ON THE (K^-,π^+) INCLUSIVE REACTIONS WITH Σ^- OR Λ PRODUCTION*

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Dedicated to Wojciech Królikowski in honour of his 70th birthday

The theory of the inclusive (K^-,π^+) reaction, in which only the pion spectrum is measured, is presented. The hyperon in the final state — either Σ^- or Λ (produced via $\Sigma\Lambda$ conversion) — is described in the effective two-channel approach, and the cross section is calculated in the coupled-channels impulse approximation. The theory is applied to the (K^-,π^+) reaction on the ^{16}O target and compared with existing data.

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1. Introduction

In describing the Σ hypernuclear states produced in the strangeness exchange reactions (K^-,π), one has to take into account the strong $\Sigma\Lambda$ conversion in nuclear matter $\Sigma\mathcal{N} \to \Lambda\mathcal{N}'$. Usually, the cross section for the (K^-,π) reaction is calculated in the impulse approximation with the Σ wave function determined by a Schrödinger equation with a complex potential whose imaginary part represents the $\Sigma\Lambda$ conversion treated as an absorption. However, as pointed out in [1], this procedure is not correct for the inclusive (K^-,π) experiments in which only the pion spectrum is measured. Here the pions may be accompanied not only by Σ hyperons but also by Λ hyperons produced in the $\Sigma\Lambda$ conversion. Thus the conversion must not be treated as absorption, because those Σ's which convert into Λ's also contribute to the pion spectrum.

In the present paper, we present a method of calculating the cross section for the inclusive (K^-,π^+) reactions. We introduce the Λ channel

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explicitly into our procedure. We do it in the simplest way by applying the effective two-channel approach [2] used in the case of the strangeness exchange reactions by Kohno *et al.* [3]. Furthermore, we apply a coupled-channels impulse approximation which corresponds to the coupled-channels Born approximation proposed by Penny and Satchler [4].

We consider the case of the (K^-,π^+) reaction because here only one direct elementary strangeness exchange process occurs, namely $K^- + P \to \pi^+ + \Sigma^-$, and Λ's appear only due to the $\Sigma^- P \to \Lambda N$ conversion. (We neglect the final state pion interaction. Otherwise, Λ's could appear also as a result of a direct Λ production, $K^- P \to \pi^0 \Lambda$, with the subsequent charge exchange, $\pi^0 P \to \pi^+ N$.)

The paper is organized as follows. In Section 2, we present the coupled-channels impulse approximation for the inclusive (K^-,π^+) reaction, and the effective two-channel approach to the final state of the hyperon in this reaction. In Section 3, we apply our theory of the (K^-,π^+) reaction to the case of the ^{16}O target, discuss our results, and compare them with the existing data. Our final expressions for the cross sections are presented in Appendix A. The method of finding analytical solutions of the coupled Schrödinger equations for the Σ and Λ component of the hyperon wave function is outlined in Appendix B.

2. Cross section for the inclusive (K^-,π^+) reaction in the coupled-channels impulse approximation

Let us consider the (K^-,π^+) reaction in which the projectile kaon, in the elementary process $K^- P \to \pi^+ \Sigma^-$, transfers its strangeness to the target proton P in the state φ and emerges in the final state as pion in the direction $\hat{k}_\pi$ with the energy E_π.

First let us consider the case when Σ^- is detected in the direction $\hat{k}_\Sigma$ (its energy is fixed by energy conservation). In the impulse approximation (with K^- and π^+ plane waves)[1], the cross section for this reaction is:

$$d^3\sigma_\Sigma / d\hat{k}_\Sigma d\hat{k}_\pi dE_\pi = \frac{E_K E_\pi M_\Sigma c^2 k_\pi k_\Sigma}{(2\pi)^5 (\hbar c)^6 k_K} |t \int d\mathbf{r} \exp(-i\mathbf{q}\mathbf{r}) \psi(\Sigma, \boldsymbol{k}_\Sigma; \mathbf{r})^{(-)*} \varphi(\mathbf{r})|^2 , \qquad (1)$$

where $\mathbf{q} = \boldsymbol{k}_\pi - \boldsymbol{k}_K$ and ψ is the Σ scattering wave function. Moments of the respective particles (in units of $\hbar$) are denoted by $\boldsymbol{k}$, and their energies (including their rest masses) by E. We assume a zero-range spin-independent

[1] If we used a pion wave function distorted by the pion optical potential, we would face the problem (raised by Oset) that part of the absorption is due to the inelastic pion scattering which contributes to the observed pion spectrum.

interaction for the elementary process $K^-P \to \pi^+\Sigma^-$ with a constant transition matrix t. Spins are suppressed in the notation.

If only the energy spectrum of pions at fixed $\hat{k}_\pi$ is measured, then this spectrum, $d^2\sigma_\Sigma/d\hat{k}_\pi dE_\pi$, is obtained by integrating cross section (2) over $\hat{k}_\Sigma$:

$$d^2\sigma_\Sigma/d\hat{k}_\pi dE_\pi = \int d\hat{k}_\Sigma[d^3\sigma_\Sigma/d\hat{k}_\Sigma d\hat{k}_\pi dE_\pi]\,. \tag{2}$$

In the Σ single particle (s.p.) model the function ψ satisfies a s.p. Schrödinger equation with a s.p. potential $V_\Sigma(r)$. This approach was followed in [5], where an imaginary potential $iW_\Sigma(r)$ was added to $V_\Sigma(r)$ to take into account the $\Sigma\Lambda$ conversion. In the present paper, we introduce explicitly the Λ channel which is coupled to the Σ channel due to the $\Sigma\Lambda$ conversion. We do it in the effective two-channel approximation, similarly as in [3] (see also [1]), in which the hyperon s.p. wave function has two components: the Σ component ψ and the Λ component χ. They satisfy the system of two coupled Schrödinger equations:

$$\left\{-\frac{\hbar^2}{2M_\Sigma}[\Delta + k_\Sigma^2] + V_\Sigma(r)\right\}\psi(\mathbf{r}) = V_x(r)\chi(\mathbf{r})\,, \tag{3}$$

$$\left\{-\frac{\hbar^2}{2M_\Lambda}[\Delta + k_\Lambda^2] + V_\Lambda(r)\right\}\chi(\mathbf{r}) = V_x(r)\psi(\mathbf{r})\,, \tag{4}$$

where V_Σ, V_Λ are the Σ, Λ s.p. (real) potentials and V_x is the s.p. coupling potential which represents the result of folding the two-body interaction responsible for the $\Sigma^-P \to \Lambda N$ conversion into the nucleon density of the nuclear core. The momenta k_Λ and k_Σ are connected by:

$$E_\Lambda = E_\Sigma + \tilde{\Delta}\,, \tag{5}$$

where $E_\Sigma = \hbar^2k_\Sigma^2/2M_\Sigma$, $E_\Lambda = \hbar^2k_\Lambda^2/2M_\Lambda$, and

$$\tilde{\Delta} = (M_\Sigma + M_P - M_\Lambda - M_N)c^2 = 80.4\ \text{MeV}\,. \tag{6}$$

(By M_Σ we denote the mass of Σ^-.)

In this two-channel model, we imagine that the hyperon (in a Σ^- or Λ state) moves around a rigid nuclear core. However, the nuclear core in the Λ state differs from the nuclear core in the Σ^- state because it has a different charge, and is most likely in an excited state. Thus only a part $\tilde{\Delta}' = x\tilde{\Delta}$ of the energy $\tilde{\Delta}$ released in the $\Sigma\Lambda$ conversion is available to Λ. This is simulated in the effective two-channel approximation by replacing $\tilde{\Delta}$ by a smaller quantity $\tilde{\Delta}' = x\tilde{\Delta}$, or equivalently by replacing M_Λ by a larger mass $M'_\Lambda = M_\Lambda + (1-x)\tilde{\Delta}/c^2$.

The use of coupled Σ and Λ channels in place of a single Σ channel modifies the calculation of the cross section for the (K^-,π^+) reaction. Here, we follow the procedure of the coupled-channels Born approximation proposed by Penny and Satchler [4] and described in detail in the textbook of Satchler [6].

To calculate $d^3\sigma_\Sigma/d\hat{k}_\Sigma d\hat{k}_\pi dE_\pi$, we define $\psi(\Sigma,\boldsymbol{k}_\Sigma;\boldsymbol{r})^{(-)}$ and $\chi(\Sigma,\boldsymbol{k}_\Sigma;\boldsymbol{r})^{(-)}$ as the solutions of Eqs. (3)–(4) which are regular at the origin, and satisfy the asymptotic conditions:

$$\left\{\psi(\Sigma,\boldsymbol{k}_\Sigma;\boldsymbol{r})^{(-)}-\mathrm{e}^{ik_\Sigma r}\right\}_{r\to\infty}\sim \mathrm{e}^{-ik_\Sigma r}/r\,, \qquad (7)$$

$$\left\{\chi(\Sigma,\boldsymbol{k}_\Sigma;\boldsymbol{r})^{(-)}\right\}_{r\to\infty}\sim \mathrm{e}^{-ik_\Lambda r}/r\,. \qquad (8)$$

With the help of these final state hyperon wave function (ψ,χ), we obtain expression (1) for the cross section $d^3\sigma_\Sigma/d\hat{k}_\Sigma d\hat{k}_\pi dE_\pi$. Notice that in the (K^-,π^+) reaction considered here, the only direct strangeness-exchange process, $K^-P\to\pi^+\Sigma^-$, produces Σ^-, *i.e.*, the ψ component of the hyperon wave function.

Now, let us consider the case when the (K^-,π^+) reaction leads to the production of Λ in the direction $\hat{k}_\Lambda$ (the Λ energy is fixed by energy conservation). We denote the cross section for this reaction by $d^3\sigma_\Lambda/d\hat{k}_\Lambda d\hat{k}_\pi dE_\pi$. To calculate it, we now define $\psi(\Lambda,\boldsymbol{k}_\Lambda;\boldsymbol{r})^{(-)}$ and $\chi(\Lambda,\boldsymbol{k}_\Lambda;\boldsymbol{r})^{(-)}$ as the solutions of Eqs. (3)–(4) which are regular at the origin, and satisfy the asymptotic conditions:

$$\left\{\psi(\Lambda,\boldsymbol{k}_\Lambda;\boldsymbol{r})^{(-)}\right\}_{r\to\infty}\sim \mathrm{e}^{-ik_\Sigma r}/r\,, \qquad (9)$$

$$\left\{\chi(\Lambda,\boldsymbol{k}_\Lambda;\boldsymbol{r})^{(-)}-\mathrm{e}^{i\boldsymbol{k}_\Lambda\boldsymbol{r}}\right\}_{r\to\infty}\sim \mathrm{e}^{-ik_\Lambda r}/r\,. \qquad (10)$$

With the help of these final state hyperon wave function (ψ,χ), we obtain for the cross section $d^3\sigma_\Lambda/d\hat{k}_\Lambda d\hat{k}_\pi dE_\pi$ the expression:

$$d^3\sigma_\Lambda/d\hat{k}_\Lambda d\hat{k}_\pi dE_\pi = \frac{E_K E_\pi M_\Lambda c^2 k_\pi k_\Lambda}{(2\pi)^5(\hbar c)^6 k_K}|t\int d\boldsymbol{r}\exp(-i\mathbf{qr})\psi(\Lambda,\boldsymbol{k}_\Lambda;\boldsymbol{r})^{(-)*}\varphi(\boldsymbol{r})|^2\,. \qquad (11)$$

Notice that in the case of (K^-,π^+) reaction considered here, we do not have direct transitions to the Λ component χ.

To obtain the energy spectrum of pions at fixed $\hat{k}_\pi$, $d^2\sigma_\Lambda/d\hat{k}_\pi dE_\pi$, when Λ is not detected, we have to integrate (11) over $\hat{k}_\Lambda$:

$$d^2\sigma_\Lambda/d\hat{k}_\pi dE_\pi=\int d\hat{k}_\Lambda[d^3\sigma_\Lambda/d\hat{k}_\Lambda d\hat{k}_\pi dE_\pi]\,. \qquad (12)$$

By applying the partial wave expansion to the Σ scattering wave functions $\psi(\Sigma, \boldsymbol{k}_\Sigma; \boldsymbol{r})^{(-)}$ and $\psi(\Lambda, \boldsymbol{k}_\Lambda; \boldsymbol{r})^{(-)}$, we obtain the final expressions for the double differential cross sections $d^2\sigma_\Sigma/d\hat{k}_\pi dE_\pi$ and $d^2\sigma_\Lambda/d\hat{k}_\pi dE_\pi$, given in Appendix A.

To get the pion spectrum measured in an inclusive experiment, $d^2\sigma_T/d\hat{k}_\pi dE_\pi$, one has to add expressions (2) and (12):

$$d^2\sigma_T/d\hat{k}_\pi dE_\pi = d^2\sigma_\Sigma/d\hat{k}_\pi dE_\pi + d^2\sigma_\Lambda/d\hat{k}_\pi dE_\pi \,. \tag{13}$$

3. Results for the ^{16}O target and discussion

We consider the case of the inclusive (K^-, π^+) reaction on ^{16}O at $p_K =$ 450 MeV/c ($\theta = 0°$) investigated at CERN by Bertini *et al.* [7]. This is the case considered in the single Σ channel approach in [5]. Here, we extend the approach of [5] to two coupled channels Σ and Λ.

Similarly as in [5], our Σ s.p. potential has the form of an attractive square well plus a repulsive surface delta bump,

$$V_\Sigma(r) = -V_{\Sigma 0}\theta(r - R) + V_1\delta(r - R)\,, \tag{14}$$

with $R = 3$ fm, $V_{\Sigma 0} = 20$ MeV, and $V_1 = 20$ MeV fm. Our $V_{\Sigma 0}$ is compatible with the model D [8] of the Nijmegen baryon-baryon interaction (see [9–11]), and our V_1 is estimated from the repulsive surface bump calculated in [12, 13].

For the target proton wave function φ, we use — as in [5] — the bound state solution of the Schrödinger equation with the proton s.p. potential

$$V_P(r) = -V_{P0}\theta(R - r) - V_{Pls}\boldsymbol{ls}\delta(r - R)\,, \tag{15}$$

with $V_{P0} = 46$ MeV and $V_{Pls} = 15$ Mev fm. This potential leads to the s.p. proton energies in the $p_{1/2}$ and $p_{3/2}$ states: $\varepsilon_P(p_{1/2}) = -12.5$ MeV and $\varepsilon_P(p_{3/2}) = -19.1$ MeV, which agree with the corresponding empirical proton energies in ^{16}O, -12.5 and -19 MeV (see [3]).

The Coulomb interaction of Σ^- and the target proton is disregarded. Inside the nuclear core its average value is $\sim \pm 4$ MeV, and we assume that it is included into $V_{\Sigma 0}$ and V_{P0}.

For Λ, we assume a square well s.p. potential,

$$V_\Lambda(r) = -V_{\Lambda 0}\theta(r - R)\,, \tag{16}$$

with the value of $V_{\Lambda 0} = 30$ MeV, suggested by empirical values of Λ binding energies and by calculations [14, 10] starting from the Nijmegen baryon-baryon interaction.

For the $\Sigma\Lambda$ coupling s.p. potential, V_x , we also assume a square well shape,

$$V_x(r) = V_{x0}\theta(r - R)\,, \tag{17}$$

with $V_{x0} = 5$ MeV. Notice that the sign of V_{x0} is irrelevant, because Eqs. (3)–(4) are invariant under the transformation $V_x \to -V_x, \chi \to -\chi$.

For the parameter $x = \tilde{\Delta}'/\tilde{\Delta}$, we use the value 0.2, *i.e.*, we have $\tilde{\Delta}' =$ 16.1 MeV.

The values of V_{x0} and x were obtained by adjusting them so as to get an agreement with the CERN data [7]. Our value of V_{x0} coincides with the central value of V_x used in [3]. With our x, we get for $\tilde{M}_\Lambda/M_\Sigma$ the value of 0.985 which almost coincides with the value 0.99 used in [3].

With the square well potentials, Eqs. (14), (16), (17), our system of coupled equations for ψ and χ, Eqs. (3), (4), has been solved analytically (see Appendix B).

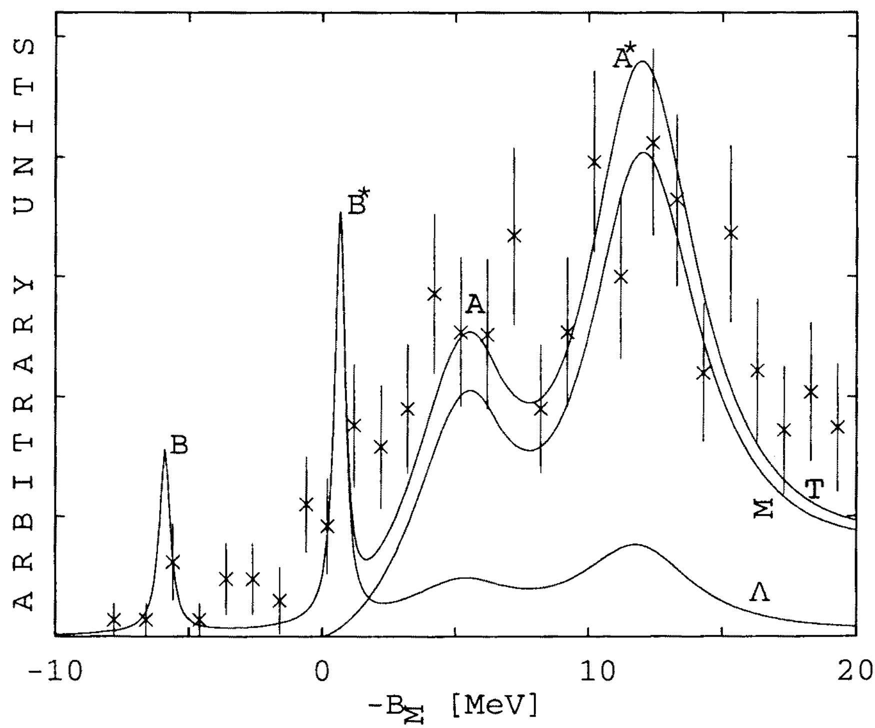

Fig. 1. Pion spectrum from from (K^-, π^-) reaction on ^{16}O at $\theta = 0^o$ at $p_K = 450$ MeV/c. See text for explanation.

In Fig. 1, we show our results for $d^2\sigma_\Sigma/ d\hat{k}_\pi dE_\pi$ (curve Σ), $d^2\sigma_\Lambda/ d\hat{k}_\pi dE_\pi$ (curve Λ), and $d^2\sigma_T/d\hat{k}_\pi dE_\pi$ (curve T), at $p_K = 450$ MeV/c and $\theta = 0°$, as functions of B_Σ, the separation (binding) energy of Σ^- from the hypernucleus produced (in the ground or excited state) with the nuclear core left in its ground state. Since the data of [7], shown also in Fig. 1, are only counting rates, our calculated results include a normalization to match the overall magnitude of the data.

In the energy range considered there are two contributions which result from the K^- interaction with $p_{1/2}$ and $p_{3/2}$ protons in ^{16}O. In the case of the $p_{1/2}$ contribution, the final state of the nuclear core is a $p_{1/2}$ hole in ^{16}O, *i.e.*, the ground state of ^{15}N, and we have $-B_\Sigma = E_\Sigma$. In the case of the $p_{3/2}$ contribution, the final nuclear core configuration is a $p_{3/2}$ hole in^{16}O, *i.e.*, an excited state of ^{15}N with the excitation energy $E^* = \varepsilon_P(p_{1/2}) - \varepsilon_P(p_{3/2}) = 6.6$ MeV, and we have $-B_\Sigma = E_\Sigma + E^*$.

The two peaks denoted by A and A^* result from the K^- interaction with the $p_{1/2}$ and $p_{3/2}$ protons respectively. The dominant contribution to the A (A^*) peak comes from the $p_{1/2}$ ($p_{3/2}$) component of the final state of Σ^-, and both peaks corespond to the p state resonance of Σ^- in V_Σ. Thus the A (A^*) peak is the substitutional state $[p_{1/2}, p_{1/2}^{-1}]_{\Sigma P}$ ($[p_{3/2}, p_{3/2}^{-1}]_{\Sigma P}$) in which Σ^- is in the continuum, with E_Σ in the vicinity of the resonance energy. Notice that because of the coupling of the Λ and Σ channels, also the cross section for Λ emission (curve Λ) shows maxima at about the same energies as the cross section for Σ emission (curve Σ). Hence the total pion spectrum in an inclusive experiment (curve T) shows substitutional states similar to the exclusive pion spectrum accompanied by Σ emission (curve Σ).

The Σ emission requires that $E_\Sigma > 0$, and thus the contribution of the K^- interaction with $p_{1/2}$ and $p_{3/2}$ protons to the Σ curve vanishes for $-B_\Sigma < 0$ and $-B_\Sigma < E^* = 6.6$ MeV respectively. Similarly, the Λ emission requires that $E_\Lambda > 0$, and the contribution of the K^- interaction with $p_{1/2}$ and $p_{3/2}$ protons to the Λ curve vanishes respectively for $-B_\Sigma < -\tilde{\Delta}' = -16.1$ MeV and $-B_\Sigma < -\tilde{\Delta}' + E^* = -9.5$ MeV [see (5)]. Consequently, the $p_{1/2}$ contribution to the inclusive pion spectrum for -16.1 MeV $< -B_\Sigma < 0$, and for the $p_{3/2}$ contribution for -9.5 MeV $< -B_\Sigma < 6.6$ MeV, is entirely due to the Λ emission.

The peak denoted by B at $B_\Sigma = 5.94$ MeV is produced by the K^- interaction with $p_{1/2}$ protons, leading to Σ^- and subsequently to Λ, both of them in the $s_{1/2}$ state. The appearance of this peak is due to the production of Σ^- (in the primary $K^- P \to \pi^+ \Sigma^-$ process) in the $s_{1/2}$ bound state which decays into Λ with $E_\Lambda > 0$ in the secondary $\Sigma\Lambda$ conversion process.

In the absence of the $\Sigma\Lambda$ coupling ($V_{x0} = 0$), our Σ potential V_Σ leads to a $s_{1/2}$ bound state with the binding energy $B_\Sigma(s_{1/2}) = 6.87$ MeV. In the presence of the $\Sigma\Lambda$ coupling, the bound state acquires a width and a shift towards smaller binding. Both the width and shift increase with increasing V_{x0} in agreement with [15].

Our present approach should be contrasted with our much less satisfying one-channel approach with a complex s.p. Σ potential [16], where a peak similar to B was obtained "by hand": first we obtained a δ type peak by calculating the transition to the discrete bound state of Σ^- in the case of pure real V_Σ, and next we smeared it out to a Breit-Wigner shape with the width equal twice the imaginary part of the eigenvalue of the complex Σ s.p. Hamiltonian.

The peak denoted by B^* is similar to the B peak. The only difference is that it is produced by the K^- interaction with $p_{3/2}$ protons instead of the $p_{1/2}$ protons in the case of the B peak.

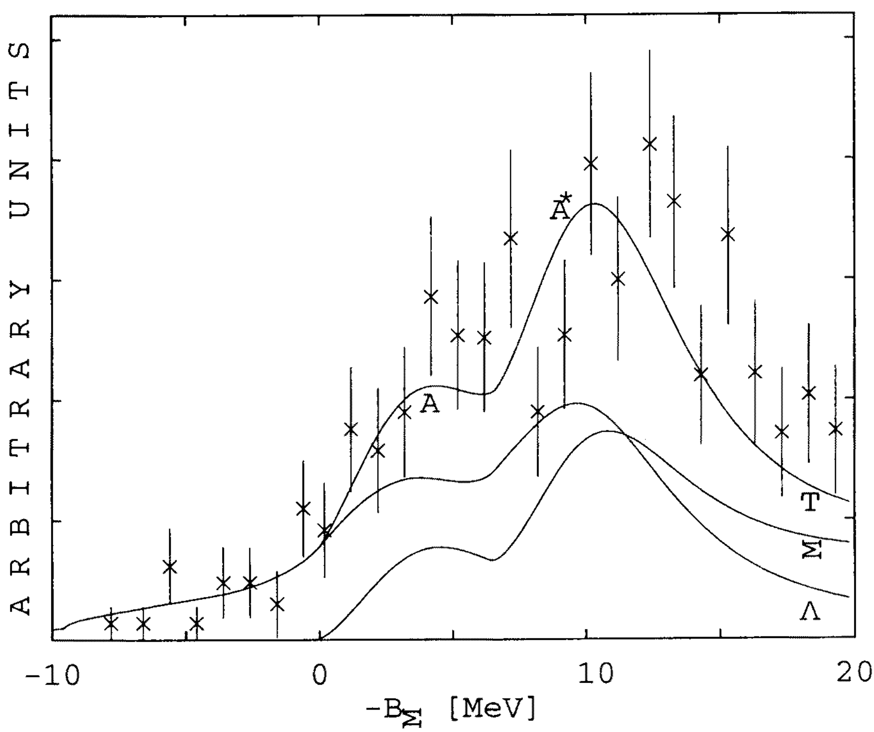

Fig. 2. Same as Fig. 1 but for $V_\Lambda = 0$.

Our results are sensitive to the s.p. Λ potential V_Λ, as is seen from Fig. 2 which shows our results obtained with $V_\Lambda = 0$. Otherwise Fig. 2 does not differ from Fig. 1 (also the normalization in both figures is the same). Comparing the two figures, we notice two effects. First, without V_Λ the peaks in the calculated pion spectrum become essentially broader. Because of this broadening, the bound state peaks B and B^* cease to be visible. Second, switching off V_Λ increases the cross section for the emission of Λ. The two effects may be understood in the following way. When Σ^- converts into Λ on which no potential V_Λ is acting, then most probably Λ will leave the hypernucleus, and we have a fast Λ emission after the first $\Sigma\Lambda$ conversion process, *i.e.*, an appreciable cross section for the strangeness exchange reaction accompanied by the emission of Λ. Such a fast Λ emission broadens the resonances and bound states in the Σ channel, and thus also the corresponding peaks in the cross section. On the other hand, a sufficiently attractive V_Λ keeps Λ within the hypernucleus where the $\Sigma\Lambda$ coupling potential may convert it back into Σ. Consequently, Λ emission becomes less probable compared to Σ emission, and the broadening of Σ resonance and bound states due to $\Sigma\Lambda$ conversion weakens.

The position of our two substitutional states A (at $B_\Sigma = -5.5$ MeV) and A^* (at $B_\Sigma = -12.0$ MeV) in Fig. 1 agrees nicely with the position of the two narrow peaks which in the analysis of the CERN data [7] were located at $B_\Sigma = -5.9 \pm 1$ MeV and -12.4 ± 1 MeV. On the other hand, our Σ bound state peaks B and B^* are too narrow and too steep compared with the CERN data. However , the CERN data are not very accurate, and it would be highly desirable to perform the experiments with an improved statistics. In the case of the strangeness exchange reaction on ^{9}Be, such experiments have been performed recently at BNL [17], and they do not reveal the peaks in the pion spectrum reported in the earlier CERN experiments [18]. In this situation it would be premature to adjust the parameters of our model so as to get a precise agreement with the existing ^{16}O data [7].

A serious shortcoming of the present approach is the effective two-channel approximation. We hope to improve this point by representing the nuclear core by a system of nucleons similarly as we did in Ref. [19].

Appendix A

Final expressions for the cross sections

Let us rewrite expressions (1), (2) and (11), (12) in a compact and a more precise form:

$$d^2\sigma_Y(l_P j_P)/d\hat{k}_\pi dE_\pi = \frac{E_K E_\pi M_Y c^2 k_\pi k_Y}{(2\pi)^5(\hbar c)^6 k_K}|t|^2 S_Y(l_P j_P)\,, \tag{A1}$$

$$S_Y(l_Pj_P) = \sum_{\mu_Y, m_P} \int d\hat{k}_Y \left| \int d\tau \exp(-i\boldsymbol{q}\boldsymbol{r}) \psi(Y, \boldsymbol{k}_Y \mu_Y; \boldsymbol{r}\xi)^{(-)*} \varphi_{l_Pj_Pm_P}(\boldsymbol{r}\xi) \right|^2, \tag{A2}$$

where $Y = \Sigma, \Lambda$, $\int d\tau$ denotes the integration over $\boldsymbol{r}$ and summation over the spin coordinate ξ. By $d^2\sigma_Y(l_Pj_P)/d\hat{k}_\pi dE_\pi$ we denote the cross section for the production of the $(\hat{k}_\pi, E_\pi)$ pion, accompanied by the emission of the hyperon Y in any direction $\hat{k}_Y$ and with any spin projection μ_Y, and with the target nucleus left with a proton hole in the l_Pj_P state with any $\mathbf{j}_P$ projection m_P.

The target proton wave function φ is

$$\varphi_{l_Pj_Pm_P}(\boldsymbol{r}\xi) = \mathcal{Y}_{l_P\frac{1}{2}j_Pm_P}(\hat{r}\xi) R_{l_Pj_P}(r)/r, \tag{A3}$$

where

$$\mathcal{Y}_{l_P\frac{1}{2}j_Pm_P}(\hat{r}\xi) = \sum_{m\mu} (l_P\tfrac{1}{2}m\mu|j_Pm_P) Y_{l_Pm}(\hat{r}) \eta_\mu(\xi), \tag{A4}$$

where η_μ denotes the spin wave function.

The partial wave expansion of $\psi^{(-)}$ is (see Appendix B):

$$\psi(Y, \boldsymbol{k}_Y\mu_Y; \boldsymbol{r}\xi)^{(-)} = \\ 4\pi \sum_{lmjm_j} i^l Y_{lm}(\hat{k}_Y)^* (l\tfrac{1}{2}m\mu_Y|jm_j) \mathcal{Y}_{l\frac{1}{2}jm_j}(\hat{r}\xi) u_{lj}(Y, k_\Sigma; r)^{(-)}/r. \tag{A5}$$

After inserting expressions (A4) and (A5), and the expansion

$$e^{-iqr} = 4\pi \sum_{LM} (-i)^L Y_{LM}(\hat{q})^* Y_{LM}(\hat{r}) j_L(qr) \tag{A6}$$

into expression (A2), a straightforward calculation leads to the following result for S_Y in terms of Wigner $3-j$ and $6-j$ symbols:

$$S_Y(l_Pj_P) = (4\pi)^2(2j_P+1)(2l_P+1) \sum_{Llj} (2j+1)(2L+1)(2l+1) \\ \times \begin{pmatrix} L & l_P & l \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} j_P & \frac{1}{2} & l_P \\ l & L & j \end{Bmatrix}^2 |\langle lj|j_L(qr)|l_Pj_P\rangle_Y|^2, \tag{A7}$$

where

$$\langle lj|j_L(qr)|l_Pj_P\rangle_Y = \int dr u_{lj}(Y, k_\Sigma; r)^{(-)*} j_L(qr) R_{l_Pj_P}(r). \tag{A8}$$

In the case of a pure central potential V_Σ (without spin-orbit coupling), considered in Section 3, functions u do not depend on j, $u_{lj}^{(-)} \to u_l^{(-)}$, and Eqs. (A5), (A7) take the simpler form:

$$\psi(Y, \boldsymbol{k}_Y \mu_Y; \boldsymbol{r}\xi)^{(-)} = 4\pi \sum_{lm} i^l Y_{lm}(\hat{k}_Y)^* Y_{lm}(\hat{r}) \eta_\mu(\xi) u_l(Y, k_\Sigma; r)^{(-)}/r\,, \tag{A9}$$

$$\begin{aligned} S_Y(l_P j_P) =& (4\pi)^2 (2j_P+1) \sum_{Ll} (2L+1)(2l+1) \\ & \times \begin{pmatrix} L & l_P & l \\ 0 & 0 & 0 \end{pmatrix}^2 |\langle l|j_L(qr)|l_P j_P\rangle_Y|^2\,, \end{aligned} \tag{A10}$$

where the expression for $< l|j_L(qr)|l_P j_P >_Y$ is identical with (A8) except that $u_{lj}^{(-)}$ is replaced by $u_l^{(-)}$.

Appendix B

Solution of the coupled equations for ψ and χ

Here, we outline the procedure of solving the coupled equations with the interactions V_Σ, V_Λ, V_x considered in Section 3.

For the hyperon wave function in the Σ channel, we use partial wave expansion (A9), and in the Λ channel an analogical expansion,

$$\chi(Y, \boldsymbol{k}_Y \mu_Y; \boldsymbol{r}\xi)^{(-)} = 4\pi \sum_{lm} i^l Y_{lm}(\hat{k}_Y)^* Y_{lm}(\hat{r}) \eta_\mu(\xi) w_l(Y, k_\Lambda; r)^{(-)}/r\,, \tag{B1}$$

When we insert expansions (A9) and (B1) into Eqs. (3), (4), we obtain:

$$\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - v_\Sigma(r) + k_\Sigma^2 \right\} u_l(Y, k_\Sigma; r)^{(-)} = v_{x\Sigma}(r) w_l(Y, k_\Lambda; r)^{(-)}\,, \tag{B2}$$

$$\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - v_\Lambda(r) + k_\Lambda^2 \right\} w_l(Y, k_\Lambda; r)^{(-)} = v_{x\Lambda}(r) u_l(Y, k_\Sigma; r)^{(-)}\,, \tag{B3}$$

where

$$v_Y(r) = (2M_Y/\hbar^2) V_Y(r),\ v_{xY}(r) = (2M_Y/\hbar^2) V_x(r)\,. \tag{B4}$$

Let us present our procedure in the case when Σ is emitted. In this case, we have for distances larger than the range R of the interaction, $r > R$:

$$k_\Sigma u_l(\Sigma, k_\Sigma; r)^{(-)} - J_l(k_\Sigma r) = \alpha^* H_l^{(2)}(k_\Sigma r)\,, \tag{B5}$$

$$k_\Lambda w_l(\Sigma, k_\Lambda; r)^{(-)} = \beta^* H_l^{(2)}(k_\Lambda r)\,, \tag{B6}$$

Here, we use the notation:

$$J_l(x) = xj_l(x),\ H_l^{(1,2)}(x) = xh_l^{(1,2)}(x)\,. \tag{B7}$$

The particular form of the partial wave expansion of ψ, Eq. (A9), was chosen to get the required asymptotic behavior:

$$\left\{\psi(\Sigma, \boldsymbol{k}_\Sigma \mu_\Sigma; \boldsymbol{r}\xi)^{(-)} - e^{i\boldsymbol{k}_\Sigma \boldsymbol{r}} \eta_{\mu_\Sigma}(\xi)\right\}_{r\to\infty} \sim \eta_{\mu_\Sigma}(\xi) e^{-ik_\Sigma r}/r\,. \tag{B8}$$

The partial wave expansion of χ, Eq. (B1), is then implied by the system of the coupled equations for ψ and χ.

In calculating the cross sections, we need $u_l(\Sigma, k_\Sigma; r)^{(-)*}$ which enters into $< l|j_L(qr)|l_P j_P >$. Thus we introduce

$$u_l(\Sigma, k_\Sigma; r)^{(+)} = u_l(\Sigma, k_\Sigma; r)^{(-)*}\,, \tag{B9}$$

$$w_l(\Sigma, k_\Lambda; r)^{(+)} = w_l(\Sigma, k_\Lambda; r)^{(-)*}\,. \tag{B10}$$

Functions $u_l^{(+)}$, $w_l^{(+)}$ satisfy the same system of equations as $u_l^{(-)}$, $w_l^{(-)}$, and represent the outgoing wave radial wave functions. In place of (B5) and (B6), we have for $r > R$:

$$k_\Sigma u_l(\Sigma, k_\Sigma; r)^{(+)} - J_l(k_\Sigma r) = \alpha H_l^{(1)}(k_\Sigma r)\,, \tag{B11}$$

$$k_\Lambda w_l(\Sigma, k_\Lambda; r)^{(+)} = \beta H_l^{(1)}(k_\Lambda r)\,. \tag{B12}$$

Let us introduce a simplified notation:

$$u_l(\Sigma, k_\Sigma; r)^{(+)} \equiv u_l(r),\ w_l(\Sigma, k_\Lambda; r)^{(+)} \equiv w_l(r)\,. \tag{B13}$$

With potentials (14), (16), (17), Eqs. (B2), (B3) for $r < R$ are:

$$\left\{\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \kappa_\Sigma^2\right\} u_l = v_{x\Sigma 0} w_l,\,, \tag{B14}$$

$$\left\{\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \kappa_\Lambda^2\right\} w_l = v_{x\Lambda 0} u_l\,, \tag{B15}$$

where

$$\kappa_Y^2 = k_Y^2 + (2M_Y/\hbar^2)V_{Y0}\,,\ v_{xY0} = (2M_Y/\hbar^2)V_{x0}\,. \tag{B16}$$

For the solution (u_l, w_l) regular at $r = 0$, we make the Ansatz:

$$u_l = A_\Sigma J_l(\kappa r),\ w_l = A_\Lambda J_l(\kappa r)\,. \tag{B17}$$

We insert this Ansatz into Eqs. (B14), (B15), use the equation

$$\left\{\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right\} J_l(\kappa r) = -\kappa^2 J_l(\kappa r) , \tag{B18}$$

and obtain:

$$(\kappa_\Sigma^2 - \kappa^2) A_\Sigma - v_{x\Sigma 0} A_\Lambda = 0 , \tag{B19}$$

$$-v_{x\Lambda 0} A_\Sigma + (\kappa_\Lambda^2 - \kappa^2) A_\Lambda = 0 . \tag{B20}$$

Nonvanishing solutions A_Σ, A_Λ of these two equations exist only if

$$(\kappa_\Sigma^2 - \kappa^2)(\kappa_\Lambda^2 - \kappa^2) - v_{x\Sigma 0} v_{x\Lambda 0} = 0 . \tag{B21}$$

This equation for κ^2 has two solutions:

$$\left.\begin{matrix}\kappa_1^2 \\ \kappa_2^2\end{matrix}\right\} = \frac{1}{2}\left(\kappa_\Sigma^2 + \kappa_\Lambda^2 \pm \sqrt{(\kappa_\Sigma^2 - \kappa_\Lambda^2)^2 + 4 v_{x\Sigma 0} v_{x\Lambda 0}}\right) . \tag{B22}$$

Notice that for vanishing coupling, we have $\kappa_1^2 = \kappa_\Sigma^2$ (and $A_\Lambda = 0$), and $\kappa_2^2 = \kappa_\Lambda^2$ (and $A_\Sigma = 0$).

For $\kappa^2 = \kappa_i^2$ the two equations, (B19) and (B20), are linearly dependent. If, *e.g.*, we use Eq. (B19), we obtain (for each of the two values of κ_i^2, $i = 1, 2$):

$$A_\Lambda^{(i)} = [(\kappa_\Sigma^2 - \kappa_i^2)/v_{x\Sigma 0}] A_\Sigma^{(i)} , \tag{B23}$$

and the corresponding two solutions (for $r < R$):

$$u_l^{(i)} = A_\Sigma^{(i)} J_l(\kappa_i r), \; w_l^{(i)} = A_\Sigma^{(i)} \frac{\kappa_\Sigma^2 - \kappa_i^2}{v_{x\Sigma 0}} J_l(\kappa_i r) , \tag{B24}$$

where $A_\Sigma^{(1)}$ and $A_\Sigma^{(2)}$ are undetermined so far.

The general solution (for $r < R$) of the system of linear differential equations (B14), (B15) is a linear combination of the two solutions, Eq. (B24), which we write in the form

$$u_l = \sum_{i=1}^{2} A_i J_l(\kappa_i r) , \tag{B25}$$

$$w_l = \sum_{i=1}^{2} A_i \frac{\kappa_\Sigma^2 - \kappa_i^2}{v_{x\Sigma 0}} J_l(\kappa_i r) . \tag{B26}$$

The two arbitrary constants A_1 and A_2 are determined by the requirement that the functions $u_l(r < R), w_l(r < R)$, Eqs. (B25), (B26), join smoothly at $r = R$ the functions $u_l(r > R), w_l(r > R)$, Eqs. (B11), (B12):

$$u_l(R - \varepsilon) = u_l(R + \varepsilon)\,, \tag{B27}$$

$$\frac{d}{dr}u_l(R - \varepsilon) = \frac{d}{dr}u_l(R + \varepsilon) - v_1 u_l(R + \varepsilon)\,, \tag{B28}$$

$$w_l(R - \varepsilon) = w_l(R + \varepsilon)\,, \tag{B29}$$

$$\frac{d}{dr}w_l(R - \varepsilon) = \frac{d}{dr}w_l(R + \varepsilon)\,, \tag{B30}$$

where $\varepsilon \to 0$. The extra term $v_1 u_l$ in Eq. (B26) (with $v_1 = (2M_\Sigma/\hbar^2)V_1$) represents the change in the slope of u_l at $r = R$ produced by the surface delta bump in V_Σ, Eq. (14).

From the four equations (B27)–(B30) one determines A_1, A_2, and also the constants α, β which appear in $u_l(r > R)$, $w_l(r > R)$, Eqs. (B11), (B12). Solving the four equations for A_1, A_2, α, β is elementary and straightforward, and we do not feel that it would be justified here to write down the relatively lengthy expressions for A_1, A_2, α, β.

The procedure in the case when Λ is emitted is similar. The only difference is that now instead of Eqs. (B11), (B12), we have

$$k_\Sigma u_l(\Lambda, k_\Sigma; r)^{(+)} = \tilde{\alpha} H_l^{(1)}(k_\Sigma r)\,, \tag{B31}$$

$$k_\Lambda w_l(\Lambda, k_\Lambda; r)^{(+)} - J_l(k_\Lambda r) = \tilde{\beta} H_l^{(1)}(k_\Lambda r)\,. \tag{B32}$$

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