [1, 2] and SLAC [3–5] and also the older ones from these laboratories [6, 7]. The spin asymmetries are measured on proton (SLAC-Yale [6], EMC [7], SMC [2] and E143 [4]), neutron (E142 [3] made on ^3He) and deuterium (SMC [1] and E143 [5]) targets. Using the experimental figures we can study phenomenologically nucleon spin structure and in particular we can determine the polarized parton distributions.

For unpolarized case several fits were performed [8–11], there were also attempts to get the spin distributions [12–18]. Some time ago Martin,

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Roberts and Stirling (MRS) [8] presented a complete fit with determination of parton (*i.e.* quark and gluon) distributions. Quite recently after the measurements from Hera [19] for the small x region the new fits with more reliable gluon contribution and modified sea were performed [9]. Our discussion how to determine the polarized structure functions using MRS fits [8] was given in Ref. [16] and in Ref. [17].

In this paper we would like to get polarized quark parton distributions starting from the unpolarized ones and using existing data on spin asymmetries. The calculated values of octet axial-vector couplings a_3 and a_8 obtained from the fit are compared with the experimental values gotten from nucleon and hyperon β-decays and modified for QCD corrections. We use these quantities to find the best fit (of course together with the χ^2 values). In order to stabilize the fits we put the restriction on a_8 (such procedure takes place also *e.g.* in Ref. [12]). We have also tried to include polarized gluons contributing in the way proposed in Ref. [18]. It comes out that the polarized gluonic degrees of freedom do not lead to any substantial improvement in a fit.

Let us start with the formulas for unpolarized quark parton distributions given (at $Q^2 = 4\,\mathrm{GeV}^2$) in the newest fit performed by Martin, Roberts and Stirling [9]. We have for the valence quarks:

$$u_v(x) = 2.704x^{-0.407}(1-x)^{3.96}(1 - 0.76\sqrt{x} + 4.20x)\,,$$
$$d_v(x) = 0.251x^{-0.665}(1-x)^{4.41}(1 + 8.63\sqrt{x} + 0.32x)\,, \tag{1}$$

and for the antiquarks from the sea:

$$2\bar{u}(x) = 0.392M(x) - \delta(x)\,,$$
$$2\bar{d}(x) = 0.392M(x) + \delta(x)\,,$$
$$2\bar{s}(x) = 0.196M(x)\,,$$
$$2\bar{c}(x) = 0.020M(x)\,. \tag{2}$$

In Eq. (2) the singlet contribution is:

$$M(x) = 1.74x^{-1.067}(1-x)^{10.1}(1 - 3.45\sqrt{x} + 10.3x)\,, \tag{3}$$

whereas the isovector part:

$$\delta(x) = 0.043x^{-0.7}(1-x)^{10.1}(1 + 64.9x)\,. \tag{4}$$

For the unpolarized gluon distribution we get:

$$G(x) = 1.51x^{-1.301}(1-x)^{6.06}(1 - 4.14\sqrt{x} + 10.1x)\,. \tag{5}$$

We assume, in an analogy to the unpolarized case, that the polarized quark distributions are of the form: $x^{\alpha}(1-x)^{\beta}P_2(\sqrt{x})$, where $P_2(\sqrt{x})$ is a second order polynomial in $\sqrt{x}$ and the asymptotic behaviour for $x\to 0$ and $x\to 1$ (*i.e.* the values of α and β) are the same (except for ΔM, see a discussion below) as in unpolarized case. Our idea is to split the numerical constants (coefficients of P_2 polynomial) in Eqs (1), (3) and (4) in two parts in such a manner that the distributions are positive defined. Our expressions for $\Delta q(x) = q^{+}(x) - q^{-}(x)$ $(q(x) = q^{+}(x) + q^{-}(x))$ are:

$$\Delta u_v(x) = x^{-0.407}(1-x)^{3.96}(a_1 + a_2\sqrt{x} + a_3 x),$$
$$\Delta d_v(x) = x^{-0.665}(1-x)^{4.41}(b_1 + b_2\sqrt{x} + b_3 x),$$
$$\Delta M(x) = x^{-0.567}(1-x)^{10.1}(c_1 + c_2\sqrt{x}),$$
$$\Delta\delta(x) = x^{-0.7}(1-x)^{10.1}c_3(1 + 64.9x). \tag{6}$$

For a moment we will not take into account polarized gluons *i.e.* we put $\Delta G = 0$. For total sea polarization *i.e.* ΔM, we assume that there is no term behaving like $x^{-1.067}$ at small x (we assume that ΔM and hence all sea distributions are integrable), which means that coefficient in this case have to be splitted into equal parts in M^{+} and M^{-}. The next term $(x^{-0.567})$ is relatively more singular then the sea contribution in the preferred fit in Ref. [17]. On the other hand when we put the coefficient in front of $x^{-0.567}$ equal to zero (*i.e.* $c_1 = 0$) $M(x)$ will behave for $x \to 0$ like $x^{-0.067}$. In this case we get the fit with higher $\chi^2/N_{\rm DF}$. That means that in spite of the fact that in the present fit the unpolarized sea behaviour is less singular for small x values, contrary to the case in Ref. [17], the model for $\Delta M(x)$ with more singular sea contribution is phenomenologicaly chosen. As we will see that influences the behaviour of $g_1^p(x)$ for small x values.

In order to get the unknown parameters in the expressions for polarized quark distributions at $Q^2 = 4\,{\rm GeV}^2$ (see Eq. (6)) we make a fit to the experimental data on spin asymmetries for proton, neutron and deuteron targets. The theoretical expressions for these asymmetries are given by:

$$A_1^p(x) = \frac{4\Delta u_v(x) + \Delta d_v(x) + 2.236\Delta M(x) - 3\Delta\delta(x)}{4u_v(x) + d_v(x) + 2.236M(x) - 3\delta(x)}(1 + R),$$

$$A_1^n(x) = \frac{\Delta u_v(x) + 4\Delta d_v(x) + 2.236\Delta M(x) + 3\Delta\delta(x)}{u_v(x) + 4d_v(x) + 2.236M(x) + 3\delta(x)}(1 + R),$$

$$A_1^d(x) = \frac{5\Delta u_v(x) + 5\Delta d_v + 4.472\Delta M(x)}{5u_v(x) + 5d_v + 4.472M(x)}(1 - \frac{3}{2}p_D)(1 + R). \tag{7}$$

Where the ratio $R = \sigma_L/\sigma_T$ (which vanishes in the Bjorken limit) is taken from [20] and p_D is a probability of D-state in deuteron wave function (equal to $(5 \pm 1)\%$ [1, 5]).

We assume that the spin asymmetries do not depend on Q^2 (it is only our first order approximation) what is suggested by the experimental data [1, 3] and phenomenological analysis [21]. We hope that numerically our results at $Q^2 = 4\,\mathrm{GeV}^2$ will not change much if the evolution of F_1 and g_1 functions will be taken into account.

Spin structure function *e.g.* g_1^p is given by:

$$g_1^p(x) = \frac{4\Delta u_v(x) + \Delta d_v(x) + 2.236\Delta M(x) - 3\Delta\delta(x)}{18}. \tag{8}$$

The obtained polarized quark distributions $\Delta u(x)$, $\Delta d(x)$, $\Delta M(x)$ and $\Delta\delta(x)$ can be used to calculate first moments. For a given Q^2 we can write the relations:

$$\begin{aligned}
\Gamma_1^p &= \frac{4}{18}\Delta u + \frac{1}{18}\Delta d + \frac{1}{18}\Delta s + \frac{4}{18}\Delta c, \\
\Gamma_1^n &= \frac{1}{18}\Delta u + \frac{4}{18}\Delta d + \frac{1}{18}\Delta s + \frac{4}{18}\Delta c,
\end{aligned} \tag{9}$$

where

$$\Delta q = \int_0^1 \Delta q(x)\,dx \quad \text{and} \quad \Gamma_1 = \int_0^1 g_1(x)\,dx.$$

We define other combinations of integrated quark polarizations:

$$\begin{aligned}
a_3 &= \Delta u - \Delta d, \\
a_8 &= \Delta u + \Delta d - 2\Delta s, \\
\Delta\Sigma &= \Delta u + \Delta d + \Delta s.
\end{aligned} \tag{10}$$

Such results for the integrated quantities (calculated at $4\,\mathrm{GeV}^2$) after taking into account known QCD corrections (see *e.g.* Ref. [22]) could be compared with axial-vector coupling constants g_A and g_8 known from neutron β-decay and hyperon β-decays (in the last case one needs SU(3) symmetry). The difference of $\Gamma_1^p(Q^2)$ and $\Gamma_1^n(Q^2)$ can be expressed by:

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{\Delta u(Q^2) - \Delta d(Q^2)}{6} = \frac{a_3(Q^2)}{6} = \frac{c_{\mathrm{NS}}(Q^2)g_A}{6}, \tag{11}$$

where $c_{\mathrm{NS}}(Q^2)$ describes QCD corrections for non-singlet quantities [22] and $g_A = 1.2573 \pm 0.0028$ (see *e.g.* Ref. [23]) is obtained from the neutron β-decay. In our paper Q^2 is constant and takes the value $4\,\mathrm{GeV}^2$.

We get from the experimental figure $a_3(4\,\text{GeV}^2) = c_{\text{NS}}(4\,\text{GeV}^2)g_A = 1.11$ and with this value we shall compare a_3 calculated from our fits. Another useful combination of $\Gamma_1^p(Q^2)$ and $\Gamma_1^n(Q^2)$ is equal to:

$$\Gamma_1^p(Q^2) + \Gamma_1^n(Q^2) = \frac{5a_8(Q^2)}{18} + \frac{2\Delta s(Q^2)}{3} \qquad (12)$$

with $a_8 = c_{\text{NS}}(Q^2)g_8$, where $g_8 = 0.58 \pm 0.03$ [23] is obtained from the hyperon β-decays. Knowing $c_{\text{NS}}(Q^2)$ we can calculate $a_8(4\,\text{GeV}^2) = 0.51 \pm 0.03$ and with this number we shall compare the results obtained from our fit. If we have had very precise experimental data and in the whole x range there would be no problems with determination of polarized quark distributions. Unfortunately that is not the case yet. Actually, from the experiment we have information on Γ_1^p and Γ_1^n. The combination Γ_1^p-Γ_1^n is directly connected to g_A experimental quantity modified by QCD corrections. On the other hand $\Gamma_1^p + \Gamma_1^n$ is the combination of a_8 and Δs and it came out in Ref. [17] that the fits are not sensitive enough to determine a_8 and Δs separately in a stable way. The values of a_8 and Δs were different for our models and for different subsets of data. To stabilize the determination of parameters we also here assume in addition that $a_8 = 0.51$ with 0.1 as artificial theoretical error.

We get the following values of our parameters (describing the polarized quark distributions in Eq. (6)) from the fit to all existing data for spin asymmetries:

$$\begin{array}{lll} a_1 = & 1.71\,, & a_2 = -6.68\,, & a_3 = & 14.2\,, \\ b_1 = -0.005\,, & b_2 = & 0.835\,, & b_3 = -3.33\,, \\ c_1 = -1.10\,, & c_2 = & 1.38\,, & c_3 = -0.03\,. \end{array} \qquad (13)$$

TABLE I

The first moments of polarized distributions (see Eqs (9) and (10)). The strange sea polarization Δs (not presented in the Table) is connected to the total sea polarization by the relation: $\Delta s = 0.196\Delta M$. We have made our fits taking all existing experimental data on spin asymmetries.

Fit	Γ_1^p	Γ_1^n	a_3	a_8	$\Delta \Sigma$	ΔM	χ^2/N_{DF}
Old fit [17]	0.139	-0.072	1.27	0.47	0.20	-0.45	1.05
New fit	0.119	-0.072	1.14	0.50	0.12	-0.65	1.01
New fit with gluons	0.123	-0.082	1.23	0.51	0.06	-0.09	1.02

In the second row of the Table I the integrated quantities: Γ_1^p, Γ_1^n, a_3, a_8, $\Delta \Sigma$ and ΔM together with the coresponding χ^2/N_{DF} that follow from our fit are presented. They can be compared (first row) with the values

obtained from our previous fit presented in Ref. [17]. We will not show the comparison of the results of the fit to CERN data only (proton + deuteron) and SLAC data (proton + neutron + deuteron). Two fits like before are in agreement with each other and with the overall fit. It is specially interesting because the CERN data are taken at much smaller x values then the SLAC data.

In Figs 1, 2 and 3 we present the comparison of our fit with the experimental asymmetries for proton (1), neutron (2) and deuterium (3) target. We see that in the case of deuterium many experimental points lie below our fitted curve especially in the large x region where the errors are relatively big. In the Fig. 2 we see that A_1^n approaches very slowly zero for $x \to 0$ what produces relatively high (negative) value of Γ_1^n. Because we have 62 points for proton spin asymmetries in comparison with 33 for deuteron and 8 for neutron it seems that our curves are dominated by the proton data. Small discrepancies between different experiments are also not excluded.
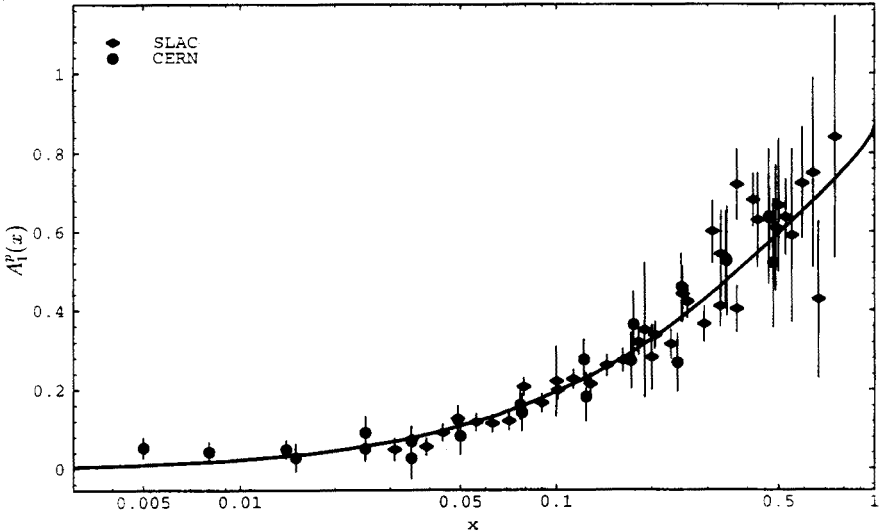

Fig. 1. The comparison of spin asymmetry on protons with the curve gotten from our fit to all existing data. Points are taken from SLAC (E80, E130, E143) and CERN (EMC, SMC) experiments.

The obtained quark distributions lead to the following integrated quantities: $\Delta u = 0.69$ ($\Delta u_v = 0.83$), $\Delta d = -0.45$ ($\Delta d_v = -0.08$) and $\Delta s = -0.13$, hence the amount of sea polarization is $\Delta M = -0.65$. This last number is not small and hence also the strange sea polarization in our model is rather big. As one can see from Table I the a_3 value seems to be in our fit very close to the predicted value 1.11. $\Delta \Sigma$ is rather small and the tendency that the model with more singular sea behaviour ($c_1 \neq 0$) produces very
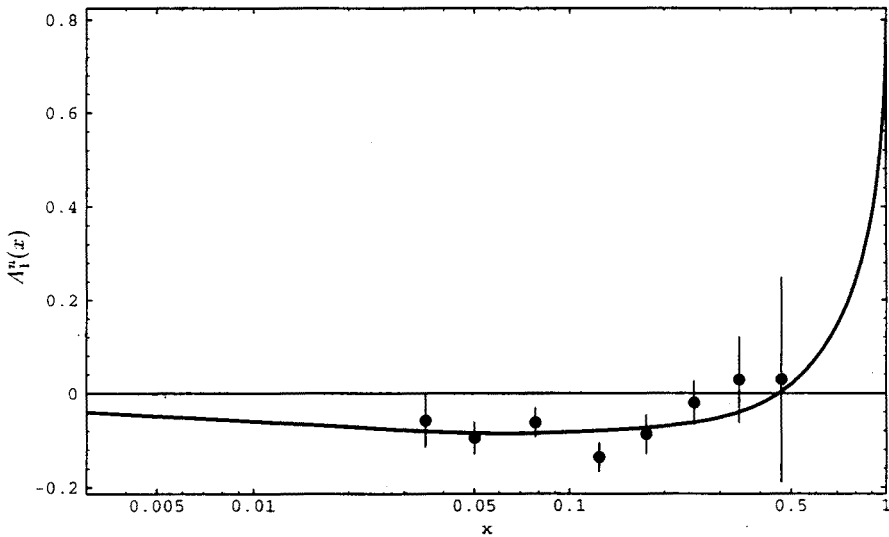
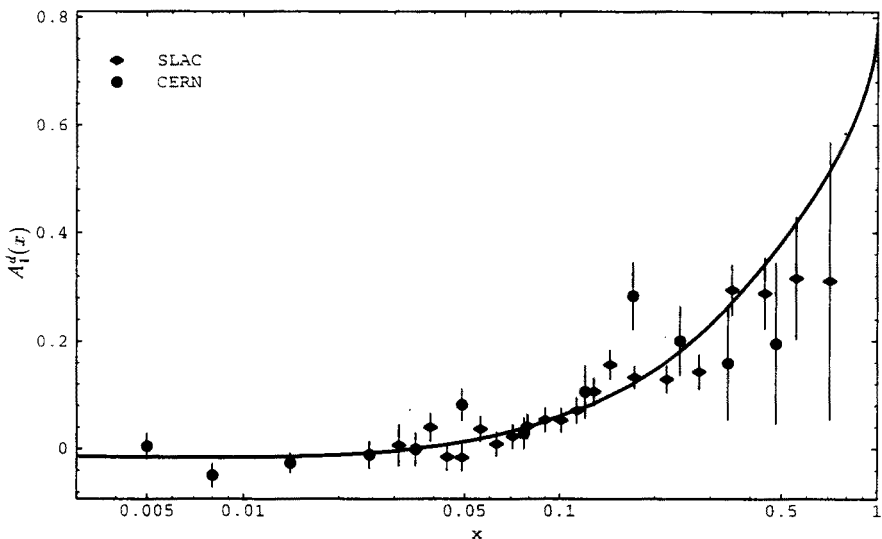Fig. 2. The comparison of spin asymmetry on neutrons (SLAC E142 data) with the curve gotten from our fit.

Fig. 3. Our prediction for deuteron asymmetry compared with the SMC and SLAC data.

small $\Delta\Sigma$ values is confirmed (this was already observed in Ref. [17]). Our integrated quantities (Γ_1^p and Γ_1^n) differ slightly from the values quoted by the experimental groups, whose figures are calculated directly from the experimental points with the assumption of Regge type behaviour at small x. But on the other hand our polarized quark distributions satisfy all the constraints taken implicitly into account in fits to the unpolarized data. The

difference in integrated quantities comes mainly due to our assumptions about small x behaviour for spin-dependent distributions. If we compare our results integrated in the interval $0.029 < x < 0.8$ (region covered in the SLAC experiments) we get $\Gamma_1^p = 0.11$ (SLAC result without the extrapolation to unmeasured region is 0.12 ± 0.01) and $\Gamma_1^d = 0.04$ (to be compared with 0.04 ± 0.005). Also we use R [20] which is not very realiable for small x and this also gives an additional error, which is rather difficult to estimate.

It is interesting to see what will happen when we relax our condition $a_8 = 0.51 \pm 0.1$. We get the fit with comparable χ^2 per degree of freedom ($\chi^2/N_{\rm DF} = 1.02$) and $\Gamma_1^p = 0.12$, $\Gamma_1^n = -0.08$, $a_3 = 1.23$, $a_8 = 0.23$, $\Delta\Sigma = 0.13$ and $\Delta M = -0.18$. We see that a_8 value ($a_8 \sim \Delta\Sigma$) is rather small in this case and also we have much smaller sea polarization.

We have also tried to include polarized gluons along the line of Ref. [18] assuming for the gluon distribution:

$$\Delta G(x) = x^{-0.801}(1 - x)^{6.06}(d_1 + d_2\sqrt{x}), \qquad (14)$$

with a new d_1 and d_2 constants which have to be fitted. The appearance of non-zero gluonic distribution affects our formulas only through the substitution: $\Delta q \Rightarrow \Delta q - (\alpha_s/2\pi)\Delta G$. In such fit ($\chi^2/N_{\rm DF} = 1.01$) we got (after integration) the negative sign of the gluonic contribution (if we take gluonic distribution with the full strength at $x = 0$). In this case Γ_1^p is equal to 0.19 because the gluonic contribution is added to the quark contribution instead of being substracted.

In the row three of Table 1 the results of the fit with gluons are shown in which gluon contribution is less singular $\Delta G(x) \sim x^{-0.301}$ for small x i.e. the coefficient in front of $x^{-0.801}$ is equaly devided between $G^+(x)$ and $G^-(x)$ ($d_1 = 0$). In this case $\chi^2/N_{\rm DF} = 1.02$ and is comparable to the fit without gluons. In this case sea polarization is very small and also $\Delta\Sigma$, as expected, is rather small (0.06). Also we get in this case for a_3 the value higher than the experimental figure. We conclude that the inclusion of gluonic contribution does not lead to the substantial improvement of the fit.

The Fig. 4 shows the comparison of $g_1^p(x)$ calculated from our fit with the experimental points (evolved to common value $Q^2 = 4\,{\rm GeV}^2$). We do not observe the growth of g_1^p for small x values in our model contrary to the previous fit in Ref. [17]. It is caused by the relatively singular behaviour of the sea contribution for small x values.

Starting from the new, improved version of the MRS fit [9] to the unpolarized deep inelastic data we have made a fit to proton, neutron (^{3}He) and deuteron spin asymmetries in order to obtain polarized quark parton distributions. To stabilize the fits we added the experimental information on octet quantity a_8. We have calculated the parameters of the polarized quark
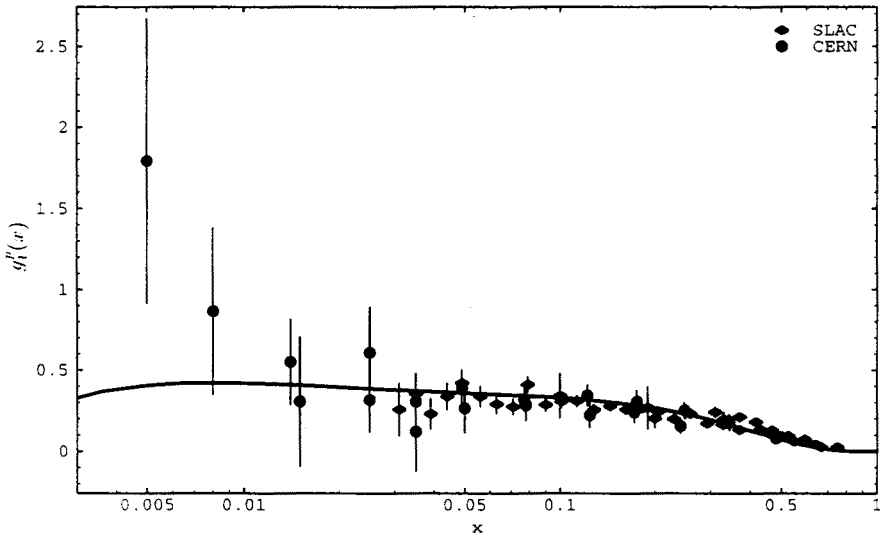
Fig. 4. The data for $g_1^p(x)$ structure function with the curve gotten using the parameters of our fit. The data points are taken from SLAC and CERN and evolved to common $Q^2 = 4\,\mathrm{GeV}^2$.

distributions using the combination of all existing proton, neutron and deuteron spin asymmetries measurements (including the new results on proton and deuteron E143 experiments and very recent SMC deuteron data from CERN). We do not need gluonic contributions to be taken into account, *i.e.* the fit with gluons is not better. The next step in front of us is to include Q^2 dependence of spin asymmetries in comparison with experimental data.

REFERENCES

[1] B. Adeva *et al.*, (Spin Muon Collaboration), *Phys. Lett.* **B302**, 533 (1993); D. Adams *et al.*, (Spin Muon Collaboration), *Phys. Lett.* **B357**, 248 (1995).

[2] D. Adams *et al.*, (Spin Muon Collaboration), *Phys. Lett.* **B329**, 399 (1994).

[3] D.L. Anthony *et al.*, (E142 Collaboration), *Phys. Rev. Lett.* **71**, 959 (1993).

[4] K. Abe *et al.*, (E143 Collaboration), *Phys. Rev. Lett.* **74**, 346 (1995).

[5] K. Abe *et al.*, (E143 Collaboration), *Phys. Rev. Lett.* **75**, 25 (1995).

[6] M.J. Alguard *et al.*, (SLAC-Yale Collaboration), *Phys. Rev. Lett.* **37**, 1261 (1976); G. Baum *et al.*, *Phys. Rev. Lett.* **45**, 2000 (1980); **51**, 1135 (1983).

[7] J. Ashman *et al.*, (European Muon Collaboration), *Phys. Lett.* **B206**, 364 (1988); *Nucl. Phys.* **B328**, 1 (1989).

[8] A.D. Martin, W.J. Stirling, R.G. Roberts, *Phys. Rev.* **D47**, 867 (1993); *Phys. Lett.* **B306**, 145 (1993); *erratum* **B309**, 492 (1993); A.D. Martin, W.J. Stirling, R.G. Roberts, *Phys. Rev.* **D50**, 6734 (1994).

[9] A.D. Martin, W.J. Stirling, R.G. Roberts, *Phys. Lett.* **B354**, 155 (1995).

[10] J.F. Owens, *Phys. Lett.* **B266**, 126 (1991); P. Chiapetta, G. Nardulli, *Z. Phys.* **C51**, 435 (1991); L.W. Whitlow *et al.*, *Phys. Lett.* **B282**, 475, (1992); M. Glück, E. Reya, A. Vogt, *Z. Phys.* **C 53**, 127 (1992); M. Glück, E. Reya, A. Vogt, *Phys. Lett.* **B306**, 391 (1993); J. Botts *et al.*, (CTEQ Collaboration), *Phys. Lett.* **B304**, 159 (1993).

[11] S.J. Brodsky, M. Burkardt, I. Schmidt, *Nucl. Phys.* **B441**, 197 (1995).

[12] T. Gehrmann, W.J. Stirling, *Z. Phys.* **C65**, 461 (1995).

[13] F. Buccella *et al.*, preprint Napoli DSF-T-95/26 (1995).

[14] C. Bourrely, J. Soffer, *Nucl. Phys.* **B445**, 341 (1995).

[15] M. Glück *et al.*, preprint Dortmund-TH 95/13 (1995).

[16] J. Bartelski, S. Tatur, *Acta Phys. Pol.* **B26**, 913 (1994).

[17] J. Bartelski, S. Tatur, CAMK preprint No. 288 to be published in *Z. Phys.*

[18] G. Altarelli, G.G. Ross, *Phys. Lett.* **B 212**, 391 (1988).

[19] T. Ahmed *et al.*, (H_1 Collaboration), Desy preprint 95-006 (1995); M. Derrick *et al.*, (ZEUS Collaboration), *Z. Phys.* **C65**, 379 (1995); M. Derrick *et al.*, *Phys. Lett.* **B345**, 576 (1995); I. Abt *et al.* (H_1 Collaboration), *Phys. Lett.* **B321**, 161 (1994).

[20] W.L. Whitlow *et al.*, *Phys. Lett.* **B250**, 193 (1990).

[21] G. Altarelli, P. Nason, G. Ridolfi, *Phys. Lett.* **B320**, 152 (1994).

[22] S.A. Larin, *Phys. Lett.* **B334**, 192 (1994).

[23] F.E. Close, R.G. Roberts, *Phys. Lett.* **B316**, 165 (1993).